SHAPE GENERATION OF STRESS OPTIMIZED SUSPENDED TANKS

A case for the use of suspended container of uniform strength has been made out in the previous chapter. In this chapter, it is proposed to present the method of solution of the governing differential equation required to generate the shape of stress optimized (fully or partly) suspended containers and to suggest design recommendations.

2.1 MEMBRANE EQUATION FOR TENSION TANK

A surface of rotation is chosen for the liquid container and this surface is obtained by rotating a plane curve about an axis lying in the plane of the curve. This curve is called the meridian of the surface and the intersection of this surface with any plane to which the axis of rotation is normal is called a parallel. For such a surface, the lines of principal curvature are along the meridian and the parallel, respectively (Figure 2.1).

Any point 'A' on the surface of rotation is fixed by the 'z' and 'r' system. The outward normal to the surface is called the 'n' direction. The notations used in the derivation are explained below with reference to Figure 2.1.

\[ z, r \] : Co-ordinates of any point A on the meridian with reference to the \((z, r)\) system.

\[ \phi \] : Angle between \(z\)-axis and normal to the surface at any point A on the surface.
FIGURE 2.1 PRINCIPAL CURVATURES
2.1.1 External load

The self weight of flexible membrane is usually small and the membrane forces introduced by virtue of this, are therefore ignored. Liquid pressure which is the predominant force on the membrane is acting normal to its surface. No other external load is assumed to act on the membrane.

2.1.2 Internal forces

The internal forces are analysed by considering the equilibrium of an element bounded by two meridians and two parallels close to each other as shown in Figure 2.2.

Since the container wall is a flexible membrane, its bending resistance is negligible and it cannot resist compressive forces. Since the container surface and the liquid loads are symmetric with reference to z-axis, the shear forces along the meridian and parallel are zero. Liquid load is resisted by developing only tension in the membrane which forms the internal forces.

2.1.3 Equations of equilibrium

The free body diagram for the elemental surface ABB₁A₁ is shown in Figure 2.2(b). From Figure 2.2(a) AB is the arc length along meridian equal to \( R_\phi \, d\phi \) and AA₁ is the arc...
FIGURE 2.2 ELEMENTAL AREA ON THE SURFACE
length along parallel equal to $R \, d\phi$. Area of element is $(R_\phi \, d\phi) \times (R_\theta \, d\theta)$. $p$ is the liquid pressure acting on the element. Let $P_\theta$ be the total resistant force acting on AB and $A_1B_1$, $P_\phi$ be the total resistant force acting $AA_1$ and $BB_1$ and $p$ be the total force due to liquid pressure acting on the elemental area $ABB_1A_1$. Resolving the force acting on the element normal to the surface (Figure 2.3).

\[
2 \frac{P_\phi}{2} \sin \frac{d\phi}{2} + 2 \frac{P_\theta}{2} \sin \frac{d\theta}{2} = P \tag{2.1}
\]

For small angles, Equation (2.1) is written as

\[
2 \frac{P_\phi}{2} - \frac{d\phi}{2} + 2 \frac{P_\theta}{2} - \frac{d\theta}{2} = P \tag{2.2}
\]

which is

\[
P_\phi \, d\phi + P_\theta \, d\theta = P \tag{2.3}
\]

Let $T_\phi$ be the meridional tension per unit length acting on faces $AA_1$ and $BB_1$ and $T_\theta$ be the hoop tension per unit length acting on AB and $A_1B_1$. Therefore,

\[
P_\phi = T_\phi \times AA_1 = T_\phi \times R_\theta \, d\theta
\]

\[
P_\theta = T_\theta \times AB = T_\theta \times R_\phi \, d\phi
\]

\[
P = p \times AB \times AA_1 = p \times R_\phi \, d\phi \times R_\theta \, d\theta
\]
TANGENT TO PARALLEL

TANGENT TO MERIDIAN

FIGURE 2.3 EQUILIBRIUM OF ELEMENTAL AREA
Substituting in Equation (2.3)

\[ T_\phi R_\theta (d\theta d\phi) + T_\theta R_\phi (d\theta d\phi) = p(R_\theta d\phi) (R_\theta d\theta) \]

Simplifying, the familiar membrane equation is obtained as below:

\[ \frac{T_\phi}{R_\phi} + \frac{T_\theta}{R_\theta} = p \] (2.4)

Another equation for \( T_\phi \) can be obtained by considering the equilibrium of the portion of the container below the parallel through the point A under consideration. Let \( P \) be the pressure intensity at this level and \( W \) be the weight of liquid below this level, as shown in Figure 2.4. In the case of the fully suspended tank of Figure 2.4(a) vertical equilibrium requires

\[ 2\pi T_\phi \sin \phi = W + \frac{1}{4}r^2 p \]

Therefore,

\[ T_\phi = \frac{W + \frac{1}{4}r^2 p}{2\pi r \sin \phi} \] (2.5)

In the case of partly suspended tank of Figure 2.4(b) the support reaction is vertically upwards and equal to the downward liquid pressure there. Again, vertical equilibrium requires

\[ 2\pi r T_\phi \sin \phi = W + \frac{1}{4}r^2 p - \frac{1}{4}r_0^2 \omega H \]

FIGURE 2.4 EQUILIBRIUM OF CONTAINER BELOW A GIVEN PARALLEL
Therefore

\[ W + \frac{\pi r^2 p - \pi r^2 WH}{2 \pi r \sin \phi} = T_\phi \] (2.6)

Equations (2.4) and (2.5) or (2.6) are sufficient to solve the unknown internal tensions \( T_\phi \) or \( T_\theta \) for a known profile of the tank. It is to be emphasized that the above equations are operative for that part of the tank which is filled with liquid. For the membrane solution to be valid everywhere the supports must have freedom of radial movement so that the assumed pattern of stress distribution exists.

2.2 SHAPE OF A TENSION TANK

If the tank shape is fully defined and the mode of support or suspension specified, it is simple to determine the unit forces \( T_\phi \) and \( T_\theta \) using the master equations derived in the previous section. However, the converse process of deriving the tank shape for a given distribution of membrane forces is by no means straightforward and is dealt with in the next section.

