Chapter Five

Ferrofluid Lubrication of Rotating Curved Rough Porous Circular Plates And Effect of Bearing's Deformation
## Contents

5.1 Introduction 135

5.2 Analysis 141

5.3 Results and discussions 146

5.4 Conclusion 163
This investigation deals with Ferrofluid based squeeze film performance in rotating curved transversely rough porous circular plates taking bearing deformation into consideration. The results suggest that the transverse surface roughness induces an adverse effect on the performance characteristics while the magnetic fluid lubricant results in an improved performance. It is found that the combined effect of rotation and deformation cause significantly reduced load carrying capacity. This reduction in load carrying capacity comes at a critical phase when higher values of porosity and standard deviation are involved. However, it is revealed that the adverse effect of porosity, deformation and standard deviation can be compensated up to some extent by the positive effect of magnetic fluid lubricant in the case of negatively skewed roughness by properly choosing curvature parameters. To compensate, the rotational inertia needs to have smaller values.

5.1 Introduction

Most of the theoretical studies dealing with the squeeze film performance assumed perfectly rigid bearing surfaces. However, under high loads a bearing may deform producing wedge effects in the lubricant film, thereby, altering the squeeze. Taking this aspect into consideration, Osterle and Seibel [1958] and Ramanaiah & Sundarammal [1982b] analyzed the effect of the bearing deformation in slider bearings. Besides, Ramanaiah and Sundarammal [1982a] analyzed the effect of bearing deformation on the squeeze film characteristics between circular and rectangular plates. Here it was proved that the bearing deformation resulted in reduced load carrying capacity and increased squeeze. Okamoto et al [1999] embarked on a theoretical analysis of bearing system considering elastic deformation effects of the housing stiffness and bearing length on the bearing performance. It was established that the bearings performance was greatly influenced by the elastic deformation of both the housing and the bearing. Gullu [2005] analyzed the performance characteristics of the thrust bearings by factoring into discussion the bearings distortion. It was shown that the bearing deformation significantly decreased the load carrying capacity of the bearing systems. Here only the deformation of pads was taken into account.

Lately, considerable attention is been paid to the use of magnetic fluid as a lubricant to modify the performance of bearing system. These fluids are found to be useful in engineering and biomedical applications.
One of the most important properties of the magnetic fluid is that it can be retained at a desired location by an external magnetic field. For some other properties of magnetic fluids, one can cast a glance at Bhat [2003]. Huang et al [2009] conducted a study on the synthesis and tribological property of Fe$_3$O$_4$ based magnetic fluids. It was found that the friction coefficients decreased considerably. Huang et al [2009] employed the co-precipitation technique to prepare Fe$_3$O$_4$ based Ferrofluids with different saturation magnetization. The computed results established that the magnetic fluids had a higher supporting capacity compared with the carrier liquid, and that the supporting capacity of magnetic fluids increased with increasing magnetization. A significant improvement in antifriction and wear resistance was observed by using magnetic fluids with an applied external magnetic field. Zueco and Beg [2010] analyzed numerically the performance of a hydromagnetic squeeze film between two rotating disks using the numerical network simulation method. Excellent comparison of NSM solutions was achieved with analytical and shooting solutions. The findings here were applied in hydromagnetic lubrication of braking devices, slider bearings, rotating machinery and electro-magnetic braking for potential spacecraft in planetary orbits. Beg et al [2011] considered various complex phenomena in magneto-fluid dynamics such as magnetic squeeze films, non-Newtonian magneto-hydrodynamics and magneto-heat transfer from conical and rotating bodies. Applications of the models
discussed spread from magnetic materials processing to physiological flows and aerospace engineering. Huang et al [2011] considered the effect of Fe$_3$O$_4$ based Ferrofluids lubrications and proved that the load carrying capacity of a Ferrofluids based film would be increased with an appropriate magnetic field intensity distribution on the rubbing surface. This turns in a significant influence on the tribological properties of Ferrofluids. The experimental results indicated that Ferrofluids turned in a good friction reduction performance. Beg et al [2012] embarked on the homotopy analysis of transient magneto-bio-fluid dynamics of micropolar squeeze film in a porous medium. A model was developed for magneto-bio-rheological lubrication. Huang et al [2012] showed that samples with thicker magnetic films could reduce friction and wear more efficiently at higher sliding velocity under the lubrication of Ferrofluids.

