ABSTRACT

The graph theory was originated in 1736 when Leonhard Euler has solved the Königsberg bridge problem. After over a century another famous problem in graph theory, the four color conjecture, was floated. Since then the graph theory has seen the development in many directions and dimensions. Now the graph theory is being applied to solve problems in many fields. As a result during the past 50 years a great deal of research has been and is being done in graph theory.

The thesis consists of six chapters. In Chapter 1, we present a literature survey along with necessary definitions and notations.

One of the interesting problems in graph theory is the study of the graph labeling techniques and their applications. Labeled graphs serve as useful models for a broad range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit designs, communication designs, data base management and models for constraint programming over finite domains. Motivated by such works we present various types of graph labelings in Chapters 2, 3, 4, and 5.

The concept of graph energy arose in chemistry where certain numerical quantities, such as the heat of formation of a hydrocarbon, are related to total \(\pi\)-electron energy which can be calculated as the energy of an appropriate molecular graph. Motivated by these we present the energy, skew energy and skew Randić energy of various graphs in Chapters 5 and 6.

In Chapter 2, we introduce the concept of strongly harmonic graphs. We then show that the complete graph \(K_n\) for \(n \geq 11\), all cycles, wheels, trees, grids, ladders, triangular ladders, fans, stars, double stars, the graphs \(K_2 + mK_1\),
cycles-cactus, triangular snakes and Mycielskian graph of the path are all strongly harmonic graphs. We will also find the upper and lower bounds for the maximum number of edges in a strongly harmonic graph of order \( n \).

In Chapter 3, we introduce the concept of square sum labeling of graphs. We then show that paths, the graph \( St(n_1, n_2, \ldots, n_k) \), the lobster, complete \( n \)-ary tree and the amalgamation of the fan with a star admit square sum labelings. Also we observe that the complete graph \( K_n \), for \( n \geq 3 \), the wheel graph \( W_n \), the fan graph \( P_{n-1} + K_1 \), the double fan graph \( P_{n-1} + \overline{K}_2 \), the friendship graph \( C_3^{(n)} \), the windmill graph \( K_m^{(n)} \), \( m > 3 \), the triangular ladder, the triangular snake graph, the double triangular snake graph, the flower graph, the graph obtained by joining all the pendant vertices of helm graph \( H_n \) with the apex vertex and the graph obtained by joining apex vertices of two wheel graphs and two apex vertices to a new vertex are not square sum graphs. Further we prove that the helm graph \( H_n \) and the graph \( (W_m : W_n) \) do not admit square sum labelings.

In Chapter 4, we introduce the concept of pentagonal sum labeling of graphs and prove the results which are analogous to the results of Chapter 3.

In Chapter 5, we show that there exists an \( (n, 2n - 3) \) strong vertex graceful graph and we find the cardinality of the set of all \( (n, 2n - 3) \) strong vertex graceful graphs. Also we find the energy, the Laplacian energy and the Laplacian-energy-like invariant of an \( (n, 2n - 3) \) strong vertex graceful graph.

In Chapter 6, we introduce skew-Randić energy of a digraph. We then find skew energy and skew-Randić energy of an \( (n, 2n - 3) \) strong vertex graceful digraph and skew-Randić energy of a complete bipartite digraph and a crown digraph.