5.0 Introduction

5.1 Basic Elements of Research Method
   5.1.1. Variables
   5.1.2. Hypotheses
   5.1.3. Tools used
   5.1.4. Sample Selection

5.2 Statistical techniques: Experimental Design
   5.2.1. Quasi-experimental Designs
   5.2.2. General Designs
   5.2.3. Choice of the design
   5.2.4. ANOVA Factorial Design
   5.2.5. Statistical technique in ANOVA

5.3 Execution of CTM
   5.3.1. Instructions to Students
   5.3.2. Time Schedule
   5.3.3. Execution
   5.3.4. Observations

5.4 Summary
5.0 Introduction:

Design is data discipline. The purpose of a research design is to impose controlled restrictions on observations of natural phenomena. It tells the investigator what to do and what not to do.

The quality of research depends upon the quality of its design. If the design has faults then the final product will have faults. Writing about the importance of experimental design Hummel (1980) had remarked "experimental design is a difficult art to master, whereas the analysis of a well-designed experiment proceeds in a fairly straightforward fashion." The above statement shows that a good design is needed for valid analysis.

5.1 Basic elements of Research Method:

The basic elements of research method are variables, hypotheses, research tools, and sample selection. Each one of the four elements are described in detail below.
5.1.1 Variables:

Several special programmes were available for developing various abilities like creativity, self-concept, divergent thinking and attitude. There were also programmes like SMSG, SMP and PSSC for developing improved instructional materials. But all of them are special programmes that can not be used in an ordinary classroom without disturbing its activities.

A programme that could be used without causing any inconvenience to the students has to be developed. CTM is such a programme. It produces cognitive and affective behaviours in pupils through mathematics using certain teacher modes or strategies. Since one of the objectives of the study is to study its effect on students' achievement and attitude it was chosen as an independent variable.

Several studies have shown that Sex is an important biological factor that influences other variables like achievement, attitude, and creativity. Hence sex was also taken as independent variable.

Motivation is a psychological factor that is of great importance in several fields like education, management, sociology and anthropology. It has great influence on a person's behaviour, Motivation towards school was taken as another independent variable.
Socio-economic factors like standard of living, parental income, caste, and parental education have much influence on child's behaviour and his school performance. Hence Parental Education was also chosen as an independent variable.

Thus there are four independent variables each of two levels. Achievement and attitude are the two dependent variables. The details of these variables are shown in the following table.

Table: 5.1

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Name of the variable</th>
<th>Nature of the variable</th>
<th>No. of levels</th>
<th>Name of Levels</th>
</tr>
</thead>
</table>
| 1.      | Treatment Creative Teaching Model | Independent | 2 | 1. Treatment ($A_1$)  
2. No treatment ($A_2$) |
| 2.      | Sex                  | Independent            | 2 | 1. Boys ($B_1$)  
2. Girls ($B_2$) |
| 3.      | Motivation           | Independent            | 2 | 1. High ($C_1$)  
2. Low ($C_2$) |
| 4.      | Parental Education   | Independent            | 2 | 1. High ($D_1$)  
2. Low ($D_2$) |

The above described variables led to the following hypotheses.
5.1.2 Hypotheses:

The hypotheses formulated for investigation are as follows:

$H_1$ : There is a significant difference between the achievements of experimental group and control group.

$H_2$ : There is no significant difference between the achievements of Boys and Girls.

$H_3$ : There is no significant difference between the achievements of low motivation group and high motivation group.

$H_4$ : There is no significant difference between the achievements of Low Parental education group and the High Parental education group.

$H_5$ : There is no significant effect of the interaction of CTM and sex on the achievement of students.

$H_6$ : There is no significant effect of the interaction of CTM and Motivation on the achievement of students.

$H_7$ : There is no significant effect of the interaction of CTM and Parental education on the achievement of students.

$H_8$ : There is no significant effect of the interaction of sex and motivation on the achievement of students.

$H_9$ : There is no significant effect of the interaction of sex and parental education on the achievement of students.
H₁₀ : There is no significant effect of the interaction of motivation and parental education on the achievement of students.

H₁₁ : There is no significant effect of the interaction of CTM, sex and motivation on the achievement of students.

H₁₂ : There is no significant effect of the interaction of CTM, sex and parental education on the achievement of students.

H₁₃ : There is no significant effect of the interaction of CTM, motivation and Parental education on the achievement of students.

H₁₄ : There is no significant effect of the interaction of sex, motivation and parental education on the achievement of students.

H₁₅ : There is no significant effect of the interaction of CTM, sex, motivation, and parental education on the achievement of students.

H₁₆ : There is no significant difference between the attitudes of experimental group and control group.

H₁₇ : There is no significant difference between the attitudes of Boys and Girls.

H₁₈ : There is no significant difference between the attitudes of low motivation group and high motivation group.

H₁₉ : There is no significant difference between the attitudes of low parental education group and high parental education group.
H20: There is no significant effect of the interaction of CTM and sex on the attitudes of students.

H21: There is no significant effect of the interaction of CTM and motivation on the attitudes of students.

H22: There is no significant effect of the interaction of CTM and parental education on the attitudes of students.

H23: There is no significant effect of the interaction of sex and motivation on the attitudes of students.

H24: There is no significant effect of the interaction of sex and parental education on the attitudes of students.

H25: There is no significant effect of the interaction of motivation and parental education on the attitudes of students.

H26: There is no significant effect of the interaction of CTM, sex and motivation on the attitudes of students.

H27: There is no significant effect of the interaction of CTM, sex and parental education on the attitudes of students.

H28: There is no significant effect of the interaction of CTM, motivation and parental education on the attitudes of students.

H29: There is no significant effect of the interaction of sex, motivation and parental education on the attitudes of students.
H₃₀ : There is no significant effect of the interaction of CTM, sex, motivation, and parental education.

5.1.3 Tools used:

The selection of an appropriate tool or instrument for measuring the variables is one of the most critical components of the research process. The following instruments were used in this study.

(i) Creative Teaching Model
    Developed by the investigator

(ii) Achievement Test
    Test administered by the District Common Examination Board; scores from school records (Guntur)

(iii) Attitude Test
    Developed by H.G. Desai

(iv) JIM Scale, (Motivation)
    Developed by Extension Service Unit, M.B. Patel College of Education

(v) Profile of Parental Education
    Developed by the investigator.

(i) Creative Teaching Model:

This is the programme developed by the investigator. It has 30 ideas, covering part of the content of ninth class mathematics of Andhra Pradesh. This Model was described in detail in chapter Four.
This programme aims at developing the thinking and feeling of the pupils. Each idea tries to develop one cognitive and one affective behaviour. The teacher uses certain selected strategies.

Each idea supplements and enriches the usual classroom instruction.

(ii) Achievement Test:

The District Common Examination conducts tests at the end of the year. The format of the test is same as that of SSC public examination. There are two papers and each paper has two parts. The marks obtained by the students in the annual examination has been taken as the achievement score.

(iii) Attitude Test:

This mathematical attitude scale was developed by H.G. Desai. It was a standardised, culture free test. It is in Gujarati. Dr. J.Z. Patel translated it into English and the investigator had translated it into Telugu. This Telugu version was administered to the students.

It has 20 statements. Each statement shows a person's feeling towards mathematics. Each subject is asked to tick the statements that he felt as appropriate. To each statement a value is given. These are provided in the Scoring key.
The scoring method is quite simple. If a particular student has ticked statements 3, 7 and 9 his total score will be $5.82 + 3.79 + 4.15 = 13.76$. The average is 4.59. This will be the attitude score of that person. A low score indicates high positive attitude towards mathematics. A high value shows low positive attitude towards mathematics.

