CHAPTER 10

VALIDATION OF THE MATHEMATICAL MODELS USED TO PREDICT BENDING RIGIDITY OF THE SILK CREPE-DE-CHINE FABRICS

10.1 INTRODUCTION

A lot of research is in progress to study the mechanical properties of the woven fabrics largely as a result of the introduction by Kawabata (1980) of set of commercial instruments for measuring the properties. The availability of these instruments has helped the research workers to engineer the design of the fabrics, keeping in view their end use requirement. Leaf et al., (1993) have suggested a model for predicting the bending rigidity of the fabrics based on the data of the various samples woven from cotton, polyester, wool and blends thereof. They help in engineering of the fabric to the specific requirement.

Conceptually, the bending of yarns originates from a pure bending of a symmetrical solid beam in classical mechanics. However, the bending of yarns differs from a solid structure. The fibers in the yarn are assumed to bend independently of each other so that the yarn bending resistance is given as the sum of the bending resistances of individual fibers and is not proportional to the moment of inertia. Thus,

\[ B = M \rho \text{ or } B = \frac{M}{K} \]  

...(10.1)

Where,

\[ B = \text{Flexural rigidity of the yarn or fabric} \]
Grosberg (1966) analysed the bending of a set of parallel plates compressed together, and fibers in the yarns analogous to these plates. The mathematical representation of this approximately is:

\[
K = \begin{cases} 
0 & \text{if } M \leq M_0 \\
\frac{(M - M_0)}{B} & \text{if } M > M_0 
\end{cases} \quad \text{(10.2)}
\]

Where,
\[
K = \text{yarn curvature} \\
M = \text{applied moment} \\
M_0 = \text{the frictional couple} \\
B = \text{yarn Flexural Rigidity}
\]

Abbott et al. (1973) showed a better approximation to the initial bending where the friction is not overcome, as

\[
K = \begin{cases} 
M - aM_0 & \text{if } M < 2M_0 \\
\frac{(M - M_0)}{B} & \text{if } M \geq 2M_0 
\end{cases} \quad \text{(10.4)}
\]

Where,
\[
a = \frac{M}{M_0} - \frac{1}{4} \left( \frac{M}{M_0} \right)^2 \quad \text{(10.6)}
\]

More recently, Huang proposed a modification of Grosberg's idealisation of yarn bending by suggesting bilinear relation.

\[
K = \begin{cases} 
\frac{M}{B'} & \text{if } M < Ma \\
\frac{Ma}{B'} + \frac{(M - Ma)}{B} & \text{if } M > Ma 
\end{cases} \quad \text{(10.7)}
\]
Where,  

\[ B' \text{ and } B \text{ are initial & final bending rigidities} \]

\[ B' > B \]

\[ M_a - \text{transition couple related to frictional couple by the relationship.} \]

\[ M_a = \left( \frac{M_o}{1 - B/B'} \right) \quad \text{...(10.9)} \]

It can be seen that the assumption of linear bending is reasonably valid for small bending deformations whereas Grosberg's idealisation of infinite bending rigidity is too simple.

Abbott's (1973) model, although reasonably accurate, is difficult to use owing to mathematical complexities arising from non-linear equations. However Huang's (1979) proposal of bilinearity seems to be reasonably accurate or yet simple to apply. In recent years, attention has been focused on the mechanical properties of the woven fabrics, largely as a result of the introduction by Kawabata (1980) of a set of commercial instruments for measuring the properties. Notable workers in this field are Grosberg, Leaf and Chen. The work of Leaf and Clement (1986) and Leaf et al (1993) makes use of the results of analysis of tensile, bending and shear properties of plain woven fabrics, based on simple models of the fabric analysis and an error in Kandil's work has been pointed out. Leaf, Yan Chen and X. Chen (1993) have given an alternative analysis of the bending behaviour of the plain woven fabrics.

Data on the application these models to predict bending rigidity of the silk fabrics is sparse. Hence, this chapter deals with the validation of the revised model suggested by Leaf and his co-workers (1993) for predicting the fabric flexural rigidity. The ratio of fabric flexural rigidity per thread to yarn flexural rigidity (i.e. \( B/b_y \)) obtained using the method suggested by Leaf
and his co-workers (1993); and Grosberg and his co-workers (1971) were compared for two sets of data on the laboratory woven Crepe-de-chine Group 1 and Group 2 fabrics.

