Idea No. 1: Elaborative thinking and imagination

Through: Sets using: Skills of search Evaluate Situations

The goal of this idea is to develop elaborative thinking and imagination in pupils. The teacher asks the class "Name the great rivers in the country." The pupils give out names and the teacher records them on the board. After getting some four or five names the teacher asks the pupils "How did you choose them? Why have you included....?" They give different reasons. Can we call this list of names a set? Some say 'yes' and some say 'no'. They discuss among themselves. Some one says it cannot be a set because we are not sure of its elements.

How can we make it a set?

How can we improve the statement?
Idea No. 2:
To encourage: Fluent thinking and Complexity.
Through: Sets using: Analogies; organised random search

The goal is to make the pupils understand the idea - union and intersection of two sets, null set, universe set. Pupils develop fluent thinking and complexity.

Suppose our class wants to play Badminton. How shall we proceed? We shall divide the class into teams. Let them be A, B, C, D. Could it be possible for a player to be on two teams say A and C. The set of players common to teams A and C is called intersection of A and C. It is empty. The teacher writes $A \cap C = \emptyset$.

What do we get if we put together all our teams? We get all our class. The set of all members - the universal set.

$A \cup B \cup C \cup D = X$

Pupils were asked to state as many such examples as they can.

Next they were asked to give some examples of sets which have some elements in common.
Idea No. 3:
To encourage: Flexible thinking and complexity
Through: Sets using strategies: Attributes, Visualisation skills

The goal of this idea is to teach the meaning of equal sets, and equivalent sets.

Suppose our family members are sitting for dinner. How many chairs do we need? As many chairs as there are persons. The set of chairs and the set of persons are equivalent.

Let us take another example. The set of students in our class who study Mathematics. The set of students who study physics. How are these two sets? The members are the same. They are equal.

Let us think we are playing a game of musical chairs. Are the set of players and the set of chairs equivalent? or they equal?

We know line segments are sets of points. Can we call $\overline{AB}$, $\overline{CD}$ equivalent sets?
Idea No. 4:
To encourage: Fluent thinking and curiosity
Through: Sets and Subsets using, Skills of search
Discrepancies
Provocative questions

We place three dolls on the table. We call upon a pupil in the class to choose a doll he likes. He may choose none, or one, or two, or all. The teacher lists all the selections.

\[
\{\}, \{1\}, \{1,2,3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{2\}, \{3\}
\]

These are called subsets of the given set.

1. Is \(\emptyset\) a subset of every set?
2. Can a set be a subset of itself?
3. Is \(\emptyset\) a subset of \(\emptyset\)?
4. Can a set be equivalent to its subset?
Idea No. 5:
To encourage: Flexibility and Complexity
Through ordered pairs using: Attributes
Provocative questions

The goal of this idea is to develop the meaning of ordered pairs. Suppose there is going to be a chess game in our school. Suppose two students from eighth class and three from ninth class have come forward to play. How many teams could be arranged?

Pupils assume imaginary names and arrange teams. The pairs are listed (Ramu, Gopal), (Venu, Raghu) etc. Then pupils were asked to prepare such pairs using sets

\[ A = \{42\} \quad B = \{5,7,9\} \]

How many pairs do we get?
What do we get if we start choosing first from B and then from A?

Are \( A \times B \), \( B \times A \) equivalent?
Could they be equal?
Idea No. 6:
To encourage: Fluent thinking and curiosity.
Through: Relations Using: Attributes
Provocative questions Discrepancies

The goal of this idea is to develop the concept of Relation.

The teacher asks the pupils to list the pairs state and its capital. There will be a set of ordered pairs.

The teacher writes the ordered pairs:
\{(Dasaratha, Rama), (Humayun, Akbar), (Arjun, Abhimanyu)\}

How are the members of the ordered pair related?
Continue the series \{(1,4), (2,5), (3,6)\}....

How could you do it?

Continue the series \{(1,5), (2,9), (3,4)\}...

Is this a relation?
Are the relations \{(1,3), (2,4), (3,5)\}
\{(a,b), (c,d), (e,f)\}
equivalent?
Are they equal?

How many sets do you need to form a relation?
Idea No. 7:

To encourage: Original thinking and imagination
Through: Functions Using: Visualisation skill
evaluate situations.