(iv) JIM Scale:

This test for measuring motivation towards school was developed and standardised by the Extension Services Unit of M.B. Patel College of Education, Vallabh Vidyanagar.

It has eighty statements in it. Some are true and some are false. The subject has to respond on a four point scale. The four points are: (1) Strongly agree, (2) Agree, (3) Disagree, (4) Strongly Disagree.

The thirty false statements are randomly interspersed among the fifty true statements. The subject is requested to mark all the statements.

For example Sr. No. 4 "To be earnest is better than to be kind" is a false statement Sr. No. 13 "Many young persons regret a lot" is a true statement.

**Scoring:** Before scoring all the false statements are omitted. Only the true fifty statements are taken into consideration. The weightages to the responses are strongly Agree +2, Agree +1,
Disagree -1, Strongly Disagree -2; After computing the total score 50 is added to it. This is the JIM score. It is the measure of a person's motivation towards school.

The JIM Scale was translated by the investigator into Telugu and was administered to the subjects. A score of 75 or low was taken to mean low motivation, and a score greater than 75 was taken to mean high motivation.

(v) Profile of Parental Education:

This is a very simple tool designed by the investigator himself under the guidance of Dr. J.Z. Patel. It contains items like name, class, section for identifying the subject. There is a table for recording parental Education. The student is required to tick the appropriate box for both father and mother. The Telugu version of the profile was administered to all the members of the sample to obtain their parental education score.

The scaling is shown below:

<table>
<thead>
<tr>
<th>Level</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>V standard</td>
<td>0</td>
</tr>
<tr>
<td>Matriculate</td>
<td>1</td>
</tr>
<tr>
<td>Graduate</td>
<td>2</td>
</tr>
<tr>
<td>Post Graduate</td>
<td>3</td>
</tr>
</tbody>
</table>

The total of the both scores gives the parental education score of a child. A score of 3 or less than 3 was taken as low and a score greater than 3 was taken as high.
5.1.4 Sample Selection:

Sampling means taking any portion of a population or universe as representative of that population or universe. A sample is said to be representative when it has approximately the characteristics of the population relevant to the research in question. When a random sample was drawn from a population it was assumed to be representative.

There are several methods of selecting samples. Some of the commonly used methods are:

1. Random Sampling
2. Quota Sampling
3. Purposive Sampling.

Random Sampling is that method of selecting a sample from a population so that all samples of fixed size $n$ have the same probability of being selected. Thus in a random sample every element of the population has an equal chance of being selected.

When a population could be divided into strata these strata are used for selecting a representative sample. In this method quotas are assigned to different strata. That is why it is called quota sampling.
Purposive sampling is a non-probability form of sampling. Under this method one selects the available sample. This method has to be followed when it is not possible to identify all the subjects of the universe or when it is not possible to disturb the subjects due to administrative reasons as in the case of a classroom experiment.

The present study was conducted at Sri Majesty Guravaiah High School, Arundelpet, Guntur, Andhra Pradesh. It is an aided, recognised and privately managed secondary school. The classes range from sixth to tenth. It is a coeducational institution.

The students mostly belong to middle class and lower middle class families. They are admitted into six class in two ways. Some take the entrance test held by the District Common Examination Board at the beginning of the year. Those who passed the test are admitted into Sixth class. Those students who have a five year study in an elementary school can directly join the Sixth class. They don't need the entrance test. Students are assigned to different sections at the time of admission such that all sections have nearly the same number of students.

There were two sections of ninth class in this school. Their number was 120. There are 62 boys and 58 girls. Out of these students the sample for the experiment was selected using the following procedure.
All the boys were given serial numbers. Using random number tables they were assigned to two groups. The same procedure was repeated with girls. Now there are two groups each having 31 boys and 29 girls. The experimental group was selected again by random process.

Now the JIM scale and Parental Profile were administered to the two groups so that the sample could be further partitioned. The number of subjects in the 16 cells are adjusted such that each cell had 7 subjects. Then the control group and experimental group each has 56 students. Of them 28 are boys and 28 are girls.

The choice of the institution and then the selection of the students made the sample purposive. Since the school chosen was a normal one (unlike private residential schools or Ashram Schools where students are admitted on the basis of their merit) it can be assumed to be representative of the population.

Certain degree of randomisation could be done in assigning subjects to the experimental and control groups. The other three variables namely sex, motivation and parental education are attributive variables. Randomization is not possible in their case.
5.2 Statistical Techniques: Experimental Design:

For testing the hypotheses an experimental design is needed. In an experimental design the investigator manipulates and controls one or more independent variables and observes the dependent variable for corresponding changes.

The designs are classified into two groups or categories: (i) Inadequate designs or quasi experimental designs, and (ii) General experimental designs.

5.2.1. Quasi-experimental Designs:

The one-group designs come under the first category. It is also known as one-shot case study. Case studies fall under this group and hence the name. In this design a group is exposed to some treatment and after a period the effect is measured. For example if a school wants to introduce a new curriculum and study its effects. After an year the student achievement is studied and found to be same or better. Symbolically it is denoted by

\[ X \rightarrow Y \]

Here the dependent variable Y is studied while the independent variable X is assumed or imagined. Sometimes conclusions could be misleading.

Another form of one-group design is the pre test-post test type. This is an improvement over the previous
method. The important characteristic of this design is that a group is compared with itself. This is theoretically sound since all the independent variables associated with the subjects' characteristics are controlled. The group is measured on the dependent variable $Y$ before the experiment. It is called Pre-test. After the experimental manipulation again $Y$ is measured. The differences in scores or $Y_a - Y_b$ are studied. Symbolically it can be shown as

$$Y_b \quad X \quad Y_a$$

Though this appears to be sound it is not that simple. The difference might have been caused by variables like history or maturity.

5.2.2 General Designs:

(i) The experimental - control group design is one of the best designs for many experimental purposes in education and psychology. The paradigm is

$$\begin{array}{c}
R \\
\sim X \\
X \quad Y \\
(\text{experimental}) \\
\sim X \\
X \quad Y \\
(\text{control})
\end{array}$$

The R placed before the design shows that subjects are to be randomly assigned to the experimental group and the control group.

There are two merits in this method: (1) the presence of a control group gives the comparability required by Science, and (2) randomization provides assurance that
the two groups are approximately equal on variables that may be related to the dependent variable.

(ii) The two group matched subjects design is another. Here instead of randomization the subjects are matched on are on one or more attributes. Symbolically

\[ \begin{array}{c|c|c|}
M^r & X & Y \\
\hline
\sim X & \sim Y \\
\end{array} \]

(Experimental)

(Control)

The suffix r shows that after matching the members of each pair must be assigned to the two groups randomly. This can be done using random numbers. Odd numbered subjects go into one group and the even numbered subjects go into another group.

(iii) Three group, before-after.

Its paradigm is

\[ \begin{array}{c|c|c|}
R & Y_b & X \\
\hline
Y_b & \sim X & Y_a \\
\end{array} \]

(Experimental)

(Control 1)

(Control 2)

This is an improvement over the previous design. It avoids the possible interactive effects of the pre-test. This is done by the second control group. If the treatment is effective then the means of experimental group and control group 2 will be significantly higher than the mean of control group 1.

(iv) Four-group, before-after (Solomon).

This satisfactory design was proposed by Solomon.
This design has powerful controls. The salient features of the previous designs are included in this one design. It is widely used by social scientists.

5.2.3 Choice of the Design:

In deciding an approach the researcher has to take into consideration several factors like available setting, nature of objectives, and time.

Two separate and independent dimensions can help the investigator in the choice of approach. Fox (1969) had suggested the following table.