2.2.1 General

The meridional and hoop forces in a tension tank depend on the radii of curvature of the given shape as seen in Equation (2.4). It should be noted from the Equations (2.5) and (2.6) that in a tension tank of any shape, \( T_\phi \) is always positive. The radius of curvature \( R_\theta \) is also always positive as it is the length of normal from the point to the axis of rotation. However, the radius of curvature \( R_\phi \) can take positive or negative values. If a tank shape with only
positive values for $R_\phi$ is chosen for the meridian, the hoop force varies from tension at the bottom to compression at the top. This is so because $p = 0$ at the water surface and Equation (2.4) now becomes

$$\frac{T_\phi}{R_\phi} + \frac{T_\theta}{R_\theta} = 0$$

Since $T_\phi$, and $R_\theta$ are positive and if $R_\phi$ is positive, $T_\phi$ has to be negative, which denotes a compressive force that the membrane is incapable of resisting. Therefore, the sign for $R_\phi$ has to be negative to keep $T_\phi$ positive at the top of a fully loaded tank. In the investigation that follows, the lower region for the tank shape is always started with a positive value for $R_\phi$. Thus the shapes generated have a positive $R_\phi$ at the bottom reaches that changes to negative values in the top portion.

2.2.2 Stress optimized shape

As discussed above, with the sign for $R_\phi$ varying from positive to negative from bottom to top of the tank, it is possible to maintain $T_\phi$ positive throughout, which aspect is a necessity in tension tanks. With no other condition imposed on the radii, it will be observed that the magnitude of $T_\phi$ and $T_\theta$ can be varying widely for points on the membrane at different levels. Therefore, if the maximum stress is made to correspond to the limiting permissible stress, the stresses elsewhere can be much less, signalling lack of economy.

In order to economise on the material, it is necessary to make $T_\phi$ and $T_\theta$ equal to each other at any point
and the same value made to prevail over the entire tank wall. This is a case of stress optimization and the equation for the shape of such a tank will have to be evolved on this basis.

2.3 SHAPE GENERATION FOR A STRESS-OPTIMIZED TANK

2.3.1 Equations governing the shape

In a stress-optimized tank, \( T_{\phi} = T_{\theta} = T \), a constant, that depends on the strength and thickness of material used. Equation (2.4) now becomes

\[
\frac{1}{R_{\phi}} + \frac{1}{R_{\theta}} = \frac{p}{T} \quad (2.8)
\]

For an analytical solution of this equation, it is modified as follows using the geometrical properties of the short arc length \( AB = dS_A \) of a meridian shown in Figure 2.5. Using the relations \( R_{\phi} d\phi_A = dS_A, r_A = R_{\theta} \sin \phi_A \) and \( P = w(H - z_A) \), Equation (2.8) is written as

\[
\frac{d\phi_A}{dS_A} \sin \phi_A + \frac{w(H - z_A)}{r_A} = \frac{1}{T} \quad (2.9)
\]

This is a nonlinear equation and a numerical solution is presented below. The immediate object of the exercise is to find the co-ordinates of point \( B(z_B, r_B) \) and the slope of the meridian at \( B(\phi_B) \) if these three values for the adjacent point \( A \) are known.
FIGURE 2.5 GEOMETRY OF ARC AB ON MERIDIAN
For the determination of the co-ordinates of the point B, the triangle ABD in Figure 2.5 is considered. AD = \(dz_A\), DB = \(dr_A\) and the slope of the chord AB = \(\phi_{AB}\). From triangle ABD,

\[
\text{Chord length } AB = \frac{DB}{\cos \phi_{AB}} = \frac{dr_A}{\cos \phi_{AB}}
\]

Since the arc length AB can be approximated to chord length AB,

\[
\text{Arc length } dS_A = \frac{dr_A}{\cos \phi_{AB}}
\]

Substituting this value of \(dS_A\) in Equation (2.9),

\[
\frac{\cos \phi_{AB} d \phi_A}{dr_A} + \frac{\sin \phi_A}{r_A} = \frac{w(H - z_A)}{T} \tag{2.10}
\]

is obtained. Also, from triangle ABD,

\[
\frac{AD}{BD} = \tan \phi
\]

Substituting the values of AD and DB,

\[
\frac{dz_A}{dr_A} = \tan \phi \tag{2.11}
\]
Moreover, it will be seen from the same Figure 2.5 that

\[ \phi_{AB} = \phi_A + \frac{d \phi_A}{2} \]  

Equations (2.10), (2.11) and (2.12) are sufficient to solve for the co-ordinates of the point B, and the slope at B as explained below.

### 2.3.2 Solution of equations

Taking z as the independent variable throughout, and assuming equal intervals of dz\(_A\), the corresponding values of dr\(_A\) and d\(\phi\)_A are now determined as follows. Since \(\phi_{AB}\) is as yet an unknown quantity, its initial value is assumed to be equal to \(\phi_A\) and dr\(_A\) is calculated from Equation (2.11). Substituting in Equation (2.10), d\(\phi\)_A is obtained. The second value for \(\phi_{AB}\) is obtained from Equation (2.12). Achieving higher degrees of accuracy for \(\phi_{AB}\) involves a process of iteration which is easily performed in computer, the iteration being terminated when the prescribed degree of accuracy is reached. The co-ordinates of the point B are calculated as follows:

\[
\begin{align*}
    z_B &= z_A + dz_A \\
    r_B &= r_A + dr_A \\
    \phi_B &= \phi_A + d\phi_A 
\end{align*}
\]

Thus, the shape of the required structure is obtained by proceeding from one point to the next adjacent point till the tank top is reached. The solution is started in the computer
by assuming the initial conditions discussed in section 2.5. But the next sub-section is devoted to explain a nondimensional approach to develop the stress optimized shape.