10.2 MATERIALS

The laboratory woven Crepe-de-chine Group 1, and Group 2 fabrics whose constructional details are given in chapter 4 (Table 4.1) were used. The geometrical and physical properties of these fabrics such as fabric sett, weight, crimp, modular length, weave angle, bending rigidity (yarns and fabrics), linear density etc. have been systematically analysed. Data on these properties along with the data obtained by cyclic hysteresis bending, and Kawabata evaluation system for fabrics (KESF) bending, which has been reported in chapters 5 and 6 for these fabrics are used.

10.3 METHODS

Experimental data on the bending rigidity by KESF and cyclic hysteresis bending is used in predicting the bending rigidity of the fabrics. The experimental data on bending parameters is shown in the Table 10.1. The predicted bending rigidity by application of various model has been shown in Table 10.2.

10.3.1 Determination of the bending rigidity using the model of Leaf et al, (1993)

Leaf, Yan Chen and X.Chen (1993) used a linear 'saw tooth' model of plain woven fabric as a basis for the analysis.

In what follows suffixes 1 and 2 denote warp and weft respectively.

\[ T_y = \text{Linear density of fibre in tex} \]
\[ p = \text{thread spacing in mm} \]
<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Fabric Samples</th>
<th>Yarn lineal density (dx)</th>
<th>Thread Spacing x 10^3 (mm)</th>
<th>Crimp % Cc</th>
<th>Modular length x 10^3 (mm)</th>
<th>Constant W measured by least square method</th>
<th>Yarn flexural rigidity (gN/m^2)</th>
<th>Calculated Contact length (mm)</th>
<th>Fabric flexural rigidity mN/mm^3</th>
<th>Calculated flexural rigidity mN/mm^3</th>
<th>Leaf and his co-workers model</th>
<th>Abbott's model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>N1</td>
<td>N2</td>
<td>P1</td>
<td>P2</td>
<td>Cc</td>
<td>Cc</td>
<td>I1</td>
<td>I2</td>
<td>K1</td>
<td>K2</td>
<td>b1</td>
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<td>I.</td>
<td>Group 1: Light weight crepe-de-chine samples</td>
<td>1. Sample A</td>
<td>20</td>
<td>44</td>
<td>9.40</td>
<td>2.20</td>
<td>13.68</td>
<td>7.70</td>
<td>25.28</td>
<td>10.16</td>
<td>0.85</td>
<td>0.51</td>
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<tr>
<td></td>
<td></td>
<td>2. Sample B</td>
<td>20</td>
<td>46</td>
<td>8.90</td>
<td>2.20</td>
<td>15.60</td>
<td>9.20</td>
<td>25.70</td>
<td>9.73</td>
<td>0.89</td>
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<td></td>
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<td>20</td>
<td>52</td>
<td>8.80</td>
<td>2.20</td>
<td>15.63</td>
<td>14.32</td>
<td>25.70</td>
<td>10.10</td>
<td>0.85</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Sample D</td>
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<td>56</td>
<td>8.70</td>
<td>2.20</td>
<td>15.72</td>
<td>21.80</td>
<td>25.72</td>
<td>10.53</td>
<td>0.82</td>
<td>0.30</td>
</tr>
<tr>
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<td></td>
<td>5. Sample E</td>
<td>20</td>
<td>76</td>
<td>8.60</td>
<td>2.20</td>
<td>16.40</td>
<td>22.64</td>
<td>25.89</td>
<td>10.52</td>
<td>0.90</td>
<td>0.29</td>
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<td>II.</td>
<td>Group 2: Medium weight crepe-de-chine sample</td>
<td>1. Sample A</td>
<td>27</td>
<td>58</td>
<td>7.60</td>
<td>2.40</td>
<td>12.80</td>
<td>7.70</td>
<td>26.85</td>
<td>8.14</td>
<td>0.88</td>
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<td>5. Sample E</td>
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<td>2.40</td>
<td>14.80</td>
<td>11.30</td>
<td>27.33</td>
<td>7.60</td>
<td>0.81</td>
<td>0.26</td>
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TABLE 10.2  COMPUTATION OF THE BENDING PARAMETERS OF THE CREPE-DE-CHINE FABRICS BY USE OF BENDING RIGIDITY OBTAINED FROM KESF BENDING TESTER