<table>
<thead>
<tr>
<th>Dimension 2</th>
<th>Dimension 1</th>
<th>(Time in which interest lies)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Past</td>
<td>Present</td>
</tr>
<tr>
<td>Intent of Research</td>
<td>Historical</td>
<td>Survey</td>
</tr>
<tr>
<td>Description</td>
<td>Simple historical</td>
<td>Simple survey case study</td>
</tr>
<tr>
<td>Comparison</td>
<td>Parallel historical</td>
<td>Multiple group survey</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table: 5.2 (contd.)

<table>
<thead>
<tr>
<th>Evaluation</th>
<th>Dimension 1</th>
<th>(Time in which interest lies)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical and criterion measure</td>
<td>Single-group or multiple group survey criterion measure</td>
<td>Single group experiment with criterion measure</td>
</tr>
</tbody>
</table>

The use of the above table in the present case leads to the choice of multiple group experimental design. Since there are four independent variables each of two levels a Factorial design is called for.

5.2.4. ANOVA: Factorial Design:

According to Kerlinger (1978) "Factorial Design is the structure of research in which two or more independent variables are juxtaposed in order to study their independent and interactive effects on a dependent variable."

In the present experiment the independent variables are treatment (A), Sex (B), Motivation (C), Parental Education (D). Each is at 2 levels. Hence it is a $2^4$ Factorial Experiment.

Factorial Analysis of variance has several advantages. It enables the researcher to manipulate and control two or more variable. Secondly variables like sex, parental education etc., that cannot be manipulated can also be controlled. A third advantage is factorial analysis is more precise
than the one-way analysis. Finally the interactive effects could be studied. This is important from a scientific point of view.

5.2.6 Statistical technique in ANOVA:

Here treatment (A), Sex (B), Motivation (C), Parental education (D) are the independent variables each at 2 levels. In all there are 16 blocks. They are shown in the following table

The F-test is based on the following assumptions:
(i) an equal unit scale is assumed for the measurement of the dependent variable.
(ii) Homogeneity of variance.

The ANOVA summary helps in testing whether the group means differ or not.

Table 5.3
ANOVA Summary: Between the Groups and within the Groups

<table>
<thead>
<tr>
<th>Source of variance</th>
<th>Sum of squares</th>
<th>df</th>
<th>Mean SS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within the groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The 0.05, 0.01 confidence levels were taken to test for significance.
### Table: 5.4

<table>
<thead>
<tr>
<th></th>
<th>$A_1$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_2$</th>
<th>$D_1$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_2$</td>
<td>$C_1$</td>
<td>$D_1$</td>
<td>$D_2$</td>
<td>$D_2$</td>
<td>$D_1$</td>
<td>$D_1$</td>
<td>$D_1$</td>
<td>$D_2$</td>
<td>$D_2$</td>
<td>$D_2$</td>
</tr>
</tbody>
</table>

2^4 Factorial design for data analysis

117
To test the Main effects and Interaction effects complete ANOVA is used. The full form is shown in the following table:

Table: 5.5
Summary of Four Way ANOVA

<table>
<thead>
<tr>
<th>Source of variance</th>
<th>SS</th>
<th>df</th>
<th>MSS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BCD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABCD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3 Execution of C T M:

5.3.1 Instructions to students:

There were very few instructions to the students. All precautions were taken to avoid Hawthorne Effect. Both the groups were treated in the usual manner. The instructions were given to both the groups. They were asked to be punctual, regular and neat in their work. Regular, and continuous effort will yield good results and better learning takes place.
5.3.2. **Time Schedule:**

The following time schedule was used in applying the treatment. Since each idea was built into the lesson plan of the investigator the time schedule provided by the District Common Examination was closely followed. Each idea required one period.

<table>
<thead>
<tr>
<th>Month</th>
<th>Idea No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>August</td>
<td>4, 5, 19, 20</td>
</tr>
<tr>
<td>September</td>
<td>6, 7, 8, 9, 10</td>
</tr>
<tr>
<td>October</td>
<td>15, 16, 17, 18</td>
</tr>
<tr>
<td>November</td>
<td>11, 12, 25</td>
</tr>
<tr>
<td>December</td>
<td>13, 14</td>
</tr>
<tr>
<td>January</td>
<td>21, 22, 23, 24</td>
</tr>
<tr>
<td>February</td>
<td>26, 27, 28, 29, 30</td>
</tr>
<tr>
<td>March</td>
<td>Examinations</td>
</tr>
</tbody>
</table>

The above arrangement was made taking into consideration the examination schedules of the department, festival holidays, special events like district science fairs, and district sports.
5.3.3 **Execution:**

The teacher chose the strategies selected by him for each idea and through the content tried to develop the appropriate cognitive and affective behaviours in the pupils. The normal classroom activities like questioning, homework, seat work, explanation at the blackboard were supplemented and enriched by the ideas.

Whenever some material is needed, it was prepared and kept ready well in advance. It was distributed to the students at the time of use. For example ideas 16, 17 need graph paper. The graph paper of suitable size was procured, the coordinate axes were drawn and kept ready. Similarly ideas 28, 29, 30 involved paper folding. The coloured lac paper was procured and cut into triangles of suitable size. These were distributed at the time of teaching those units.

5.3.4 **Observations:**

During the implementation of the CTM the investigator found the appropriate behaviour developed through the strategies specially chosen using mathematics as content. They were recorded at the time of the experiment. The following description contains them in all detail.

**Idea No. 1:**

When the teacher put the question 'Name the great river in the country' pupils came up with a number of names. All of them were written on the board. The teacher made no comment. The list was 'Ganga, Krishna, Kaveri, Brahmaputra'.
Another student stood up and said 'Kinnerasani'. Some one objected saying it was narrow and small. The student who gave the answer replied 'It is great Kavi Samrat has written a book of poems on it.' There arose a discussion. One student remarked 'we must first say what we mean by great.' Without that no one can say what could be included or not. Then they began framing rules. One rule was 'It must be a long river'. In the light of this they framed the set \{Ganga, Brahmputra, Krishna, Godavari\}. Another rule was 'there should be water throughout the year.' A new list was made \{Ganga, Brahmputra\}

Here the teacher employed the skills of search and evaluate situations. Pupils used imagination and improved upon the notion of set. They wrote in their note books a rule is needed to specify the elements of a set.

Idea No. 2:

In this lesson through union and intersection of sets fluent thinking was developed. When the teacher asked for teams to play they came up with lists. Teacher wrote on the board. $A = \{\text{Gopal, Venkatesh}\}$, $B = \{\text{Krishna, Ramu}\}$ etc. until all the members were exhausted. There were some girls teams like $C = \{\text{Sita, Sujata}\}$ and mixed teams like $\{\text{Venu, Rama}\}$.
Next they searched for the players who are common to two teams. The set remained empty, teacher wrote then the teacher told them of the special symbol $\emptyset \cdot A \cap B = \emptyset$. Intersection is the set of all elements that are common to two sets or more. Then the teacher gave another example, $E$ be the set of all even numbers and $O$ the set of odd numbers what is $E \cap O$?

They put all the teams together and got a big list of names the whole class. The teacher explained 'The set of all members under consideration has a special name universal set. It is denoted by another symbol $\mu$.'

Here pupils displayed Fluent thinking in giving a number of examples for empty set and forming intersections which are not empty. They worked with sets whose members are letters like $P = \{a,b,c\} \quad Q = \{b,c,d\} \quad$ etc.