The procedure adopted by Verma [1986] was modified by Bhat and Deheri [1993] to analyze the effect of magnetic fluid lubricant on the action of squeeze film in curved porous circular disks. Moreover, the analysis incorporated in Bhat and Deheri [1991] was developed by Bhat [2003] to study the effect of magnetic fluid lubricant on the squeeze film performance between porous curved plates in the presence of an external magnetic field oblique to the lower plate.

The squeeze film performance between curved circular plates lying along the surfaces determined by the secant function under the presence
of a magnetic fluid lubricant was studied and analyzed by Bhat and Deheri [1993]. Hsu et al [2008] discussed the squeeze film characteristics between rotating circular disks with an electrically conducting lubricant in the presence of a transverse magnetic field. It was concluded that, on the whole, the use of electrically conducting fluid in the presence of a transverse magnetic field resulted in an improvement of performance characteristics in rotating circular disks. Lin et al [2004] dealt with the performance of a squeeze film between two curved circular plates in the presence of a transverse magnetic field, using of an electrically conducting fluid.

Sukla et al [1982] investigated the performance characteristics of squeeze film bearings with power law lubricants considering the effect of consistency variations. Various bearing geometries were also considered with rigid surfaces as well as the compliant layers.

Several methods have been discussed to deal with the effect of surface roughness and the performance characteristics of squeeze films. Tzeng and Saibel [1967] employed a stochastic approach to model the random roughness which in turn, was extended by Christensen and Tonder [1969a]; [1969b]; [1970] to study the effect of surface roughness in general. These studies recognized the random character of surface roughness. A number of investigations deployed the stochastic model of Christensen and Tonder to analyze the effect of surface roughness.
[Ting [1972], Prakash and Tiwari [1983], Guha [1993], Gupta and Deheri [1996]]. Prajapati [1991] analyzed the squeeze film performance in rotating porous rough circular plates with elastic deformation and proved that the combined effect of elastic deformation and roughness was comparatively adverse.

Saber and Gamal [2006] analyzed the stability of the plane cylindrical journal bearing having an elastic shell. Here the effect of elastic deformation on its dynamic stability was evaluated. It was concluded from this work that increasing the elasticity of the bearing liner resulted in an increase in bearing stability. Recently, Hsu et al [2009] investigated the combined effect of surface roughness and rotational inertia on the squeeze film behaviour in parallel circular disks. Muhsin and Hussein [2011] studied the effects of elastic deformation and thermal distortion on the performance characteristics of conventional hydrodynamic journal bearings. It was established that the thermohydrodynamic effect was more than elastohydrodynamic effect especially, at high journal speed.

Although, the transverse surface roughness induces an adverse effect in general, the investigations carried out by Patel and Deheri [2003], Deheri et al [2007] and Shimpi & Deheri [2010b] suggested that the negatively skewed roughness resulted in a relatively better performance. Therefore, it was deemed appropriate to study the effect of surface roughness and bearing deformation on the squeeze film performance in
curved porous rotating circular plates taking a magnetic fluid as the lubricant.

5.2 Analysis

The geometry and configuration of the bearing system is shown in Figure 1 which consists of the circular disks, each of radius a.

*Figure 1 Configuration of the bearing system*
Both the disks are assumed to be elastically deformable and their contact surfaces are considered to be transversely rough. The upper disk moves towards the lower disk normally with uniform velocity $\dot{h} = \frac{dh}{dt}$.

Both the disks are assumed to have transversely rough surfaces. The characteristic of bearing surface roughness is adopted from Christensen and Tonder [1969a]; [1969b]; [1970] whose details are provided in Chapter 3.