The goal of this idea is to develop original thinking and imagination. The teacher draws the diagram.

```
4
3
2
1
```

<table>
<thead>
<tr>
<th>Function Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 2 1 0</td>
</tr>
</tbody>
</table>

Let us imagine a strange machine that gives out numbers 0, 1, 2, 3 etc. When we put 1, 2, 3, 4....

What do we get if we put 20 into the machine?
What is happening to the numbers in the machine?

The teacher asks the pupils to mark some of these ordered pairs on a graph paper. The set of points corresponding to these ordered pairs is called the graph of its function.

How does function differ from a relation?
Idea No. 8:

To encourage: flexible thinking and risk taking
Through: Linear Graph Using: organised Random Search
Evaluate situations.

This idea is to help the pupils understand a line and an equation associated with that line. Each Pupil takes a graph paper with coordinate axes marked on it. They are asked to mark a point (2,3) on it. Now each student is provided with a transparent paper having a black-line. The teacher says "Mark any other point on the graph paper. It is your choice. Now place the transparency such that the black line passes through (2,3) and also your point.

In how many ways can you arrange the line?
Where does the line intersect the y-axis?
Suppose you chose (2,7) for your point.
Then how is the line?
Take the line joining the points (2,3), (4,5).
What is its slope?
What is the equation to the line?
Idea No. 9:
To encourage: Flexible thinking and Imagination
Through: Simultaneous Equations using: provocative questions attributes

The goal of this idea is to develop flexible thinking and imagination in pupils. Each child is provided with a graph paper and two transperancies each having a black line on them. Now the pupils are asked to arrange the lines such that they intersected at a point.

What are the equations of two lines?
Where do they intersect?
Idea No. 10:

To encourage: elaborative thinking and curiosity
Through: Simultaneous equations using: Provocative questions
Examples of change

The goal of this idea is to develop curiosity and elaborative thinking in children. They are provided with a graph paper and two transperancies with black lines on them. The pupils are asked to arrange the two lines on the graph paper such that they do not intersect.

What are the equations of two lines?
What are their slopes?
What are the coefficients of two equations?
Do you see any peculiarity?
Idea No. 11:
To encourage: Fluent thinking and Imagination
Through: Matrices Using: Attributes
Provocative questions
Organised random search

The pupils are asked to get the prices of different brands of tea and of different quantities. By suitable questioning the pupils are encouraged to tabulate them. They will get an array.

<table>
<thead>
<tr>
<th>Brand/Quantity</th>
<th>250 gm.</th>
<th>500 gm.</th>
<th>1000 gm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Next they are asked to prepare a table for marks scored by 4 or 5 students.

These arrays of numbers arranged in rows and columns are called MATRICES. The above matrix records information, hence it is called information matrix.

Code Matrix: Take the following matrix

\[
\begin{pmatrix}
A & B & C & D & E & F & G & H & I & J & K \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11
\end{pmatrix}
\]
Consider the matrix

\[
\begin{pmatrix}
3 & 1 & 7 & 5 \\
2 & 1 & 3 & 11
\end{pmatrix}
\]

What letters do the matrix indicate?
What are the words?

**Route Matrix:**

<table>
<thead>
<tr>
<th></th>
<th>Hyd</th>
<th>Del</th>
<th>Mad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyd</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Del</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Mad</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Can you give any suitable names to the above matrices?

Prepare some more examples.
Idea No. 12:
To encourage: Flexible thinking and curiosity
Through: Matrices Using: organised Random Search
Skills of Search

The teacher writes the following two equations on the board. Waits for a few seconds. The pupils think about what to do with them.

\[ 5x + 2y = 7 \quad \text{What do they represent?} \]
\[ 3x + y = 4 \quad \text{What can you find out?} \]

Can you show the equations in a different forms?

They will attempt to write in different forms. Slowly their answers are improved.

\[
\begin{pmatrix}
5 & 2 \\
3 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
7 \\
4
\end{pmatrix}
\]

The pupils were asked to write the equations when the matrix is given.
Idea No. 13:
To encourage: Fluent thinking and Risk taking
Through: Matrices. Using: Analogies; Discrepancies

The teacher writes two matrices on the board.
He writes two numbers on the right end. What are the various things that you can do with these two numbers?

Pupils come up with a variety of suggestions. They add, subtract, multiply, divide etc.

Can we add two matrices? What are the possibilities? Can we subtract one matrix from the other? What shall we do?

Make up your own examples.