Idea No. 3:

Here the goal is to teach the ideas of equality and equivalence using strategies attributes and visualisation skill. Pupils developed Flexible thinking and met the challenge of comparing two line segments of unequal length. They considered the set of chairs $C$ and the set of family members $F$. They said 'as many $\sim$ chairs are needed as there are persons'. One of the students said $C$ and $F$ have the same number of elements/members. Next they considered the question
'Could we call them equal or same?' After discussion they rejected the idea saying 'The elements are different.' Sets having the same number of elements are equivalent sets. This they noted in their notes.

Afterwards they took up the sets of students studying physics, and students studying mathematics. They named them as M, P. On comparison they were found to have the same elements because all the students studied both the subjects. Hence M = P. Sets having the same elements are equal sets.

Next they faced the challenge of a complex situation. The teacher drew the two line segments $\overline{AB}$, $\overline{CD}$ one long and another short. Can we call them equal sets? In a little while they said they are not. The elements (points) we different.

The next question is more puzzling 'are they equivalent?' Some students were confused. They replied 'cannot say.' Some said 'They are not equivalent $\overline{CD}$ is longer than $\overline{AB}$ and hence it should have more points in it.' Another student said 'it is impossible. We cannot count.'

The teacher said 'I shall give you a cue.' He joined $\overline{CA}$, and $\overline{DB}$ they met at E. The teacher said 'now you have A corresponding to C and B corresponding to D. Then he took X on $\overline{AB}$. Can you find the point corresponding to X?
After some discussion one student joined EX and produced it to meet CD. Now they said it is possible to find a corresponding point X' for each point X. This arrangement is called one-one correspondence. The teacher wrote on the board 'Two sets are equivalent if there is a one-one correspondence between their elements.' Pupils noted it in their books.

Pupils were asked to compare $E = \{2, 4, 6, \ldots \}$ and $O = \{1, 3, 5, \ldots \}$.

**Idea No. 4:**

Here the teacher through provocative questions made the pupils inquisitive. They made several responses. They made all the possible selections from the three dolls.

- no selection $\{\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2, 3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$

The teacher listed them on the board. These are called subsets they are formed out of the elements of the given set. Next they pondered over the question 'Can a set be subset of itself?'

They searched the list and found $\{1, 2, 3\}$ the set itself.

In the same manner they answered the other questions. Noted the answers

- $\emptyset$ is a subset of every set
- Every set is a subset of itself
- $\emptyset$ is a subset of $\emptyset$
The last question was somewhat strange. Here there is a discrepancy. Can a set be equivalent to itself?

To aid their reasoning the teacher gave an example:

\[ N = \{1, 2, 3, 4, \ldots\} \quad E = \{2, 4, 6, 8, \ldots\} \]

Is \( E \) a subset of \( N \)?

Students said 'yes'. Each element of \( E \) is an element of \( N \).

'How can we decide whether \( E \) and \( N \) are equivalent or not.'

Some one recalled the idea of one-one correspondence.

Then teacher asked the students to make a one-one correspondence. Find the method by which we can get the elements of one set from the other.

To help the students teacher wrote

\[ N = \{1, 2, 3, 4, 5\} \quad E = \{2, 4, 6, 8, 10\} \]

Now the students could see the relation 'from the elements of \( N \) they could get the elements of \( E \) by multiplying by 2.'

\[
\begin{array}{c c c c}
1 & 2 & 3 & 4 \\
2 & 4 & 6 & 8 \\
\end{array}
\]

They noted the correspondence in their books. It is possible with infinite sets.

\[ A = \{1, 2\} \quad B = \{1, 2, 3, 4\} \]

\( A \nsubseteq B \) and \( A, B \) are not equivalent
Idea No. 5:

This idea is related to ordered pairs. Here they are challenged with finding the difference between set \( \{a, b\} \) and ordered pair \((a, b)\).

Pupils give the teams for chess when asked by the teacher. The pairs are listed (Ramu, Gopal), (Venu, Raghu) etc., the first member is from the eighth class and the second member is from the ninth class. 'What is the advantage of this procedure?' is the provocative question.

Pupils discuss, think and reply.

'We will know who belonged to which class'

Next pupils were asked to prepare pairs from sets \( A = \{1, 2\} \), \( B = \{5, 7, 9\} \). They write the pairs in their notebooks.

How many do we get?

Some count and some guess. \( 2 \times 3 = 6 \)

\( \{(1,5), (1,7), (1,9), (2,5), (2,7), (2,9)\} \)

Next they are asked to write starting with \( B \).

This time they were quick

\( \{(5,1), (7,1), (9,1), (5,2), (7,2), (9,2)\} \)

Teacher asks 'are these two sets equivalent?'

They recalled the previous lesson and said 'yes'. The teacher said the first set of ordered pairs is denoted by \( A \times B \) and the second set by \( B \times A \).

Are they equal?
They came up with different answers. Some say 'yes' while some others 'no'.

They gave the reason \((1,5) \neq (5,1)\)
Pupils wrote in their note books
\[ A \times B \neq B \times A \]

Here the teacher has ascribed the order property to members of the pair. Hence the name ordered pair. In a set order is not important \(\{2,3\} = \{3,2\}\)

In this idea pupils studied sets and ordered pairs and came up with the distinction between them.

Idea No. 6:

The goal of this lesson is to develop the concept of Relation. For this the teacher used the strategies attributes, provocative questions and discrepancies.

The teacher writes on the board

<table>
<thead>
<tr>
<th>State</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gujarat</td>
<td></td>
</tr>
<tr>
<td>Andhra Pradesh</td>
<td></td>
</tr>
<tr>
<td>Tamilnadu</td>
<td></td>
</tr>
<tr>
<td>Karnataka</td>
<td></td>
</tr>
</tbody>
</table>

Pupils gave out the names and the teacher filled the blanks.

Can you write them as ordered pairs. Pupils wrote the ordered pairs like (Andhra Pradesh, Hyderabad) etc. and read them.
Can you state the rule?
They said the second name is the capital of the first state in a pair.

Next the teacher gave the set.
(Dasaratha, Rama) etc., father - son relation.

Teacher followed this with a discrepancy:
\{(1,4), (2,5), (3,6), \ldots, (9, \_ )\}

What should be in the gap?
After a while they found out the relation. The second member is than the first member by 3.

Then they gave the answer 12.

They made some relevant responses regarding relations.
\{(1,3), (2,4), (3,5)\}, \{(a,b), (c,d), (e,f)\}

"Are these relations equal? Are they equivalent?" are the provocative questions. This raised their curiosity.

Pupils recorded 'a relation is a set of ordered pairs'
Two sets are needed to make a relation.

Idea No. 7:

Here the teacher employed visualisation skill to develop imagination and original thinking.

He drew a function machine and put numbers 1,2,3 into it. Out of the machine 0,1,2 came up.
What happens if we put 20?

Pupils imagined what was going on inside the machine.

They said the numbers are reduced by 1. Then they argued 20 would get reduced by 1, that is 19 will be the answer.

Next they took a graph paper and marked the ordered pairs.

When they got the set of points the teacher asked them to give an appropriate name. Several names like line, set etc., were given.

The teacher said 'The set of points corresponding to these ordered pairs is called graph.' The set of ordered pairs is called a function. Next they considered the question how a function differed from a relation. What is their similarity?

They took a relation \((1,3), (1,4), (2,5), (2,9)\)
and a function \((1,0), (2,1), (3,2), (4,3)\)

They observed the two sets for comparison. The teacher made a table on the board. The pupils gave replies.

<table>
<thead>
<tr>
<th>Similarity</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Both are sets of ordered pairs.</td>
<td>1. The first coordinates are different.</td>
</tr>
</tbody>
</table>

The teacher wrote "A function is a relation whose first coordinates are distinct."
Pupils copied it in their notes.

Idea No. 8:

After learning that an ordered pair represents a point, in this lesson they studied the idea of an equation associated with a line.