It is assumed that the upper plate lying along the surface determined by

$$Z_l = h_0 \left[ \frac{1}{1 + Br} \right]; \quad 0 \leq r \leq a$$

approaches with normal velocity $\dot{h}_0 = \frac{dh_0}{dt}$ to the lower plate lying along the surface

$$Z_l = h_0 \left[ \frac{1}{1 + Cr} - 1 \right]; \quad 0 \leq r \leq a$$

where $h_0$ is the central distance between the plates, $B$ and $C$ are the curvature parameters of the corresponding plates. The central film thickness $h(r)$ then, is defined by [Bhat [2003]]

$$h(r) = h_0 \left[ \frac{1}{1 + Br} - \frac{1}{1 + Cr} + 1 \right]; \quad 0 \leq r \leq a \quad (1)$$

Assuming axially symmetric flow of the magnetic fluid between the annular plates under an oblique magnetic field

$$\vec{H} = H(r) \left( \cos \theta(r,z), 0, \sin \theta(r,z) \right)$$
whose magnitude $H$ is a function of $r$ vanishing at $r = 0, \alpha$; the angle of inclination $\theta$ of the magnetic field as in Bhat [2003]; the modified Reynolds equation governing the film pressure $p$ under the usual assumptions of hydromagnetic lubrication, takes the form [Prajapati [1995], Bhat and Deheri [1991]; [1993]]

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r g(h) \frac{\partial}{\partial r} \left( p - 0.5 \mu_0 \bar{\mu} H^2 \right) \right] = 12 \mu_0 H_0 + 4 \Phi(h)$$

(2)

with

$$g(h) = (h + p' p_a \delta)^3 + 3 \alpha (h + p' p_a \delta)^2 + 3 (\alpha^2 + \sigma^2) (h + p' p_a \delta) + 3 \sigma^2 \alpha + \sigma^3 + \varepsilon + 12 \phi H_0$$

and

$$\Phi(h) = (h + p' p_a \delta)^3 + 3 (\alpha^2 + \sigma^2) (h + p' p_a \delta) + 3 \sigma^2 \alpha + \sigma^3 + \varepsilon;$$

where

$$H^2 = kr^2(a - r)$$

(3)

while $\mu_0$ is permeability of free space, $\bar{\mu}$ is the magnetic susceptibility of particles and $\mu$ is the viscosity of the lubricant, $\phi$ is the permeability of porous facing, $H_0$ is the thickness of porous medium, $\delta$ is the local elastic deformation of the porous facing and $p_a$ is the reference ambient pressure.

With the aid of the following non-dimensional quantities,

$$P = - \frac{h_0^3 p}{\mu_0 a^2}, R = \frac{r}{a}, \tilde{\sigma} = \frac{\sigma}{h_0}, \tilde{\alpha} = \frac{\alpha}{h_0}, \tilde{\varepsilon} = \frac{\varepsilon}{h_0^3}, \psi = \frac{\phi H_0}{h_0^3}, \kappa = \frac{12 \mu h}{h_0^3}$$

$$\tilde{B} = B a^2, \tilde{C} = C a^2, \mu^* = - \frac{\mu_0 \bar{\mu} h^3}{\mu h}, \tilde{\rho} = p' p_a, \tilde{\delta} = \frac{\delta}{h}, S = \frac{3 \rho \Omega^2}{p_a}$$

($\rho$ being density of lubricant and $\Omega$ is angular velocity)
integrating the stochastically averaged Reynolds' equation (2) under the boundary conditions

\[ \left( \frac{\partial P}{\partial R} \right)_{R=0} = -\frac{\mu^*}{2}, \quad P(1) = 0 \]  

