CHAPTER 2

Hydrodynamic Lubrication of secant shaped Porous Slider

2.1 Introduction:

Problem of the hydrodynamic lubrication of non-porous inclined plane slider bearing is a classical one. Prakash and Vij analysed the same problem when the slider was non-porous and the stator had a porous facing backed by a solid wall. Cameron introduced and analysed the hydrodynamic lubrication of a non-porous secant shaped slider bearing discussing the practical advantages of such a shape. Porous bearings have become very popular because of their self lubricating properties, design simplicity and being more stable than the equivalent conventional bearings. Slider bearings are designed to support axial loads. Various applications of slider bearings include hydroelectric generators and gas turbines.

The purpose of this chapter is to discuss the lubricating characteristics of secant shaped slider bearing when the slider has a porous
Secant shaped porous slider bearing

Fig. 1 Geometry and co-ordinates
Fig. 2 Configuration of problem for various $\bar{\beta}$
facing backed by a solid wall so as to give strength to the porous surface and not to allow the lubricant to flow out of the bearing surface. The lubricant is assumed incompressible. Film thickness is assumed to be of the form

\[
h = h_0 \sec \beta x \quad (1)
\]

where \( h_0 \) is the outlet film thickness and \( \beta \) is the curvature parameter and \( \beta L < \pi/2 \). Bearing is assumed to be infinite in z-direction.

2.2 Analysis:

The geometry and coordinates of the problem is shown in Fig. 1. The effect of the curvature parameter on the film shape is presented in Fig. 2. The modified Reynolds equation for a porous bearing as derived by Rouleau\(^{(37)}\), after neglecting the side leakage effect is

\[
\frac{d}{dx} \left( h^3 \frac{dp}{dx} \right) = -6\mu U \frac{dh}{dx} + 12 \frac{\partial p^*}{\partial y} \bigg|_{y=0} \quad (2)
\]

Expressing the film thickness \( h \) in dimensionless
form as

\[ \tilde{h} = h/h_0 = \sec \tilde{\rho}_x \]  

(3)

where \( \tilde{\rho} = \rho L \) and \( X = x/L \)

Pressure \( p^* \) within the porous matrix satisfies

the Laplace equation

\[ \frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} = 0 \]  

(4)

Related boundary conditions are

\[ p = p^* \quad \text{at} \quad y = 0 \]  

(5)

\[ \frac{\partial p^*}{\partial y} = 0 \quad \text{at} \quad y = -H \]  

(6)

Expanding left hand side of (6) by Taylor series

and neglecting higher order derivatives and using

boundary condition (5) and (4) results in

\[ \frac{\partial p^*}{\partial y} \bigg|_{y=0} = -H \frac{\partial^2 p}{\partial x^2} \]  

(7)
Substituting from (7) into (2) and rearranging it to get

\[
\frac{d}{dx} \left[ (h^3 + 12 \Phi H) \frac{dp}{dx} \right] = -6 \mu U \frac{dh}{dx} \tag{8}
\]

Putting \( h \) from (1) into (8) and integrating twice with respect to \( x \) with boundary conditions \( p = 0 \) at \( x = 0 \) and \( x = L \), the dimensionless pressure distribution is given by

\[
P = \frac{ph^2}{\mu U L} = -6 \int_0^x \frac{\cos^2 \bar{x} \, dx}{1 + 12\psi \cos^3 \bar{x}}
\]

\[
+ 6 C \int_0^x \frac{\cos^3 \bar{x} \, dx}{1 + 12\psi \cos^3 \bar{x}} \tag{9}
\]

where \( \psi = \frac{\Phi H}{h_o^3} \),

\[
c = \int_0^1 \frac{\cos^2 \bar{x} \, dx}{1 + 12\psi \cos^3 \bar{x}} \tag{10}
\]
The dimensionless load carrying capacity of the slider is given by

\[ \bar{W} = \frac{Wh^2}{\mu UBL^2} = \int_0^1 P dX \]

\[ = 6 \int_0^1 \frac{X \cos^2 \beta X}{1 + 12 \Psi \cos^3 \beta X} dX \]

\[ - 60 \int_0^1 \frac{X \cos^3 \beta X}{1 + 12 \Psi \cos^3 \beta X} dX \]  \hspace{1cm} (11)