2.3.3 Nondimensional parameters

In order to facilitate designing a tank for a given capacity, certain nondimensional parameters are worked out as follows:

Taking \( z_A = \gamma_A H \) and \( r_A = \beta_A H \)

\[
dz_A = \frac{\alpha_A}{\gamma_A} H \quad \text{and} \quad dr_A = \frac{\beta_A}{\beta} H
\]

Substituting these values in Equation (2.10),

\[
\cos \phi_{AB} \frac{d \phi_A}{\beta_A} + \frac{\sin \phi_A}{\beta_A} \frac{wH^2}{T} = \frac{wH^2}{T} (1 - \alpha_A)
\]

Calling the nondimensional parameter \( \frac{wH^2}{T} \)

as the shape constant \( C \), Equation (2.10) becomes

\[
\cos \phi_{AB} \frac{d \phi_A}{\beta_A} + \frac{\sin \phi_A}{\beta_A} = C(1 - \alpha_A) \quad (2.14)
\]
Also, Equation (2.11) becomes

\[
\tan \phi_{AB} \quad \frac{dz_A}{dr_A} \quad \text{becomes} \quad \frac{d^2 A}{d^2 A}
\]

Therefore \( \tan \phi_{AB} = \frac{d^2 A}{d^2 A} \) (2.15)

Assuming the shape constant \( C \) and taking a known incremental value for \( d^2 A \), the values of \( d^2 A \) and \( d^A \) can be solved using the three Equations (2.12), (2.14) and (2.15). This enables the shape of the meridian to be generated following the approach outlined in section 2.3. The tank obtained by revolving the meridian will have a unique shape having fixed values for the volume, \( v \) and surface area \( s \). Thus parameters \( C, v \) and \( s \) are interconnected. Also the height of the tank will be unity since \( a \) becomes 1.0 when \( z \) reaches the value \( H \). Such a tank may be called a nondimensional tank and is designated by its volume \( v \).

The value of \( v \) and \( s \) are obtained as \( \Sigma \pi b^2 \Delta a \) and \( \Sigma 2 \pi b \Delta s \) respectively. If \( V \) and \( S \) are the volume and surface area of the actual tank respectively the relationships between these \( v \) and \( s \) are derived as follows:

\[
V = \Sigma \pi r^2 \Delta z
\]

\[
= \Sigma \pi (bH)^2 \quad (H \Delta a)
\]

\[
= H^3 \Sigma \pi b^2 \Delta a
\]
Therefore

\[ V = vH^3 \quad (2.16) \]

and

\[ S = \pi \sum r \Delta S = \pi \sum (8H)(H \Delta s) = H^2 \pi \sum 8 \Delta s \]

Therefore

\[ S = sH^2 \quad (2.17) \]

It is to be noted that if a particular nondimensional shape designated by its volume \( v \) is chosen the actual tank shape obtained from this will be geometrically similar to it.

For a given volume \( V \) and the chosen \( v \), the height of tank is obtained from the Equation (2.16). To find the tank wall thickness \( t \), the \( C \) value corresponding to the chosen nondimensional shape is used, as below:

\[ \frac{wh^2}{C} \]

\[ C = \text{------} \quad \text{in which} \quad T \]

\[ T = \sigma t, \quad \sigma \text{ being the allowable tensile stress in the material.} \quad \text{Therefore} \]

\[ \frac{wh^2}{C \sigma} \quad (2.18) \]
2.3.4 Initial Condition

The type of tank generated depends upon the initial values of $\beta$ at $a = 0$. In a fully suspended tank, $\beta = 0$ and $\phi = 0$ at $a = 0$. Because of symmetry at the origin, $R_\phi = R_\beta$ and Equation (2.8) becomes

$$\frac{2 wH^2}{R_\phi T}$$

Taking $R_\phi = r_\phi H$, where $r_\phi$ is the radius of curvature of the nondimensional tank shape at $a = 0$,

$$\frac{2 wH^2}{r_\phi T} = C$$

Therefore

$$r_\phi = \frac{2}{C}$$ (2.20)

Knowing $r_\phi$ from Equation (2.20) and assuming the first incremental value $d\alpha$, the corresponding incremental value for $\beta$ and $\phi$, namely, $d\beta$ and $d\phi$ can be determined using the geometry of a circle with intersecting chords on the assumption that the tank shape up to this point is spherical. Thus

$$(d\beta)^2 = (2r_\phi - d\alpha) d\alpha$$

Ignoring terms of trivial value, $d\beta = \sqrt{(2r_\phi - d\alpha)}$ and
\[
\frac{d\beta}{d\phi} = \frac{r_{\beta}}{r_{\phi}}
\]

In a partly suspended and partly supported tank, \( \beta_0 \) is chosen and \( \phi \) is equal to zero upto and inclusive of \( \beta = \beta_0 \). From Equation (2.10),

\[
\cos \phi_{AB} \frac{d\phi}{dA} \frac{wH}{dr} = \frac{wH}{T} \]

Since the left hand side of equation is equal to \( \frac{1}{R_{\phi}} \) of Equation (2.8)

\[
\frac{1}{R_{\phi}} = \frac{wH}{T} \tag{2.21}
\]

As before, taking \( R_{\phi} = r_{\phi} H \),

\[
\frac{1}{r_{\phi}} = \frac{wH^2}{T} \quad \text{which is equal to} \quad C
\]

Therefore

\[
r_{\phi} = \frac{1}{C} \tag{2.22}
\]

Alternatively the above results could be obtained as follows:

In Figure 2.4(b), at the starting point for the meridian at \( z = 0 \), \( r = r_0 \) and \( R_{\beta} \) is equal to infinity since
the normal is perpendicular to the base. Substituting this in Equation (2.8)

\[
\frac{1}{R_{\phi}} = \frac{p}{T} = \frac{wH}{T}
\]

Expressing in nondimensional form,

\[
\frac{1}{r_{\phi}} = \frac{WH^2}{T} = C
\]

Therefore

\[
r_{\phi} = \frac{1}{C}
\]

2.4 DESIGN CONSIDERATIONS - FULLY SUSPENDED TANK

This section deals with some of the major design considerations that govern the shape and economy of the tank. The treatment is made separately for fully suspended and partly suspended tanks and the design procedure is explained in each case by an illustrative example.