The teacher employed the strategies organised random search and evaluate situations. The students are provided with a marked graph paper and a transparent paper with a black line on it.

The teacher asked the students to mark the point (2,3) which they did easily. Next they marked any point of their choice and placed the line passing through it. For the question how many lines could they get they all replied only one. One of the students said by changing the second point he could get a different line. The pupils exhibited flexible thinking. The teacher concluded the discussion by writing 'Given two points there exists only one line passing through them.' pupils noted down this point in their books.

For the question where did the line intersect the Y-axis each one gave a different answer depending on his choice of the second point. One student said that his line did not intersect the Y-axis.

Next teacher asked the students 'how your line will be if you take (2,7) as second point?'
Pupils made different guesses. Some were correct and some were wrong. The teacher asked them to verify. They found that the line is vertical and parallel to Y-axis.

Then teacher explained the idea of a slope and asked them to find the slopes of their lines which they could do.

Then teacher questioned.

Find the equation to the line joining points (2,3) and (4,5).

Some proceeded to find the slope and the equation of the line using point-slope form. Others used the two-point form. Some guessed and said Y = x + 1. Then they were asked for the reason. They explained that by observing the coordinates of both the points they could say. The pupils exhibited risk taking and flexible thinking.

Idea No. 9:

In this lesson Flexible thinking and imagination were developed through the study of the properties of a pair of lines. Each child was provided with a graph paper. They are asked to mark the coordinate axes. After marking the axes they were encouraged to place the two transperancies with black lines in various positions such they intersect.

The teacher asked the following provocative questions.

What are the equations of the two lines?
Each child noticed where his lines intersected the coordinate axes and gave the equations to those lines. They wrote down the equations in their note books. Some of the pairs of equations were $3x + 2y = 6$ and $2x + 3y = 5$ another such pair was $y - x = 0$ and $y + x = 2$.

Next they were instructed to find the coordinates of the point of intersection. For the above two pairs they gave $(1, 2, 1.2)$ and $(1, 1)$. Each student was asked to find such pairs and their intersection as many as they can in fifteen minutes.

"How is the ordered pair of the point of intersection related to the pair of equations? After some discussion among themselves they said 'The point lies on both the lines. Its coordinates must satisfy both the equations'. They recorded in their note books.

The coordinates of the point of intersection of two lines satisfies both the equations of the lines.

Pupils displayed imagination and flexible thinking.

Idea No. 10:

This lesson further explored the idea of simultaneous equations.

It explores the provocative question 'What happens if the lines do not intersect?'

Pupils gave out various answers: "The lines are parallel."
"There is no ordered pair which satisfies both the equations."

They became inquisitive.

Then the teacher advised them to write the equations of one such pair of lines and observe for any peculiarities. They gave such pairs like

\[
\begin{align*}
2x + 3y &= 4; \quad 2x + 3y = 7 \\
x + y &= 6; \quad x + y = 3
\end{align*}
\]

The teacher listed some of them on the board. The children further improved their answers and said the equations have the same coefficients for \(x\) and \(y\). Another one said the equations are differing in only the constant term.

The teacher wrote their finding on the board.

Equations of parallel lines differ in the constant term.

The next provocative question is "What can you say about their slopes?"

They reflected for a moment and said:

"Two lines are parallel if their slopes are equal"

"If two lines are parallel then their slopes are equal."

The teacher recorded:

"If \(m_1\) and \(m_2\) are slopes of two lines and \(m_1 = m_2\) then the lines are parallel." Also if the lines are parallel then \(m_1 = m_2\). Students noted the proposition in their books.
Idea No. 11:

In this lesson different types of matrices were considered. Teacher used organised random search starting from a familiar example like tea and its various brands. Students used their imagination and gave several responses.

Teacher wrote the table for different brands of Tea. They got an array of numbers.

Next the teacher asked the pupils to make such an array for marks scored by students sitting on each bench. The students using their imagination prepared a table. Thus some ten tables were prepared.

Teacher said "These arrays of numbers are called Matrices."

Then he asked them to find a name for these matrices. Some of them are marks matrix, Tea matrix, Price matrix.

Then teacher said "Since all of them contained information they are known as information matrices."

Suppose you wanted to send a secret message. How do you do it? This is the provocative question.

After a little reflection they came up with several answers.

"Write it in code", "use pictures,"
"use a strange language."
"Now let us prepare a simple code," the teacher said. He wrote the English Alphabet on the board. Beneath each letter he wrote a number:

A B C D E F G H I J K........
1 2 3 4 .. .. ..

After 4 the students took on and completed the code. Then teacher gave the matrix

\[
\begin{pmatrix}
3 & 1 & 7 & 5 \\
2 & 1 & 3 & 11
\end{pmatrix}
\]

He asked the class to find its meaning. They decoded it and said

C A G E
B A C K

Next the teacher asked them to write in code

G I V E
M O R E
F O O D

The students could write easily. "Since the matrix is in code it is called code matrix" the teacher said.

The teacher said 'the following figure shows how Delhi, Hyderabad and Madras are connected.' He drew the figure on the board. Then he made a blank table.

<table>
<thead>
<tr>
<th></th>
<th>Hyd</th>
<th>Del</th>
<th>Mad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyd</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Del</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mad</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The teacher asked the students to fill the nine cells with numbers showing number of paths connecting the two towns. As an example he filled the last cell on the first row with 2. The students gave the relevant numbers immediately.

What do we call this matrix?
They said "path matrix", "route matrix."

Idea No. 12:
This idea explains the use of matrices for writing simultaneous equations. The teacher used organised random search and skills of search for this purpose. The teacher wrote on the board.

\[ 5x + 2y = 7 \]
\[ 3x + y = 4 \]

and waited for a few seconds. The pupils became curious. The teacher asked What do they represent?

Most of the pupils said 'They represent a pair of lines.'

The next question was what can you find out?
Some said 'We can find the line', 'we can find their point of intersection' etc.

Here the teacher used the skills of search "Can you show the equations in another form?"

The students wrote them in their books and showed the results. Some of them were:
The teacher further questioned them what about the variables x, y?

They improved their answer and wrote

\[
\begin{pmatrix}
5 & 2 \\
3 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
=
\begin{pmatrix}
7 \\
4
\end{pmatrix}
\]

Then teacher showed them the proper way of writing

\[
\begin{pmatrix}
5 & 2 \\
3 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
=
\begin{pmatrix}
7 \\
4
\end{pmatrix}
\]

Then they wrote some examples of their own and changed them into matrix form.

Similarly they were successful in writing the equations when the matrix form was given.

**Idea No. 13:**

In this lesson the teacher used analogies and discrepancies to teach matrix addition.

The teacher wrote two matrices on the left side of the board and two numbers on the right side:

\[
\begin{pmatrix}
5 & 2 \\
1 & 3
\end{pmatrix},
\begin{pmatrix}
3 & 4 \\
2 & 1
\end{pmatrix}
\]

7, 8

"What are the various things that you can do with numbers 7, 8?" the teacher asked.
Pupils gave a number of replies:

"add", "subtract", "multiply", "take the power like \(7^8\)," "Divide \(\frac{7}{8}\)." Thus they generated a number of ideas. Next teacher asked "Can you add the two matrices on the board."

They gave answers, some of them funny:

\[
\begin{pmatrix}
5 & 2 & 3 & 4 \\
1 & 3 & 2 & 1
\end{pmatrix}, \quad
\begin{pmatrix}
5 & 2 \\
1 & 3 \\
3 & 4 \\
2 & 1
\end{pmatrix}, \quad
\begin{pmatrix}
8 & 6 \\
3 & 4
\end{pmatrix}
\]

They discussed about the answers. One student said "the first could not be called addition." Finally they agreed with the third. The teacher said that was the proper method.