Can we add \( \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 4 & 6 \\ 7 & 8 & 9 \end{pmatrix} \)

Why not? What is the condition?
Idea No. 14:

To encourage: Flexible Thinking and Complexity.
Through: Matrices Using: attributes
organised random search.

Two persons Gopal and Krishna bought potatoes and Brinjals at two different markets say X and Y. The quantities they bought and the rates are given below:

<table>
<thead>
<tr>
<th>Potatoes</th>
<th>Brinjals</th>
<th>Market X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gopal</td>
<td>2 kg.</td>
<td>3 kg.</td>
<td>Pot 3.00</td>
</tr>
<tr>
<td>Krishna</td>
<td>4 kg.</td>
<td>5 kg.</td>
<td>Brinj 2.00</td>
</tr>
</tbody>
</table>

How much Gopal paid at market X?
What shall he pay if he purchased at Y?
How about Krishna?
Try to write them in matrix form.
Idea No. 15:

To encourage: Flexible thinking and Imagination

Each pupil is given a graph paper. They are asked to mark the coordinate axes, and the line

What is the equation of \( L \)?
Put \((0,0)\) in the equation of line. What do you get?
Put \((1,1)\) in the equation
Put \((3,2)\) in the equation

How do the results differ?
What can you say about the points above the line?
How about the points below the line?

The two regions are called half-planes.

\[ 2x + 3y > 6 \]

Gives a half-plane.
Idea No. 16:

To encourage: Elaborative thinking and Complexity
Through: Inequations Using: organised Random Search
Provocative Questions

The goal of the idea is to develop elaborative thinking and complexity.

Find the equation of the line through (3,4) with slope 3.

Mark the line on a graph paper. Shade the half plane containing the origin.

What is the inequality that is associated with the half plane?

Now choose a point and slope. Frame your own problem.
Idea No. 17:

To encourage: Fluent thinking and complexity.
Through: Inequations using: Organised Random Search
Provocative questions
Analogies

Pupils are given graph papers on which the first quadrant was marked.

What do you need to form a quadrilateral?
After exploration they find two more lines are needed to form a quadrilateral.

What are the vertices of the quadrilateral?
Shade the quadrilateral region.

Find the values of the function
\[ f(x,y) = 2x + 3y \] at various points of the region.

Can you guess where it could be maximum?
Verify:
Idea No. 18:
To encourage: Elaborative Thinking and Complexity.
Through: Inequations using: examples of change
organised Random Search
Evaluate Situations

Suppose you have Rs. 15 with which you have to buy pens and ball pens. Total number of articles should not exceed 6. Cost of a pen is Rs. 5 and the cost of a ball pen is Rs. 2.

If $x$ represents the number of pens and $y$ represents the number of ball pens what can you say about $x + y$?

What is $5x + 2y$?

Represent the situation on a graph paper.

How many pens and ball pens can you buy?
Idea No. 19:

To encourage: Elaborative thinking and complexity.
Through: Statistics using: Organised random search
Discrepancies

The following are the marks scored by seven students:
40, 45, 50, 52, 52, 60, 72

What is their average? Students add the marks and find the average.

Can you do it differently? For example can you guess? 40 is too small and 62 is too big. Let us guess 52.

What are the deviations?
-12, -7, -2, 0, 0, 8, 20

What is the sum of deviations? 7. Average is 1.
Mean = 52 + 1 = 53.
Assume 50 as mean and compute the true mean. What do you get?
Idea No. 20:

To encourage: Elaborative Thinking and Complexity
Through: Statistics using: analogies
(Standard Deviation) skills of search

The marks scored by seven students are
40, 45, 50, 52, 52, 60, 72

Find the Mean.
Pupils calculate the mean as 53.
What are the deviations of scores from 53?
-13, -8, -3, -1, -1, 7, 19.

What is the sum of deviations? (zero)
169 + 64 + 9 + 1 + 1 + 49 + 361
Mean of the square is 93.42.

What is the square root? 9.66
This is called Standard Deviation.
Idea No. 21:

To encourage: Fluent thinking and Imagination
Through: Permutations using: Analogies
Provocative Questions

Suppose you are given three pens of three different colours. In how many ways can you select two pens? Let the pens be denoted by R, B, G. The arrangements are RB, RG, BG, BR, GR, GB.