2.4.1 Economic consideration

In order to economize the use of material for the tank construction, the ratio \( p \) of the tank volume \( V \) to the volume of material \( V_m \) is now considered. Since \( V_m = surface area of tank S \times thickness t \).
\[
\rho = \frac{V}{V_m} = \frac{V}{St} \tag{2.23}
\]

Substituting for \(V\), \(S\) and \(t\) from Equations (2.16), (2.17) and (2.18) respectively,

\[
\rho = \frac{(vH^3)C\sigma}{(sH)^2 wH^2} = \frac{vC\sigma}{swH} \tag{2.24}
\]

Substituting for \(H\) from Equation (2.16)

\[
\rho = \left(\frac{vC\sigma}{sw}\right)^{1/3} = \left(\frac{v^{4/3}C}{s}\right)^{1/3} \tag{2.25}
\]

Therefore

\[
\rho = \eta \frac{\sigma}{wv^{1/3}} \tag{2.26}
\]

where \(\eta\) is another nondimensional parameter equal to \(\frac{v^{4/3}C}{s}\)

For a given material defined by \(\sigma\), liquid \(w\) and a tank volume \(V\), the shape requiring minimum volume of material is obtained when \(\rho\) reaches a maximum value. From Equation (2.24) \(\rho\) is a maximum when \(\eta\) is a maximum, since \(\sigma\), \(w\) and \(V\) are fixed already. Since \(\eta\) is a function of \(v\), \(s\) and \(C\), \(\eta\) can be related to any one of these three quantities. As \(v\) is found to be a convenient design parameter this is chosen to relate it. Any one point in \((v - \eta)\) graph of Figure 2.6 is
FIGURE 2.6 VARIATION OF \( \eta \) WITH \( v \)
obtained by assuming a certain value for $C$, working out the corresponding values for $v$ and $s$ and finding $n$ as

$$\frac{v^{4/3} C}{s} \quad .$$

Using different $C$ values, other points in the graph are obtained. It is seen from this graph that the value of $n$ is a maximum when $v$ is about 1.5. Also the deviation in $n$ value from the maximum is seen to be negligible from $v = 1.0$ to 2.0. Since the higher values for $v$ result in flatter tank shape as seen in Figure 2.7, from the point of view of economy alone, a value of $v = 1.0$ is recommended.

The coordinates of points defining the tank shape namely, $\alpha$ and $\beta$ are calculated using Equations (2.12), (2.14) and (2.15) through a computer programme explained in subsection 2.3.2. These values of $\alpha$ and $\beta$ are tabulated for each set of $C$ and $v$ values in Table 2.1. The corresponding curves are shown plotted in Figure 2.8 in which the curve representing the recommended value of $v = 1$ is also included. Other curves shown there will enable the designer to have his choice varied to suit his sense of aesthetics or any other practical consideration.

2.4.2 Design Example

Table 2.2(a) gives the tank parameters/dimensions for the same tank capacity and tank shape using two different materials. The permissible stress $\sigma = 36$ Kg/cm$^2$ could stand for plastics and $\sigma = 1000$ Kg/cm$^2$ for steel sheets such as stainless steel. Item(i) is for a volume of 10 m$^3$. The thickness of the tank wall works out to 4.5 mm for $\sigma = 36$.
FIGURE 2.7 CHOICE OF SHAPE

V = 480 litres
\( \sigma = 44.4 \text{ kg/cm} \)
SHAPE CONSTANT \( V = 0.25 \)

V = 510 litres
\( \sigma = 34.4 \text{ kg/cm} \)
SHAPE CONSTANT \( V = 1.00 \)
## TABLE 2.1 PARAMETERS $\alpha$ AND $\beta$ (FULLY SUSPENDED)

<table>
<thead>
<tr>
<th>$v$ correspond to ponding</th>
<th>$\beta$ Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11)</td>
</tr>
<tr>
<td>0.00</td>
<td>0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000</td>
</tr>
<tr>
<td>0.10</td>
<td>0.213 0.221 0.235 0.248 0.259 0.280 0.299 0.321 0.341 0.378</td>
</tr>
<tr>
<td>0.20</td>
<td>0.272 0.285 0.308 0.327 0.345 0.377 0.406 0.439 0.470 0.524</td>
</tr>
<tr>
<td>0.30</td>
<td>0.298 0.316 0.347 0.373 0.397 0.440 0.478 0.521 0.560 0.631</td>
</tr>
<tr>
<td>0.40</td>
<td>0.304 0.327 0.367 0.401 0.431 0.484 0.532 0.584 0.632 0.717</td>
</tr>
<tr>
<td>0.50</td>
<td>0.297 0.326 0.376 0.417 0.454 0.518 0.574 0.636 0.693 0.793</td>
</tr>
<tr>
<td>0.60</td>
<td>0.286 0.321 0.379 0.428 0.471 0.546 0.611 0.682 0.748 0.862</td>
</tr>
<tr>
<td>0.70</td>
<td>0.280 0.318 0.383 0.439 0.488 0.572 0.647 0.727 0.801 0.929</td>
</tr>
<tr>
<td>0.80</td>
<td>0.287 0.326 0.395 0.455 0.509 0.602 0.685 0.774 0.856 0.997</td>
</tr>
<tr>
<td>0.90</td>
<td>0.314 0.350 0.418 0.481 0.538 0.639 0.729 0.826 0.916 1.071</td>
</tr>
<tr>
<td>1.00</td>
<td>0.368 0.397 0.461 0.524 0.583 0.689 0.785 0.889 0.986 1.154</td>
</tr>
</tbody>
</table>
FIGURE 2.8 NONDIMENSIONAL TANK PARAMETERS (FULLY SUSPENDED)
TABLE 2.2 DESIGN EXAMPLE

(a) Tank capacity and shape are same. Materials are different.

(i) \( V = 10 \text{ m}^3 \); \( v = 0.25 \)

<table>
<thead>
<tr>
<th>Parameter/dimension</th>
<th>( \sigma = 36 \text{ kg/cm}^2 ) (plastics)</th>
<th>( \sigma = 1000 \text{ kg/cm}^2 ) (Stainless steel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of tank wall</td>
<td>4.50 mm</td>
<td>0.157 mm</td>
</tr>
</tbody>
</table>

(In both cases, tank height = 3.42 m and top radius = 1.26)

(ii) \( V = 100 \text{ m}^3 \); \( v = 1.00 \)

<table>
<thead>
<tr>
<th>Parameter/dimension</th>
<th>( \sigma = 36 \text{ kg/cm}^2 ) (plastics)</th>
<th>( \sigma = 1000 \text{ kg/cm}^2 ) (Stainless steel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of tank wall</td>
<td>15 mm</td>
<td>0.525 mm</td>
</tr>
</tbody>
</table>

(In both cases, tank height = 4.65 m and top radius = 3.675 m)

(b) Tank capacity and material same. Shapes are different.