(4)

one obtains the expression for non-dimensional pressure distribution as

\[ P = \frac{\mu^* R^2 (R - 1)}{2} - \lambda_1 \log \left[ \frac{A_1 + A_2 R + A_3 R^2}{A_1 + A_2 + A_3} \right] \]

\[ + \lambda_2 \left[ \tan^{-1} \left( \frac{2A_2 R + A_2}{\sqrt{4A_1 A_3 - A_2^2}} \right) - \tan^{-1} \left( \frac{2A_3 + A_2}{\sqrt{4A_1 A_3 - A_2^2}} \right) \right] \]  

(5)

where

\[ \lambda_1 = \frac{3}{A_3} + \frac{2D_1}{A_3} \left( \frac{S}{\kappa} \right); \]

\[ \lambda_2 = \frac{2}{A_3 \sqrt{4A_1 A_3 - A_2^2}} \left[ 3A_2 + 2 \left( \frac{S}{\kappa} \right)(2A_3 D_2 - A_2 D_1) \right]; \]

\[ D_1 = \frac{6A_3^2 B_1 - 3A_1 A_3 B_3 - 4A_2 A_3 B_2 + 3A_2^2 B_3}{A_3^2}; \]

\[ D_2 = \frac{3A_1 A_2 B_3 - 4A_1 A_3 B_2}{A_3^2}; \]

\[ A_1 = (1 + \bar{p} \bar{\delta})^3 + 3\bar{\alpha}(1 + \bar{p} \bar{\delta})^2 + 3(\bar{\alpha}^2 + \bar{\sigma}^2)(1 + \bar{p} \bar{\delta}) + 3\bar{\sigma}^2 \bar{\alpha} + \bar{\alpha}^3 + \bar{\epsilon} + 12\psi; \]

\[ A_2 = 1.5(\bar{B}^2 - \bar{C}^2) \left[ (1 + \bar{p} \bar{\delta})^3 + 2\bar{\alpha}(1 + \bar{p} \bar{\delta})^2 + (\bar{\alpha}^2 + \bar{\sigma}^2)(1 + \bar{p} \bar{\delta}) \right]; \]

\[ B_1 = (1 + \bar{p} \bar{\delta})^3 + 3(\bar{\alpha}^2 + \bar{\sigma}^2)(1 + \bar{p} \bar{\delta}) + 3\bar{\sigma}^2 \bar{\alpha} + \bar{\alpha}^3 + \bar{\epsilon} \]

and
\[ B_2 = 1.5(B^2 - \bar{C}^2) \left[ (1 + \bar{p} \bar{\delta})^3 + (\bar{a}^2 + \bar{\sigma}^2)(1 + \bar{p} \bar{\delta}) \right]; \]

The load carrying capacity in dimensionless form then, is calculated from

\[ W = -\frac{h_0^3 w}{\mu h_0 a^4} = \int_0^1 RP \, dR \]

which leads to

\[ W = \frac{\mu^*}{40} + L_1 + L_2 \log \left[ \frac{A_1 + A_2 + A_3}{A_1} \right] \]

\[ + L_3 \left[ \tan^{-1} \left( \frac{2A_3 + A_2}{\sqrt{4A_1A_3 - A_2^2}} \right) \right] - \tan^{-1} \left( \frac{A_2}{\sqrt{4A_1A_3 - A_2^2}} \right) \]

where in

\[ L_1 = \frac{X_1(A_3 - A_2) - X_2 \sqrt{4A_1A_3 - A_2^2}}{2A_3}; \]

\[ L_2 = \frac{A_2X_2 \sqrt{4A_1A_3 - A_2^2} + X_2(A_2^2 - 2A_1A_2)}{4A_3^2}; \]

\[ L_3 = \frac{A_2X_1 \sqrt{4A_1A_3 - A_2^2} + X_2(2A_1A_3 - A_2^2)}{2A_3^2}; \]

\[ X_1 = \lambda_1 - \left( \frac{S}{\kappa} \right) \left( \frac{D_1}{A_3} \right); \]

\[ X_2 = \frac{1}{A_3 \sqrt{4A_1A_3 - A_2^2}} \left[ 3A_2 - 2 \left( \frac{S}{\kappa} \right) (2A_3D_2 - A_2D_1) \right]. \]
5.3 Results and discussion

It is easily observed that Equation (5) presents the non-dimensional pressure distribution while Equation (6) accounts for the dimensionless load carrying capacity. It is clearly seen that the non-dimensional pressure increases by

$$0.5\mu^*[R^2(1 - R)]$$

while the load carrying capacity registers an increase

$$0.025\mu^*$$
as compared to the case of conventional lubricants.