The position of centre of pressure \( \bar{X} \) is given by

\[ \bar{X} = \frac{1}{\bar{W}} \int_0^1 X P dX \]

\[ = \frac{2}{\bar{W}} \left[ \int_0^1 \frac{X^2 \cos^2 \beta X}{1 + 12 \Psi \cos^3 \beta X} dX \right] \]

\[ - 6 \int_0^1 \frac{X^2 \cos^3 \beta X}{1 + 12 \Psi \cos^3 \beta X} dX \]  \hspace{1cm} (12)
The dimensionless frictional drag $F$ exerted by the moving slider is given by

$$F = \frac{F_0}{\mu U BL} = 3 \int_{0}^{1} \frac{\cos \beta x \, dx}{1 + 12 \psi \cos^3 \beta x}$$

$$- 3 \left( \int_{0}^{1} \frac{\cos^2 \beta x \, dx}{1 + 12 \psi \cos^3 \beta x} \right) + \sin \beta x \frac{\sin \beta x}{\beta}$$

Equation (13)

The coefficient of friction $\bar{F}$ is obtained from

$$\bar{F} = \frac{F}{W}$$

Equation (14)

2.3 Results and Discussion:

Parameters affecting the performance characteristics of the bearing are $\psi$ and $\beta$. Figure 3 gives a graphical representation of dimensionless pressure $P$ against $X$ for various values of curvature parameter $\beta$ and permeability parameter $\psi$. Fig. 4 is a graphical representation of dimensionless load capacity $\bar{W}$ against $\beta$ for various values of $\psi$. 
It is seen that both $P$ and $\bar{W}$ drop off as $\psi$ increases. Also for each $\bar{P}$ there is a value of $\lambda$ at which $P$ is maximum. For each $\psi$ there is a value of $\bar{\rho}$ for which $\bar{W}$ is maximum. When $\bar{\rho}$ is kept constant and the value of the permeability parameter $\psi$ is increased, the increased flow within the porous matrix modifies the velocity gradients and causes the pressure curve to peak closer to the inlet edge (Fig. 3), which also results in the centre of pressure moving closer to inlet edge (Table 1). As $\bar{\rho}$ increases, it is observed from fig. 3 that various pressure curves cross each other and the point of intersection shifts away from the inlet edge. Rate of this shift is nonuniform. This may be due to the fact that nearer the inlet edge the film thickness increases more sharply. The case $\bar{\rho} = 0$ corresponds to parallel plate slider, for which it is well known that load capacity is zero which may also be noticed in fig. 4.

As the curvature parameter $\bar{\rho}$ increases, centre of pressure is negligibly affected by the permeability parameter $\psi$ shifting only normally towards inlet edge (Table 1). However, increasing $\bar{\rho}$ affects the centre of pressure to shift
substantially towards the outlet edge. Also for smaller values of \( \bar{\gamma} \) and larger values of \( \psi \) centre of pressure is observed to be moving towards inlet edge. This provides one more degree of freedom for pivoting the bearing.

For various \( \psi \) and \( \bar{\gamma} \) the dimensionless frictional drag \( \overline{F} \) is tabulated against \( \bar{\gamma} \) in Table 2. The effects of \( \psi \) and \( \bar{\gamma} \) on \( \overline{F} \) are almost identical to those on \( \overline{X} \).

Fig. 5 is a plot of coefficient of friction \( \overline{f} \) against \( \psi \) for various values of \( \bar{\gamma} \). It shows that other factors being same, the porous slider has higher coefficient of friction than a non-porous one. For a fixed value of \( \bar{\gamma} \) as \( \psi \) increases, the rate of increase in \( \overline{F} \) becomes more pronounced.
Fig. 3 Pressure distribution for various $\bar{\beta}$ and $\psi$
Fig. 4 Load capacity versus $\bar{p}$ for various $\psi$
Fig. 5 $\frac{f}{h_o}$ versus $\psi$ for various $\bar{\beta}$
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Table 2: Frictional drag $\Phi$ for various $\beta$ and $\Psi$.  