\( V = 10 \text{ m}^3 \); \( \sigma = 36 \text{Kg/cm}^2 \)

<table>
<thead>
<tr>
<th>Parameters/dimensions</th>
<th>( v = 0.25 )</th>
<th>( v = 1.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>3.42</td>
<td>2.16</td>
</tr>
<tr>
<td>Top Radius (m)</td>
<td>1.26</td>
<td>1.70</td>
</tr>
<tr>
<td>Surface area (m(^2))</td>
<td>22.20</td>
<td>18.73</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>4.5</td>
<td>3.2</td>
</tr>
<tr>
<td>Solid Volume (m(^3))</td>
<td>0.1</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Kg/cm² and 0.157 mm for σ = 1000 Kg/cm². The same calculations are repeated in item(ii) for a tank capacity of 100 m³ which, though realistic for steel tank, is on the higher side in respect of plastic tank. The thickness now are t = 15 mm for σ = 36 Kg/cm² and t = 0.525 mm for σ = 1000 Kg/sq.cm. For a plastics tank of 100 cubic meter, This thickness of 15 mm compares very favourably with the usual thickness of about 16 mm for 10m³ capacity HDPE plastics water tanks supported on flat base.

In order to study the effect of large variations in v on the economy of the tank considering only the solid volume of material consumed, Table 2.2(b) is prepared for a volume of 10 m³ and permissible stress of 36 Kg/cm² but for two different shapes defined by v = 0.25 and v = 1.00. From the results it is seen that there is wide variation of about 67 per cent in the solid volume pointing to the necessity of sticking to a value closer to v = 1.00, unless some extraneous consideration other than economy dictates.

2.5 DESIGN CONSIDERATION - PARTLY SUSPENDED TANK

A tank which is partly resting on a flat base and partly suspended from a ring beam as shown in Figure 2.4 is considered now. Prescribing values for the bottom radius B₀ and shape contact C, the shape of nondimensional tank can be generated using Equations (2.12), (2.14) and (2.15). The solution can be started by assuming the first portion of the curve AB (B close to A) to be an arc of a circle whose radius is obtained from the Equation (2.22).
2.5.1 Economic considerations

Since every value of $\delta_0$ results in a family of curves for different values of $C$, it becomes necessary to make a design recommendation regarding the choice of a suitable non-dimensional tank shape designated by its volume $v$ for the chosen $\delta_0$. This is attempted by considering the factor $\rho$ influencing the solid volume of tank material, as for fully suspended one. As before

$$\rho = \frac{v}{V_m}$$

and

$$V_m = Sxt$$

The surface area of partly suspended tank is written as

$$S = \pi r^2 + \sum 2\pi r \Delta S$$

Expressing $r_0$, $r$ and $\Delta S$ in terms the tank height $H$,

$$S = \pi (\delta_0 H)^2 + \sum 2\pi (\delta H) (H \Delta s)$$

$$= H^2 (\pi \delta_0 \sum 2\pi \delta \Delta s)$$

$$= H^2 (s_0 + s)$$

where $s_0$ is the surface area of the base and $s$ is the surface area of the suspended portion of the nondimensional tank shape.

Therefore

$$\rho = \frac{v}{V_m} = \frac{vH^3}{H^2(s_0 + s)t} = \frac{v}{(s_0 + s)t}$$
Substituting for $t$ from the Equation (2.18)

\[
\rho = \frac{vH}{(s_0 + s) \cdot \frac{C \sigma}{wH^2}} \cdot \frac{v}{s_n + s} \cdot \frac{wH}{c_a} \cdot \frac{1}{2}
\]

Substituting for $H$ from the Equation (2.16),

\[
\rho = \frac{v}{(s_0 + s) \cdot \frac{C \sigma}{w} \cdot \frac{v^{1/3}}{\sqrt[3]{\frac{v}{3}}}} = \frac{\frac{v^{4/3} \cdot C}{\frac{\sigma}{w}}}{(s_0 + s) \cdot \frac{w}{\sqrt[3]{\frac{v}{3}}}}
\]

Therefore

\[
\rho = \eta_0 \frac{\sigma}{w v^{1/3}} \quad (2.25)
\]

where $\eta_0$ is defined by

\[
\eta_0 = \frac{v^{4/3} \cdot C}{\frac{\sigma}{w}} \quad (s_0 + s)
\]

For a given $\sigma$, $w$, and $V$, $\rho$ is maximum when $\eta_0$ is maximum. For an assumed value of $\beta_0$ and $C$, the values of $v$, $s$ and $s_0$ are uniquely determined. As for a suspended tank it is proposed to relate $v$ and $\eta_0$ to obtain an economic shape.

For any value of $\beta_0$, $C$ values are varied and corresponding $\eta_0$ values are computed. A graph is now drawn connecting $v$ and $\eta_0$ for the chosen $\beta_0$ and the value of $v$
giving maximum value for $\eta_0$ is obtained. This is repeated for different values of $\beta_0$ and $(\eta_0 - \nu)$ curves are shown plotted in Figure 2.9. It is seen from the Figure that the values of $(\eta_0)_{\text{max}}$ increase with the values of $\beta_0$ in the range considered in this Figure. Two widely different values for $\eta_0$, namely $\beta_0 = 0.25$ and $\beta_0 = 0.70$ are chosen, the corresponding $\nu$ values that make $\eta_0$ maximum found out from the Figure 2.9 and the actual tank shape for a total volume of $V = 100 \text{ m}^3$ are shown in Figure 2.10. The tank shapes corresponding to the $(\eta_0)_{\text{max}}$ values of various $\beta_0$ values are given in Figure 2.11 and the coordinate values given in Table 2.3.