It is revealed that for a porous bearing with smooth surfaces this investigation reduces to the study of Bhat and Deheri [1993] dealing with the performance of a magnetic fluid based squeeze film between circular plates in the absence of rotation and deformation. Besides, setting the magnetization parameter $\mu^*$ to be equal to zero for a porous bearing with smooth surfaces, one can have the results of Prakash and Vij [1973] when no rotation and deformation occur. Moreover, considering the magnetization parameter to be equal to zero for a porous bearing with smooth surfaces one can obtain the findings of Murty [1975] in case, there is no deformation and rotation. Lastly, for a porous bearing with smooth surfaces this study leads to the analysis of Bhat [2003] when there is no bearing deformation.
A close scrutiny of the expression for non dimensional pressure distribution reveals that the bearing deformation significantly distorts the profile of the pressure distribution. It is seen that an adverse effect is introduced by the transverse surface roughness while the elastic deformation make this negative effect more significant. Possibly, this may be because that the bearing surface roughness obstructs the motion of the lubricant which results in reduced load carrying capacity.

The effect of magnetization presented in Figures 2-9 makes it clear that the load carrying capacity increases sharply with increase in magnetization parameter. This increase in the load carrying capacity is relatively less in the case of variance.

Figure 2 Variation of Load carrying capacity with respect to $\mu^*$ and $\bar{a}$
Figure 3 Variation of Load carrying capacity with respect to $\mu^*$ and $\alpha$

Figure 4 Variation of Load carrying capacity with respect to $\mu^*$ and $\bar{\varepsilon}$
Figure 5 Variation of Load carrying capacity with respect to $\mu^*$ and $\psi$

Figure 6 Variation of Load carrying capacity with respect to $\mu^*$ and $S/k$
Figure 7 Variation of Load carrying capacity with respect to $\mu^*$ and $\bar{B}$

Figure 8 Variation of Load carrying capacity with respect to $\mu^*$ and $\bar{C}$
The effect of standard deviation on the distribution of load carrying capacity is given in Figures 10-14. It is clearly visible that the load carrying capacity falls significantly with increasing values of standard deviation. Further, it is interesting to note that the effect of standard deviation is not that significant when considered with the case of deformation. Of course, this scenario changes when relatively higher values of rotation are involved.
Figure 10 Variation of Load carrying capacity with respect to $\sigma$ and $\bar{\alpha}$

Figure 11 Variation of Load carrying capacity with respect to $\sigma$ and $\bar{\varepsilon}$
Figure 12 Variation of Load carrying capacity with respect to $\sigma$ and $\psi$

Figure 13 Variation of Load carrying capacity with respect to $\sigma$ and $s/\kappa$
The effect of the variance depicted in Figures 15-18 establishes that variance (positive) decreases the load carrying capacity while the load carrying capacity increases due to variance (negative). Moreover, the effect of $\delta$ and $\psi$ on the variation of load carrying capacity with respect to the variance is nominal.

Figure 14 Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\delta$
Figure 15 Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $\bar{\epsilon}$

Figure 16 Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $\psi$
Figure 17 Variation of Load carrying capacity with respect to $\bar{a}$ and $S/k$

Figure 18 Variation of Load carrying capacity with respect to $\bar{a}$ and $\bar{\delta}$
The fact that the negatively skewed roughness tends to increase the load carrying capacity significantly, can be observed from Figures 19-21. Mostly, the skewness follows the path of variance so far as its effect on load distribution is concerned. However, the effect of rotational inertia with respect $\bar{\varepsilon}$ to is opposite to that of Shimpi and Deheri [2010b]. It is found that the effect of rotational inertia on the variation of load carrying capacity with respect to skewness is quite significant contrary to the findings of Shimpi and Deheri [2010b].