There are three chairs and three persons.
In how many ways can you seat them?
How do you write the product 6.5.4.3.2.1?
Students explore various possibilities.
It is denoted by 6!
Each arrangement of a set of objects is called a permutation. The number of permutations of 3 things taken 3 at a time is \(3P_3 = 3 \times 2 \times 1 = 6\).

What is 0!?
Idea No. 22:

To encourage: Flexible thinking and Complexity.

Through: Meru Prastar Using: Organised random search Evaluates Situations

Pupils from their previous algebra work, knew the expansions of \((a + b)^2, (a + b)^3\) etc. Using this knowledge they develop formulae for \((a + b)^4, (a + b)^5\).

List the coefficients

What is \((a + b)^0\)?

\[
(a + b)^1 = a + b \\
(a + b)^2 = a^2 + 2ab + b^2 \\
(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3
\]

What could be the coefficients of \((a + b)^4\)?

What could be the coefficients of \((a + b)^5\)?

The arrangement of numbers in Ancient Hindu Literature is called 'Meru Prastar.'
Idea No. 24:

To encourage: Elaborative thinking and imagination
Through: Order relation Using: organised Random Search

Examples of change

The teacher tests the following pairs of numbers:

1, 3 which is bigger?
3, 4
-2, 0 The first two cases are simple. But
-3, -2 the students are puzzled in the last two cases.

Let us take the number line

-3 -2 -1 0 1 2 3 4

What do you notice about the numbers?
Idea No. 24:

To encourage: Flexible Thinking and Complexity.
Through: Order relation using: Skills of search

Visualisation skill

Teacher writes on the board 36 24
State the relation between the numbers.
Add 10 to both sides
46 34
What is the relation?
Give some examples of your own.
Suppose \( a > b \). Add \( c \) both sides
\( a + c \) \( b + c \) What is the relation?
15 > 12 multiply by 2. What happens?
If we multiply by -2 what happens?
State the Rule.
Idea No. 25:

To encourage: Flexible thinking and curiosity.
Through: Logarithms using: Abalogies
      Skills of search

The teacher writes the following numbers on the board:

\[ 16 = 2^4, \quad 81 = 3^4, \quad 100 = 10^2 \]

Pupils recall numbers that go into the blanks. Another way of writing the above number is:

\[ \log_2 16 = 4, \quad \log_3 81 = 4, \quad \log_{10} 100 = 2 \]

What is \( \log_2 1 \)?

Can you state the rule?

Though it is possible to choose any positive number other than 1 as base the widely used bases are 10 and \( e \).

\( e \) is an irrational number like \( \pi \).
Idea No. 26:
To encourage: Flexible thinking and Imagination
Through: Geometry Using: Attributes
Skills of Search
Visualisation skill

Each pupil was given three transperancies with black lines drawn. They were asked to arrange the lines in different ways. The figures they get are shown on the board.

(i) (ii) (iii)

What can you say about angles 1 and 2 in figure (1)?
How are they in fig. (2)?
Measure and verify your guess.
These angles are called alternate angles. When the lines are parallel the alternate angles are equal.
Idea No. 27:
To encourage: Flexible thinking and curiosity
Through: Geometry Using: Visualisation skill
(concurrency of altitudes)

Pupils are provided with triangles cut from colour-paper.

They are asked to fold the triangle along the dotted line. Repeat it for alternate sides. What do you find?

State the result in a sentence.
Idea No. 28:

To encourage: Flexible thinking and curiosity

Through: Geometry (concurrency of perpendicular bisectors)

Using: Visualisation skill

Skills of search

Pupils are provided with triangles cut from colour paper.

They are asked to fold the triangle at the mid-point of the side.

Repeat it for the other two sides. What do you find?

State the result in a sentence.
Idea No. 29:

To encourage: Flexible thinking and curiosity.
Through: Geometry Using: Visualisation skill
(Coincurrence of medians) Skills of search

Pupils are provided with triangles cut from colour paper.

They are asked to fold along the dotted line. (From the corner to the middle point of opposite side).

Repeat it for the other two sides. What do you find?

State it in your own words.
Idea No. 30:

To encourage: Flexible thinking and curiosity

Through: Geometry Using: Visualisation skill
(Concurrency of angle bisectors) Skill of search

Pupils are asked to fold the paper triangle such that the two edges meet along a vertex.

That is the angle bisector. Repeat it for the other two vertices.

What do you find?