2.6 COMPUTER PROGRAMME

It has been mentioned in section 2.3 that the shape generation involves an iterative procedure which is easily done by a simple computer programme. Logic involved in the computer programme, flow chart and the actual iterative procedure are explained in this section.

2.6.1 Flow Chart

The flow chart given in this sub section is based upon the procedure explained earlier in sub section 2.3.2 for generating non-dimensional tank shapes for different values of $\beta_0$ and $C$ and shown in Figure 2.12.

2.6.2 Iterative procedure for shape generation

The shape generation involves a process of iteration which is explained now in detail. The Equations (2.10), (2.11) and (2.12) are needed in this computation. The object is now to find the co-ordinates of the point $B(z_B, r_B)$ from
FIGURE 2.9 VARIATION OF $\eta_0$ WITH $v$ AND $R_0$
\begin{align*}
\beta_0 &= 0.25 \\
C &= 3.51 \\
v &= 1.6 \\
\sigma &= 1000 \text{ kg/cm}^2 \\
t &= 0.46 \text{ mm} \\
\end{align*}

\begin{align*}
\beta_0 &= 0.50 \\
C &= 3.82 \\
v &= 2.2 \\
\sigma &= 1000 \text{ kg/cm}^2 \\
t &= 0.34 \text{ mm} \\
\end{align*}

\begin{align*}
\beta_0 &= 0.70 \\
C &= 3.76 \\
v &= 3.2 \\
\sigma &= 1000 \text{ kg/cm}^2 \\
t &= 0.28 \text{ mm} \\
\end{align*}

\text{FIGURE 2.10 DESIGN OF PARTLY SUSPENDED TANK}
FIGURE 2.11 NONDIMENSIONAL TANK PARAMETERS (PARTLY SUSPENDED)
<table>
<thead>
<tr>
<th>v corresponding to C</th>
<th>1.70</th>
<th>1.90</th>
<th>1.60</th>
<th>1.70</th>
<th>1.80</th>
<th>2.00</th>
<th>2.00</th>
<th>2.20</th>
<th>2.40</th>
<th>2.50</th>
<th>2.70</th>
<th>3.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12) (13)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a )</td>
<td>0.100</td>
<td>0.200</td>
<td>0.250</td>
<td>0.300</td>
<td>0.350</td>
<td>0.400</td>
<td>0.450</td>
<td>0.500</td>
<td>0.550</td>
<td>0.600</td>
<td>0.650</td>
<td>0.700</td>
</tr>
<tr>
<td>0.1</td>
<td>0.385</td>
<td>0.471</td>
<td>0.492</td>
<td>0.537</td>
<td>0.582</td>
<td>0.632</td>
<td>0.672</td>
<td>0.721</td>
<td>0.769</td>
<td>0.816</td>
<td>0.861</td>
<td>0.920</td>
</tr>
<tr>
<td>0.2</td>
<td>0.510</td>
<td>0.585</td>
<td>0.627</td>
<td>0.667</td>
<td>0.716</td>
<td>0.747</td>
<td>0.795</td>
<td>0.842</td>
<td>0.881</td>
<td>0.927</td>
<td>0.991</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.601</td>
<td>0.666</td>
<td>0.650</td>
<td>0.685</td>
<td>0.721</td>
<td>0.768</td>
<td>0.792</td>
<td>0.838</td>
<td>0.882</td>
<td>0.917</td>
<td>0.961</td>
<td>1.030</td>
</tr>
<tr>
<td>0.4</td>
<td>0.674</td>
<td>0.731</td>
<td>0.696</td>
<td>0.727</td>
<td>0.758</td>
<td>0.803</td>
<td>0.818</td>
<td>0.863</td>
<td>0.904</td>
<td>0.933</td>
<td>0.975</td>
<td>1.049</td>
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<tr>
<td>0.5</td>
<td>0.737</td>
<td>0.785</td>
<td>0.733</td>
<td>0.758</td>
<td>0.784</td>
<td>0.827</td>
<td>0.833</td>
<td>0.875</td>
<td>0.913</td>
<td>0.936</td>
<td>0.974</td>
<td>1.054</td>
</tr>
<tr>
<td>0.6</td>
<td>0.794</td>
<td>0.834</td>
<td>0.764</td>
<td>0.784</td>
<td>0.803</td>
<td>0.844</td>
<td>0.840</td>
<td>0.879</td>
<td>0.914</td>
<td>0.929</td>
<td>0.963</td>
<td>1.050</td>
</tr>
<tr>
<td>0.7</td>
<td>0.849</td>
<td>0.882</td>
<td>0.793</td>
<td>0.807</td>
<td>0.820</td>
<td>0.858</td>
<td>0.843</td>
<td>0.879</td>
<td>0.910</td>
<td>0.917</td>
<td>0.946</td>
<td>1.040</td>
</tr>
<tr>
<td>0.8</td>
<td>0.907</td>
<td>0.931</td>
<td>0.823</td>
<td>0.831</td>
<td>0.838</td>
<td>0.873</td>
<td>0.846</td>
<td>0.878</td>
<td>0.905</td>
<td>0.903</td>
<td>0.927</td>
<td>1.029</td>
</tr>
<tr>
<td>0.9</td>
<td>0.969</td>
<td>0.985</td>
<td>0.859</td>
<td>0.861</td>
<td>0.861</td>
<td>0.892</td>
<td>0.854</td>
<td>0.882</td>
<td>0.904</td>
<td>0.892</td>
<td>0.910</td>
<td>1.019</td>
</tr>
<tr>
<td>1.0</td>
<td>1.041</td>
<td>1.048</td>
<td>0.904</td>
<td>0.899</td>
<td>0.892</td>
<td>0.919</td>
<td>0.869</td>
<td>0.892</td>
<td>0.909</td>
<td>0.888</td>
<td>0.900</td>
<td>1.015</td>
</tr>
</tbody>
</table>
START

READ $\alpha(1), \beta(1), \varphi(1)$

$L = 1$

PRINT $\alpha(L), \beta(L), \varphi(L)$

$\text{IS } \beta(1) > 0$ ?