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Fabric Samples</th>
<th>Yarn linear density dtx</th>
<th>Diameter of the yarn x 10^4 mm</th>
<th>Thread spacing x 10^3 mm</th>
<th>Modular length x 10^3 mm</th>
<th>Yarn flexural rigidity mN/mm²</th>
<th>Constant 'K' measured by least square method x 10^4</th>
<th>Contact length (mm)</th>
<th>Fabric flexural rigidity mN/mm²</th>
<th>Calculated flexural rigidity mN/mm²</th>
<th>Leaf and his co-workers model</th>
<th>Abbott's model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Group 1: Light weight crepe-de-chine samples</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1. Sample A,</td>
<td>20</td>
<td>44</td>
<td>6.50</td>
<td>8.10</td>
<td>9.40</td>
<td>2.20</td>
<td>25.26</td>
<td>10.16</td>
<td>0.160</td>
<td>0.252</td>
<td>83.36</td>
<td>17.52</td>
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<tr>
<td>2. Sample B,</td>
<td>20</td>
<td>46</td>
<td>5.60</td>
<td>8.30</td>
<td>9.90</td>
<td>2.60</td>
<td>25.70</td>
<td>10.90</td>
<td>0.160</td>
<td>0.248</td>
<td>75.14</td>
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<td>3. Sample C,</td>
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<td>5.70</td>
<td>8.60</td>
<td>9.80</td>
<td>2.60</td>
<td>25.70</td>
<td>10.40</td>
<td>0.160</td>
<td>0.239</td>
<td>81.23</td>
<td>14.68</td>
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<td>5.70</td>
<td>9.30</td>
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<td>25.72</td>
<td>10.55</td>
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<td>0.185</td>
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<td></td>
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<td>9.90</td>
<td>7.60</td>
<td>2.40</td>
<td>26.85</td>
<td>8.14</td>
<td>0.164</td>
<td>0.356</td>
<td>4.24</td>
<td>5.33</td>
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<tr>
<td>2. Sample B,</td>
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<td>6.40</td>
<td>9.90</td>
<td>7.40</td>
<td>2.40</td>
<td>26.97</td>
<td>8.11</td>
<td>0.164</td>
<td>0.354</td>
<td>4.66</td>
<td>7.00</td>
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<tr>
<td>3. Sample C,</td>
<td>27</td>
<td>71</td>
<td>6.60</td>
<td>10.40</td>
<td>7.10</td>
<td>2.40</td>
<td>27.33</td>
<td>7.82</td>
<td>0.184</td>
<td>0.255</td>
<td>1.86</td>
<td>13.33</td>
</tr>
<tr>
<td>4. Sample D,</td>
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<td>82</td>
<td>6.50</td>
<td>11.20</td>
<td>8.60</td>
<td>2.40</td>
<td>27.33</td>
<td>7.51</td>
<td>0.184</td>
<td>0.246</td>
<td>8.15</td>
<td>8.56</td>
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<tr>
<td>5. Sample E,</td>
<td>27</td>
<td>85</td>
<td>6.50</td>
<td>11.40</td>
<td>6.60</td>
<td>2.40</td>
<td>27.33</td>
<td>7.60</td>
<td>0.184</td>
<td>0.230</td>
<td>12.70</td>
<td>4.55</td>
</tr>
</tbody>
</table>

Note: Flex. Rig = Flexural rigidity, B, = Measured flexural rigidity, B, = Calculated flexural rigidity

* = B(1-0.56C), ** = B(1-0.56C)
The bending rigidity of the fabric is given by

\[ B_{t1} = \frac{b_1 p_2}{p_1 (l_1 - 2c_1)} \text{ for warp way} \]  
...(10.10)

and \[ B_{t2} = \frac{b_2 p_1}{p_2 (l_2 - 2c_2)} \text{ for weft way} \]  
...(10.11)

In order to estimate the fabric flexural rigidity, the values of contact length should be known. These are given by

\[ c_1 = K_1 (d_1 + d_2) \Theta_1 \]  
...(10.12)

\[ c_2 = K_2 (d_1 + d_2) \Theta_2 \]  
...(10.13)