Next the teacher told the students to write a few matrices and add. This they did in their note books and showed the results. They were correct.

Then they faced the discrepancy. The teacher told the students to add:

\[
\begin{pmatrix}
1 & 2 \\
3 & 4
\end{pmatrix} + \begin{pmatrix}
5 & 4 & 6 \\
7 & 8 & 9 \\
3 & 4 & 2
\end{pmatrix}
\]

After a while they said they could not do it.

The teacher gave another example:

\[
\begin{pmatrix}
1 & 2 & 4 \\
5 & 6 & 7
\end{pmatrix} + \begin{pmatrix}
3 & 4 \\
5 & 6 \\
9 & 2
\end{pmatrix}
\]

Again they said 'it cannot be done'. They probed more into the matter and said 'They must be of the same kind.' They must have the same number of rows and columns.'
In this lesson students displayed fluent thinking and risk taking.

Idea No. 14:

This lesson teaches matrix multiplication. Organised random search and attributes are the strategies used by the teacher.

The teacher started with a very common place example and proceeded with the method of matrix multiplication. He wrote on the board two tables.

<table>
<thead>
<tr>
<th>Potatoes</th>
<th>Brinjals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gopal</td>
<td>2 kg.</td>
</tr>
<tr>
<td>Krishna</td>
<td>4 kg.</td>
</tr>
</tbody>
</table>

Teacher asked 'How much Gopal paid at market X?' 'How much at market y?' Pupils calculated the amounts in their note books.

Some of them wrote

\[ 2 \times 3 + 3 \times 2 = 6 + 6 = 12. \]
\[ 2 \times 2.50 + 3 \times 3.00 = 5 + 9 = 14 \text{ etc.} \]

'Write it in the matrix form' the teacher suggested.

After some effort they could write

\[
\begin{bmatrix}
x & y \\
gopal & 12 & 14 \\
krishna & 22 & 25 \\
\end{bmatrix}
\]

Next the teacher asked them to take matrices like

\[
\begin{pmatrix}
1 & 3 \\
5 & 2 \\
\end{pmatrix}, \quad \begin{pmatrix}
7 & 4 \\
9 & 2 \\
\end{pmatrix}
\]

and multiply.
Students wrote their own examples did the multiplication and shown the results to the teacher. They were correct.

In this lesson the teacher started with the common place example of finding the cost of vegetables and proceeded to find the product of $2 \times 2$ matrices.

Students could easily switch on to the intricate method of forming inner products. One student asked 'why cannot we just multiply the corresponding element?' just as we did in addition?' The students could not answer that doubt. The teacher said that such a procedure could not solve the market problem.

Idea No. 15:

This lesson is about inequations. Students are quite familiar with equations and here they learn about inequations. Teacher drew the following diagram:

The students drew the figure in their graph papers.

What is the equation of L?

Some of the answers were

$$\frac{x}{2} + \frac{y}{2} = 1,$$
$$2x + 3y = 6,$$
$$2x + 3y - 6 = 0$$
All the forms were correct. "Put the values (0,0), (1,1), (3,2) in the expression 2x + 3y - b". They gave the answers: -6, -1, 0

"What do they indicate?" Pupils discuss among themselves use their imagination and said "the points (0,0), (1,1) do not belong to the line. (3,2) is on the line."

Then teacher advised 'mark the points (0,0) and (1,1) See how they are!'

The pupils after marking the points found them below the line.

How did the point divide the plane? How many parts?
Some replied: 'two' others said 'three'.
They said "the points below the line, the points on the line, the points above the line."

The teacher said "The two regions are called half planes."

"Mark the half plane below the line as $H_1$ and above the line $H_2$. What could be the equation of $H_1$?"

The students found it difficult to answer this question. Then teacher gave the cue. There are three possibilities:

$2x + 3y < 6$, $2x + 3y = 6$, $2x + 3y > 6$

'What could be $2x + 3y = 6$?' Pupils said that it was the line. "What about the first and third." "They must be that of the half planes $H_1$ and $H_2"$ guessed some. This lesson produced flexible thinking through imagination.
Idea No. 16:

This lesson deepens the idea of an inequation and its associated half plane. The teacher employed the strategies organised random search and provocative questions to develop elaborative thinking and complexity.

The Teacher wrote on the board:

- point $(3,4)$, slope $3$
- Equation to the line?

Pupils did the calculation in their note books and replied. The first wrote the general form $y - y_1 = m(x - x_1)$ and substituted the values to get the equation $y = 3x - 5$ or $y - 3x + 5 = 0$.

Then the teacher suggested to them to draw the line on a graph paper. This all of them could do correctly and checked their work with their teacher. They shaded the half plane containing origin.

Next came the provocative question "What is the inequation that is associated with the shaded half plane?"

The students have two possibilities before them:

- $y > 3x - 5$ or $y < 3x - 5$

Each one represents one half plane.

The pupils argued since the shaded half-plane has the origin the inequation must be satisfied by $(0,0)$. The substitution led to:

- $0 > -5$ or $0 < -5$
The second statement was false. They concluded $y > 3x - 5$ is the required inequation.

To further develop the idea the teacher asked the students to choose a point and a slope which they noted in their note books. With this data they framed a problem. Some chose the half plane containing origin and some the half plane not containing the origin. Most of them could solve their problem successfully.

Idea No. 17:

In this lesson maximisation of a function $f(x,y) = 2x + 3y$ subject to certain restrictions like $x > 0$, $y > 0$, provocative questions and analogies are the strategies employed. Pupils were asked to mark the I quadrant. Then teacher asked "what inequations are satisfied?"

Some students complained
"We don't understand the question." Then teacher reframed:
"What can you say about the points in the first quadrant?"

Most of them answered both of its coordinate, namely $x$ and $y$ are positive.

Some of them wrote
$x > 0, \ y > 0$

Through provocative question
'How do you form a quadrilateral with the two axes?'
The pupils argued since a quadrilateral has four sides two more lines are needed.

Then on his graph paper each one drew two other sides. 'What are their equations?' was the next question they tried to answer.

They could find the equations of the two sides. They could also find the four vertices of the quadrilateral, and shaded the region. Starting with origin the teacher found the values of $f$ at two or three different points. The provocative question

'What could be the point at which $f(x, y) = 2x + 3y$ is maximum?'

Some of the pupils tried some more points. Some thought are of the vertices should work.

Teacher asked them to verify their guesses which they did and accordingly accepted or rejected their guess.

Here the students were confronted with a complex situation. The quadrilateral region contained an infinite number of points. How to identify the correct one? They came to the conclusion that direct checking is not possible.

One student substituted the coordinates of the origin $(0, 0)$ and got the minimum $0$. Hence argued the student the farthest point in the region should give the maximum.

They checked the opposite vertex and found the answer.
Idea No. 18:

In this lesson application of inequations for solving simple problems was considered. Teacher employed organised Random Search and examples of change to produce elaborative thinking and complexity.

Teacher wrote the following problem:

A person has Rs. 15 with which he wants to buy pens and ball pens. The cost of a pen is Rs. 5. A ball pen costs Rs. 2. The total number of articles purchased should not exceed 6. How many pens and ball pens could he buy?

The teacher said "We don't know the number of pens and number of ball pens. So let us call them x and y. Using these x, y formulate the problem." The teacher used the organised random search. Can x take values 3, 4, -5? Pupils thought over and x and y must be positive integers. x > 0, y > 0.

The next question was

'What can you say about x + y'

Some students had difficulty in understanding the question. The teacher posed another question 'What is meant by x + y'. They could recognise it as the total number of articles purchased. They said x + y ≤ 6. Teacher recorded the statement on the board.