YES

$R = 1/C$

$L = 2$

$\Delta \beta = \sqrt{2(\Delta \alpha)(R)}$

$\Delta \varphi = \frac{\Delta \beta}{R}$

$\alpha(L) = \alpha(1) + \Delta \alpha$

$\beta(L) = \beta(1) + \Delta \beta$

$\varphi(L) = \varphi(1) + \Delta \varphi$

PRINT $\alpha(L), \beta(L), \varphi(L)$

$A_1$

NO

$R = 2/C$

$L = 2$

$\Delta \beta = \sqrt{2(\Delta \alpha)(R)}$

$\Delta \varphi = \frac{\Delta \beta}{R}$

$\alpha(L) = \alpha(1) + \Delta \alpha$

$\beta(L) = \beta(1) + \Delta \beta$

$\varphi(L) = \varphi(1) + \Delta \varphi$

PRINT $\alpha(L), \beta(L), \varphi(L)$

$A_1$
FIGURE 2.12 FLOW CHART FOR NON DIMENSIONAL SHAPE

\[ x = \varphi(K) + \frac{\Delta \varphi}{2} \]

\[ \Delta \beta = \Delta \alpha \frac{\cos x}{\sin x} \]

\[ \Delta \varphi = \frac{\Delta \beta}{\cos x} \left\{ c \left[ 1 - \alpha(K) \right] - \frac{\sin \varphi(K)}{\beta(K)} \right\} \]

\[ y = \varphi(K) + \frac{\Delta \varphi}{2} \]

**YES**

\[ \text{IS} \] \( |x - y| < \varepsilon \) \( > \) \( 0 \) ?

**NO**

\[ L = K + 1 \]

\[ \alpha(L) = \alpha(K) + \Delta \alpha \]

\[ \beta(L) = \beta(K) + \Delta \beta \]

\[ \varphi(L) = \varphi(K) + \Delta \varphi \]

PRINT \( \alpha(L), \beta(L), \varphi(L) \)

**YES**

**NO**

\[ \text{IS} \] \( \alpha \) \( ? \) \( 1 < 0 \) ?

STOP
the known position \((z_A, r_A)\) of a close neighbouring point A, the slope at A \(\phi_A\) being a known quantity. Since the slope of the chord \(\phi_{AB}\) is unknown, the initial value of \(\phi_{AB}\) is assumed to be equal to \(\phi_A\) which implies that the first approximation for \(d\phi\) is zero since \(\phi_{AB} = \phi_A + \cdots\). Using Equations (2.10) and (2.11), the values of \(dr\) and \(d\phi\) are computed for the assumed value of \(dz\) which is a convenient fraction of tank height. The position of B is located at \(C_1\) (Figure 2.13). Since \(d\phi\) is known the second approximate value of \(\phi_{AB}\) is obtained from the Equation (2.12). \(dr\) and \(d\phi\) are again computed from the Equations (2.10) and (2.11) and the second position of B is located at \(C_2\). The iteration is continued till the difference between the assumed values of \(\phi_{AB}\) and that of calculated one is within the prescribed degree of accuracy.

The number of iteration depends on the slope of the curve. Reasonable accuracy is obtained within seven cycles for flatter slopes and they are much less for steeper slopes. The details of iteration involved in the shape generation for three different shapes defined by \(C\), equal to 7.45, 5.76 and 4.12 and for two arcs defined by \(a = 0.09\) to 0.10 and \(a = 0.89\) to 0.90 for each of the three cases are given in Table 2.4.

Since the shape generation is a numerical one, the accuracy of results obtained depends on the values of \(dz\) chosen. The assumption made in the analysis is that the arc length is equal to the chord lengths. This is nearly true when the value of \(dz\) is small. When \(dz = H/100\), the error as defined in the next section is less than one per cent for suspended tank and it is less than three per cent for partly
(a) BOTTOM PORTION WITH POSITIVE CURVATURE
(b) TOP PORTION WITH NEGATIVE CURVATURE

FIGURE 2.13 ITERATION PROCEDURE IN SHAPE GENERATION
### TABLE 2.4 ITERATION INVOLVED IN SHAPE GENERATION

<table>
<thead>
<tr>
<th>c</th>
<th>v</th>
<th>d a</th>
<th>d b</th>
<th>d *</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.45</td>
<td>0.25</td>
<td>0.09</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2036</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2126</td>
</tr>
<tr>
<td>7.45</td>
<td>0.25</td>
<td>0.49</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3099</td>
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<td></td>
<td></td>
<td></td>
<td>0.3137</td>
</tr>
<tr>
<td>5.76</td>
<td>0.50</td>
<td>0.09</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2364</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2476</td>
</tr>
<tr>
<td>5.76</td>
<td>0.50</td>
<td>0.89</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.4779</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.4811</td>
</tr>
<tr>
<td>4.12</td>
<td>1.00</td>
<td>0.09</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2851</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2993</td>
</tr>
<tr>
<td>4.12</td>
<td>1.00</td>
<td>0.89</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.7242</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.7290</td>
</tr>
</tbody>
</table>
suspended tank. Hence this value is recommended for shape generation.

2.7 CHECKING ACCURACY OF COMPUTATION

Since the shape of a stress optimized tank has been generated by solving the differential equation following a numerical approach, it is desirable to check the accuracy of computation which is done by considering the equilibrium of the container. Let \( r_0 \) and \( r_t \) be radius at bottom and top of the container respectively and let \( T \) be the membrane force at the top of the container. The vertical component of \( T \) must balance the weight shared by the suspended portion of the container

\[
2\pi r_t T \sin \phi = W - \pi r_0^2 \omega H
\]

Expressing this equation in terms of the nondimensional tank parameters and taking \( u = \sin \phi \)

\[
2\pi (8_t H)T u = \omega VH^3 - \pi (8_0 H)^2 \omega H
\]

\[
= \omega H^3 (v - \pi \beta^2)
\]

Simplifying,

\[
\frac{\omega H^2}{T} = \frac{2\pi \beta}{2} u
\]

The right hand side quantities are all evaluated using an assumed value for \( C \). It must be recognised that the left hand side quantity also gives \( C \) as defined earlier. Therefore, \( C \) value assumed originally and computed now and
designated as CC can be compared to give a measure of the error involved in generating the shape. Therefore

\[ CC = \frac{2\pi \beta t u}{v - \pi \beta \delta^2} \]

\[ \text{Percent Error} = \frac{C - CC}{C} \times 100 \]

For the same value of dz obtained as a fraction of H, the per cent error varies with different tank shape defined by different C values. These are seen in Table 2.5 for three different C values and for three different dz values expressed as fraction of H. It can be concluded that when \( dz = H/100 \), the per cent error is less than three for a wide range of C values, 2 to 8 which perhaps defines the practical range.