![Figure 19 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\psi$](image)

Figure 19 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\psi$
Figure 20 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $S/k$

Figure 21 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\bar{\delta}$
The bearing deformation has a considerable adverse effect on the performance of bearing system, as can be seen from Figures 22-23. These two figures make it clear that the combined effect of porosity and deformation is significantly adverse even for nominal values of rotational inertia. However, for a wide range of the values of deformation its effect with respect to porosity is negligible, so far as the load carrying capacity is concerned.

Figure 22 Variation of Load carrying capacity with respect to $\psi$ and $S/k$
Figure 23 Variation of Load carrying capacity with respect to $\psi$ and $\delta$

Besides, Figure 24 underlines that the rotational inertia significantly increases the load carrying capacity which is compatible with the findings of Shimpi and Deheri [2010b].

Lastly, the almost opposite nature of the effect of the two curvature parameters on the performance of the bearing systems appears to be reflected in Figures 25-26.
Figure 24 Variation of Load carrying capacity with respect to $s/\kappa$ and $\delta$

Figure 25 Variation of Load carrying capacity with respect to $\bar{B}$ and $\bar{C}$
Some of the figures witnessed here tend to suggest that the combined adverse effect of standard deviation $\bar{\sigma}$, rotational inertia $S/\kappa$ and porosity $\psi$ is fraction sharp when large values of deformation are involved. Moreover, equally adverse is the combined effect of standard deviation, bearing deformation and porosity when larger values of rotational inertia are in place. The combined adverse effect of standard deviation and porosity gets more manifested with the aid of bearing deformation effect. It is also noticed that the bearing deformation effect turns out to be more and more significant when higher rotational inertia is taken into consideration.
A close look at the discussion of Shimpi and Deheri [2010b] signifies that the overall performance is comparatively better in the present case. Unlike the annular geometry, here the combined effect of rotational inertia and deformation is not that sharp. Lastly, it is observed that a suitable choice of the curvature parameters may pave the way for a relatively better performance in comparison with that of Shimpi and Deheri [2010b].

5.4 Conclusion

It is revealed that the adverse effect of porosity and standard deviation can be reduced to some extent by the magnetic fluid lubricant, at least in the case of negatively skewed roughness even for higher values of bearing deformation and rotational inertia. The occurrence of bearing deformation tends to indicate strongly that the roughness must be given due consideration while designing this type of bearing system, even if, magnetization parameter and curvature parameters are suitably chosen. Furthermore, this investigation offers the suggestion that the bearing can support a load even in absence of flow, although, there are several factors adversely affecting the bearing system.
Contents

6.1 Introduction 166

6.2 Analysis 169

6.3 Results and discussions 174

6.4 Conclusion 182
This chapter aims to analyze the performance of a magnetic fluid based rough short bearing incorporating deformation effect. The associated stochastically averaged Reynolds' equation is solved with suitable boundary conditions to obtain the expression for pressure distribution which results in the calculation of load carrying capacity. The expression for the friction is obtained for both the plates. It is seen that the load carrying capacity increases nominally due to the magnetic fluid lubricant. Further, the film thickness ratio increases the load carrying capacity. It is found that the load carrying capacity increases as the ratio of the length to outlet film thickness increases while this trend is reversed in case of magnetization. Besides, it is noticed that the friction remains unaltered due to the magnetic fluid lubricant. Further, it is interesting to know that the deformation also un-alters the friction. It is suggested that the negative effect of the standard deviation can be neutralized up to certain extent by the combined positive effect of magnetization parameter, the film thickness ratio and the ratio of the length to outlet film thickness especially when deformation is relatively less. Therefore, it offers some scopes for extending the bearing's life period. Lastly, the bearing can support a load even in the absence of flow unlike the case of conventional lubricant.