'What can you say about the total cost?' They said "It cannot exceed Rs. 15."

"State it in a mathematical form" is the challenge.

After a while they came up with the inequation

\[5x + 2y \leq 15.\]
Then they went on to represent the situation on a graph paper. Most of the students could get the quadrilateral region and shaded it.

Next they searched for the possible values of $x$ and $y$ which are in the shaded region.

$x - 0, 1, 2, 3$
$y - 6, 5, 2, 0$

which is the best combination?

They discussed the various possibilities:

(0, 6) he cannot buy a pen.
(3, 0) he cannot buy a ball pen and also the total number of articles is only three
(2,2) Though they could get both still the total number of articles is only 4.
(1, 5) Here are gets both the articles and also the maximum number of articles allowed and hence the best solution.

The students displayed elaborative thinking by improving their answer.

Idea No. 19:

This idea develops elaborative thinking and complexity through statistics. The teacher used the strategies organised random search and discrepancies.

Teacher wrote the following on the board:

40, 45, 50, 52, 52, 60, 72

What is the average? Can you guess?
They are quite familiar with the notion of average. They made a guess.

The responses are 40, 45, 52, etc.

Then there was some discussion. They thought 40 was too small. 52 was some where in the middle. It was good. They felt.

Then the teacher asked how much each value differed from 52.

They wrote the deviations:
-12, -7, -2, 0, 0, 8, 20

What is the total deviation?
The sum was found 7.

The average of deviations is \( \frac{7}{7} = 1 \).

This much must be added to our guess.
Hence \( \bar{x} = 52 + 1 = 53 \).

'Can you check it,' the teacher asked. The students used the familiar method and got the result.

Next the teacher asked 'what will happen if we chose 40, or 60 as our guess.' Some said 'we get a wrong answer.'

'It does not matter' etc. How can one decide which answer is correct?

They repeated the procedure with 40 and 60 and got the same result. They came to the conclusion they could use any number for assumed mean and could get the correct answer.
Here the pupils filled the gaps in their knowledge through trial and verification. Also they could improve their usual method of finding the mean. In this method they are dealing with smaller numbers than those in the data. They sought alternatives in the choice of assumed mean.

Idea No. 20:

This lesson deals with standard deviation. The teacher used analogies and skills of search.

The marks scored by seven students are
40, 45, 50, 52, 52, 60, 72

What is their mean?

Students recalled the earlier method, found the deviations and the mean. The answer was 53.

Next the teacher asked them to find the deviations from the arithmetic mean that is 53. The teacher noted the means on the board as the students read out.

-13, -8, -3, -1, -1, 7, 19.

What is the sum? Zero.

Will it be so in every problem?

Some guessed 'yes' others were doubtful. The teacher asked those who doubted to try a problem of their choice.

They found that it was again zero. Now they were convinced.

'To avoid this let us square and add' the teacher said.
What is the mean of square deviations. This they found by dividing the sum by 7.

They got 93.42.

The teacher said "Since we have squared the numbers in the beginning it is reasonable now to take the square root."

They found it easily as they were familiar with the methods of finding square root.

The teacher wrote Standard deviation \( \sigma \) = 9.66.

Here the teacher used the method similar to the earlier method of finding arithmetic mean. Pupils improved the situation of getting the sum zero by squaring the deviations. They used their imagination in again finding the square root.

**Idea No. 21:**

This is a lesson in Algebra, namely permutations. The pupils considered the number of ways in which a group of \( n \) different things could be arranged. An arrangement of a different objects in a given order is called a permutation of the \( n \) objects.

The teacher started with a set of three objects namely pens of colours Red, Blue and Green. Now the teacher asked them 'take two pens and arrange them in different ways.'

Students use their imagination and make the following symbols.
R for a red pen, G for a green pen and B for a blue pen.

Then they wrote the different arrangements in their note books and showed them to the teacher.

They are RB, RG, BG, BR, GR, GB. The number is 6.

The teacher said 'Each arrangement is called a permutation. Here the number of permutations is six. The number of permutations of 2 things out of 3 is 6.'

This can be written in a short form

$3P_2$ or some people write \( P(3,2) \)

Some students said the coordinates of \( P \) is \((3,2)\). To avoid confusion the first method was adopted.

Next the students imagined the value of $4P_2$. They made some guesses and then checked them by writing out the permutations with four symbols \( \{a, b, c, d\} \).

Next the class considered the problem of seating three persons in three chairs. This time instead of writing down all the permutations the teacher used provocative questions and pupils used their imagination and gave answers.

In how many ways can you fill the first chair?

"Three ways. Any one of the three could sit."

In how many ways can the second chair be filled?
"Since already one person is sitting in the first chair it can be done in two ways."

The last case was simple. The remaining person sits.

The teacher asked them to answer the original question.

\[ 3 \times 2 \times 1 = 6 \] they said.

The teacher said 'there is a special symbol for this product because we have to write such products often.' He wrote on the board:

\[ 3! \] "Three factorial"

What is 5! ?

What is 2 !, 1! and 0!?

The children have no trouble in replying the first question 2, and 1.

The question regarding 0! is a challenge. Some of the responses are:

0, it cannot be found, has no meaning etc.

Then teacher said "0! is defined as 1 by mathematicians. Such a special definition is useful in calculations."

Pupils recorded that in their note books.

In this lesson pupils developed fluent thinking and Imagination.

The teacher asked provocative questions and used analogies in defining n!
Idea No. 22:

This idea develops Flexible thinking and complexity. The strategies used are organised random search and evaluate situations.

The lesson started with the formulae for \((a + b)^2\) and \((a + b)^3\) with which the students were quite familiar from their study of algebra in earlier classes.

The teacher wrote them in the middle of the board. Then he proceeded to write the other lines as shown:

\[
\begin{align*}
(a + b)^0 &= 1 \\
(a + b)^1 &= a + b \\
(a + b)^2 &= a^2 + 2ab + b^2 \\
(a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
(a + b)^4 &= \\
\end{align*}
\]

Then he wrote on the right side the coefficients. Then the teacher asked the students to fill the gaps in the right side array. Now they had an intricate problem. A sort of a puzzle. They could see the pattern and said the second row contains two numbers and they must be ones. The first one was quite easy it has to be 1.

Then the teacher asked them to explore the further row. This was a bit intricate.

The teacher asked 'How many terms must be there in the fourth row?'
From the pattern they could guess it as 5. Someone said 'The first and last must be ones.' The teacher filled in as they answered. Next they tried to fill the gap below 1 and 3. They studied the second, third rows for clues. One girl student said '4'. It was rapidly followed by others 6 and again 4.

The teacher related a little history relating the number array. In ancient sanskrit books it was called 'Meru Prastav.' It helps in writing down the coefficients of a binomial expansion.

Next the pupils took up the expansions on the left-side and filled the gaps. As their imagination caught on the proceeded to write the expansion of \((a + b)^5\).

**Idea No. 23:**

The lesson teaches the order relation. Here the students try to decide the larger of the given two numbers. The organised random search began with two positive numbers. This was quite easy.

For example

| 1, 3 | Three larger than 1 |
| 10,12 | Twelve larger than 10 |

The teacher listed some more examples and asked them to represent symbolically. They wrote \(3 > 1, 12 > 10\) etc., in their note books and showed their work for scrutiny. It was correct.

Next they were confronted with the pair -2, 0. There was confusion. They thought zero is nothing it must be smaller.
Teacher said "let us take the help of a number line. Mark the numbers we were discussing."

Each drew a number line in his/her note book and marked the numbers they worked with earlier.

The teacher said "observe all the cases what do you find?"