2.8 SHAPE GENERATION BY A SEMI-GRAPHICAL METHOD

A hybrid method involving the result output of analytical work and a graphical approach to generate the shape of a tank for a given volume is developed now. This method requires for its data the height of the tank H and the safe membrane tension T permissible in the tank wall. To enable the designer to calculate H and T, recourse is taken to the following: For each value of \( \beta_0 \) the shape constant C is varied and the volume of non-dimensional tank shape v is obtained for each value of C. A graph connecting C and v is drawn and shown in Figures 2.14 and 2.15. The values of \( \beta_0 \) are varied and similar plots are obtained.
TABLE 2.5 INFLUENCE OF 'dZ' ON THE ACCURACY OF SHAPE GENERATION

<table>
<thead>
<tr>
<th>dZ as Fraction of H</th>
<th>H/25</th>
<th>H/50</th>
<th>H/100</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Assumed C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>0.23</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>θ</td>
<td>0.34</td>
<td>0.36</td>
<td>0.37</td>
</tr>
<tr>
<td>u</td>
<td>0.79</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>Computed C</td>
<td>7.47</td>
<td>7.49</td>
<td>7.47</td>
</tr>
<tr>
<td>Error (Percent)</td>
<td>0.38</td>
<td>0.56</td>
<td>0.36</td>
</tr>
<tr>
<td>b) Assumed C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>0.48</td>
<td>0.49</td>
<td>0.50</td>
</tr>
<tr>
<td>θ</td>
<td>0.50</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>u</td>
<td>0.89</td>
<td>0.89</td>
<td>0.88</td>
</tr>
<tr>
<td>Computed C</td>
<td>5.88</td>
<td>5.83</td>
<td>5.79</td>
</tr>
<tr>
<td>Error (Percent)</td>
<td>2.19</td>
<td>1.22</td>
<td>0.62</td>
</tr>
<tr>
<td>c) Assumed C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>0.97</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>θ</td>
<td>0.76</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td>u</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Computed C</td>
<td>4.23</td>
<td>4.18</td>
<td>4.15</td>
</tr>
<tr>
<td>Error (Percent)</td>
<td>2.79</td>
<td>1.43</td>
<td>0.73</td>
</tr>
</tbody>
</table>
$v = \frac{v}{H^3}$

$C = \frac{W}{H^2}$

FIGURE 2.14 VARIATION OF $v$ WITH C (FULLY SUSPENDED)
FIGURE 2.15 VARIATION OF $v$ WITH $c$ (PARTLY SUSPENDED)

$V = vH^3$

$C = \frac{wH^2}{T}$

$\beta = 0.8$

$0.7$

$0.6$

$0.5$

$0.4$

$0.3$

$0.2$

$0.1$

$0$

$1$

$2$

$3$

$4$

$5$

$6$

$7$

$0$

$1$

$2$

$3$

$4$

$5$

$6$

$7$
Now the designer is free to choose any shape defined by \( v \) and bottom radius and the corresponding value of \( C \) is obtained from the above Figures. The height of tank \( H \) and the membrane tension \( T \) are now calculated using the equations 2.16 and 2.18 which are included in the above Figure also.

### 2.8.1 Construction procedure

For convenience the Equation (2.8) is rewritten below:

\[
\frac{1}{R_\phi} + \frac{1}{R_\theta} - \frac{p}{T} = \frac{W(H - z_A)}{T}
\]

The construction is done in stages of several arcs similar to \( AB \) of Figure 2.5. From the above equation

\[
R_\phi = \frac{1}{(p/T - 1/R_\theta)} \frac{\sin \phi_A}{R_A}
\]

At \( A, p = W(H - z_A), \) and \( R_\theta = \frac{W(H - z_A)}{T} \) can be calculated.

Referring to Figure 2.5, a line \( AC \) is drawn at \( A \) making an angle \( \phi_A \) with the 'r' axis. The line \( AC \) is the tangent to the curve at \( A \). Let the normal at \( A \) meet the \( z \) axis at \( E \). \( AE \) represents \( R_\theta \). A length \( AF \) equal to \( R_\phi \) is measured from \( A \) and the point \( F \) is located. An arc of a circle \( AB \) is drawn from \( F \) and radius equal to \( AF \). \( d\phi_A \) is
the chosen increment of $\phi_A$ which can be made as small as feasible. Hence the next adjacent point B is fixed. This construction procedure is repeated till the top of the tank is reached. $R_\phi$ is taken equal to $R_\theta$ in the case of fully suspended tank and the first arc length is drawn accordingly. For a partly suspended tank, the curve is started by assuming $R_\theta$ to be equal to infinity.

**2.8.2 Snag in the construction procedure**

Since the arc AB has to be constructed by using a compass, the method fails when $R_\phi$ is large. In such cases, the curve is constructed by means of chords. The method is explained below.

Since $R_\phi$ is calculated and $d\phi$ is assumed, the arc length AB is calculated as $R_\phi d\phi$. AC is the tangent making an angle of $\phi_A$ with the horizontal. Chord line AB is drawn making an angle $\phi_A + d\phi_A/2$ and the point B is marked taking chord and arc lengths are same. The slope of curve at B is $\frac{d}{d\phi} A$.

**2.8.3 Example**

The graphical method is explained by working out an example for a volume of 450 litres. Choosing $v = 0.25$ and $\beta_0 = 0$ the corresponding $C = 7.45$, the height of the tank works out to 1.2 m and the membrane tension $T = 200$ Kg/m. The shape thus generated is shown in Figure 2.16 and $z$ and $r$ values are given in Table 2.6.
FIGURE 2.16 SHAPE GENERATION BY GRAPHICAL METHOD
TABLE 2.6 SHAPE GENERATION BY GRAPHICAL METHOD

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