After a while some pupils remarked the larger number is always on the right.

Then teacher said use this test. They could mark the positions -2, 0 and said that 0 was greater than -2.

The teacher gave some more examples like -30, -10; -5, 2 etc. The pupils now answered even without looking at the number line. They could visualise the relative positions.

Here teacher used organised random search when he began with a familiar example like 10, 12 and went on to explore negative integers and zero. Pupils used imagination and improved their idea of small and great. They exhibited elaborative thinking.

Idea No. 24:

This lesson further expands the concept of order relation. Teacher through strategies like skills of search and visualisation skill produces complexity and flexible thinking behaviours in children.
Teacher writes on the board 36 > 24. What is the relation between them? Pupils from their past experience say 'greater than'

36 > 24

"What happens if we add say 10?" Pupils replied that, we get 46 and 34. What did they observe? The order relation remained the same.

46 > 34

Teacher asked them to try a few examples like

3 > -5
-2 > -9 etc.

After a few examples the students came to the conclusion that adding a number does not alter the order relation.

Teacher asked 'Can you state your conclusion using 0, b, c for numbers?' They thought over the problem, compared the method used in the previous cases and wrote

a > b, for any c, a + c > b + c

Next they proceeded to see what happened in the case of multiplication.

Teacher said 'Take any pair of numbers with an order relation.' He chose

15 > 12

"Let us multiply by 2 and see what happens"

Pupils got 30 and 24. They also observed 30 > 24 and the order relation remained.
Teacher asked 'what happens when multiplied by -2.'
Some hesitated. Some quickly said 'remains the same.'
To settle the issue teacher advised them to multiply.
They got -30 and -24.
What is the order relation? It has changed
-30 < -24 the students said.

Now the teacher asked them to explore a few more cases
and state the result. After a little reflection they said.

"When multiplied by a positive number the relation is unaltered. When multiplied by a negative number the direction is changed." Then the teacher urged them to state it in terms of a, b, c.

After a few attempts they could write in their books.

If \( c > 0 \), \( a > b \) then \( ac > bc \)
If \( c < 0 \), \( a > b \) then \( ac < bc \)

The teacher used skills of search in finding the relation after the addition and multiplication. Students exhibited flexible thinking and complexity in exploring the problem.

Idea No.25:

The content is the meaning of logarithm. Teaching strategies are analogies and skills of search.

Teacher started the search by writing some numbers as powers:
\[ 16 = 2^4, \quad 81 = 3^4, \quad 100 = 10^2 \]
Pupils could readily say the numbers that went into blanks.
Then teacher said 'mathematician found a different way of saying the above things.'
He wrote:

\[
\log_2 16 = 4, \quad \log_3 81 = 4 \quad \log_{10} 100 = 2
\]

2, 3, 10 etc. are called bases. The numbers on the right are called logarithms. Then teacher wrote on the board.

If \( \log_a N = x \) then \( N = \quad \)

Pupils became curious. They looked at the previous examples for clues. Finally they said \( N = a \)

Next teacher asked 'take the number 256 and write its logarithm to base 2, 4, 16.' They explored first expressing as a power

\[
256 = 2^8, \quad 256 = 4^4, \quad 256 = 16^2
\]

\[
\log_2 256 = 8, \quad \log_4 256 = 4, \quad \log_{16} 256 = 2
\]

Next they were confronted with the question could they use 1 for a base.
They tried \( 4 = 1 \) etc. found it could not be done.

Then the teacher asked them to refine the formula using conditions.

\( a > 0, \quad a \neq 1, \quad N > 0 \) if \( \log_a N = x \) then \( N = a^x \)

Then teacher explained that e and 10 are mostly used.
e is an irrational number like
Here pupils exhibited curiosity in trying to write 4 as a power of 1. They also displayed flexible thinking in writing a definition of logarithm.

Idea No. 26:

This is a lesson in Geometry using the strategies skills of search, visualisation and attributes. Here the alternate angle property of parallel lines was investigated.

The pupils toyed with the transperancies having lines. They manipulated them as they liked. They copied the shapes they got in their note books. After looking into their note books the teacher copied some of them on the board:

Teacher marked angles a, b in the figures (i) and (ii). He asked the pupils to measure them.

They measured and found the angles to be equal in figure (i) and unequal in figure (ii).
What did you observe?

They said 'The lines are parallel in the first case. They are not so in case (ii).'

Teacher told them that these angles have got a special name alternate angles.

State the result in a sentence.

After some effort they said.

If two lines are parallel and another line cuts them, then the alternate angles are equal.

In this lesson students developed imagination and flexible thinking. They were building mental images while they played with the three black lines producing different figures. Then they categorised the figures as those having parallel lines and others not having a pair of parallel lines. They could also imagine the need of a third line for forming alternate angles.

Idea No. 27:

This lesson is about concurrency of the attitude of a triangle. The teacher provided the pupils with triangles of appropriate size.

Here the teacher mainly relied on the strategies of visualisation and skills of search. The pupils became curious were eager to know what to do.
Then teacher advised them to fold it along the dotted line which was an altitude. The pupils observed the fold carefully. After unfolding they were asked to repeat it for other vertices.

Some of the students who were more observant could do it easily. Others needed the help of their teacher.

In the end they found all the creases met at a point. Then teacher said 'the point is called orthocenter' and wrote it on the board. Then they were asked to state the result. After some guessing and changes they could state.

"The altitudes of a triangle meet at a point called orthocenter."

During the lesson pupils were inquisitive and tried to find the correct method of folding, they thought of different ways of folding. Finally they could find the right method.

Idea No. 28:

This lesson explores another concurrency property of the triangles. The strategies used are visualisation and skills of search. The activity is somewhat similar to the idea No. 27. But the fold must be the perpendicular bisector of a side. Pupils are asked to fold the triangle at the middle point of a side such that the two parts coincided. This gave the perpendicular bisector.
Without any more instruction from the teacher they folded the other two sides. They said that the three perpendicular bisectors met at a point. The teacher gave the name **circumcenter**.

He wrote it on the board. Students after some attempt stated the results as

The perpendicular bisectors of the sides of a triangle meet at a point called circumcenter.

The teacher checked the notes and corrected the statements wherever necessary.

This idea developed flexible thinking and curiosity in the children.

**Idea No. 29:**

This lesson is about the concurrency of medians of a triangle. The strategies used were visualisation skill and skills of search. When asked the pupils could recall that median is the line joining the middle point of a side to its opposite vertex.

To help them the teacher provided a dotted line on the triangle supplied to them. When once they saw how the fold was they could easily do the other two.

They found that the medians were concurrent. Teacher wrote on the board **centroid**.
Finally pupils stated

The three medians of a triangle meet at a point called centroid.

The inquisitiveness of the students in folding and the attempts they made in writing the statement developed flexible thinking.

Idea No. 30:

It deals with another concurrency property. In this lesson it was established that the angle-bisectors of a triangle are concurrent. Students carried the paper folding activity with much enthusiasm. Since it is the angle that should be bisected the paper was folded at the vertex such that the two sides of the angle coincided.

The students after carrying out the paper folding activity found the meeting point. The teacher told them that it was called incenter. The pupils stated the result in their note books which were checked by the teacher.

Pupils could think of the various kinds of lines like altitudes, medians, and perpendicular bisectors. They displayed inquisitiveness in knowing and doing things.
5.4 Summary:

This experiment was conducted with a two group design. There were four independent variables each of two levels. $2^4$ Factorial Design was used. The time schedule and execution of the programme were described in this chapter. The experiences of the investigator, the reactions of the students and how the teacher strategies worked were described in detail for each idea. The whole programme went on smoothly and the pupils expressed satisfaction of the teaching.