Where, \( K_1 \) and \( K_2 \) are multipliers; a satisfactory approximation for the weave angle \( \Theta \) is

\[ \Theta = 1.85 C_y^{\frac{1}{2}} \]  
...(10.14)

and \( C_y1 = (l_1 - p_2)/p_2, C_y2 = (l_2 - p_1)/p_1 \)  
...(10.15)

\[ d = 4.44 \times 10^{-2} (T_y/P_f)^{\frac{1}{2}} \]  
...(10.16)

To determine the value of \( (K_1 \) and \( K_2) \), \( B_t (K_1, K_2) \) is the value of \( B \) calculated from equation (10.10) by using a particular pair of values of \( K_1 \) and \( K_2 \) and let \( B_e \) be the corresponding measured value.
The method of least square suggest that \( K_1 \) and \( K_2 \) be chosen so as to minimise,

\[
S (K_1, K_2) = \sum \left[ B_e - B_t \left( K_1, K_2 \right) \right]^2
\]

...(10.17)

\[
= \sum \left( b_1 p_2 \frac{b_2 p_2}{p_1 \left[ l_1 - 2K_1 (d_1 + d_2) \Theta_1 \right]} \right)^2
\]

...(10.18)

and \( \delta S / \delta K_1 = 0 \)

i.e., \( \Sigma B_{ei} p_1 \left[ l_1 - 2k_1 (d_1 + d_2) \Theta_1 \right] - \Sigma b_1 p_2 = 0 \)

Hence,

\[
\left| K_1 \right| = \frac{\sum b_1 p_2 - \sum B_{ei} p_1 l_1}{2 \sum B_{ei} p_1 (d_1 + d_2) \Theta_1}
\]

...(10.19)

and \( \left| K_2 \right| 

\[
= \frac{\sum b_2 p_1 - \sum B_{ei} p_2 l_2}{2 \sum B_{ei} p_2 (d_1 + d_2) \Theta_2}
\]

...(10.20)

The corresponding values of \( B_t \) are shown in Table 10.2.

10.3.2 Determination of fabric bending rigidity values using the model of Abbott (1968)

\[
B/b = \frac{1}{1 - 0.56 C_1}
\]

Where,

\[
B = \text{Fabric bending rigidity divided by the number of yarns being bent}
\]

\[
b = \text{yarn bending rigidity}
\]

\[
c = \text{contact length}
\]

The ratio is calculated and shown in Table 10.2.
CALCULATED RIGIDITY Vs MEASURED RIGIDITY
Be1 Vs Bt1 (Group 1, Warp Way)

\[
\frac{\text{Calculated Flexural Rigidity (mNmm}^{2}/\text{mm)}}{\text{Measured Flexural Rigidity (mNmm}^{2}/\text{mm)}}
\]

Fig. 10.1

CALCULATED RIGIDITY Vs MEASURED RIGIDITY
Be1 Vs Bt1 (Group 2, Warp Way)

\[
\frac{\text{Calculated Flexural Rigidity (mNmm}^{2}/\text{mm)}}{\text{Measured Flexural Rigidity (mNmm}^{2}/\text{mm)}}
\]

Fig. 10.2
10.4 RESULTS AND DISCUSSION

10.4.1 Prediction of the bending rigidity of the Crepe-de-chine samples

Bending rigidity obtained by KESF system and cyclic hysteresis bending show a similar trend, i.e the warp way bending rigidity increases and weftway bending rigidity decreases. This is obviously because of the increase in warp sett and twist in the weft yarn (Table 10.1). There is a good correlation between the calculated flexural rigidity, and value of rigidities obtained by the cyclic hysteresis bending tester. Correlation of the predicted bending rigidity by various workers also shows a perfect significant correlation of (0 + 99) (Table 10.2). Estimation of the predicted bending value by KESF parameters shows a similar trend, but the predicted values are on the higher side. This may be attributed to the curvature of bending used in Kawabata bending tester. In KESF bending the yarns are subjected to bending curvature of between 0.05 min⁻¹ to 2.5 mm⁻¹ whereas in case of cyclic hysteresis bending tester the yarns are bent at very low curvature of 3 cm⁻¹.

Figures 10.1 and 10.2 show theoretical (Bₜ) and experimental (Bₑ) values in warp direction. There is a perfect correlation between the measured (Cyclic hysteresis bending parameters) and predicted values.

10.5 CONCLUSIONS

Bending rigidity of the silk fabric could be predicted by using the data of the bending rigidity obtained for the yarns. The model suggested by Leaf et al, (1993) and Abbott (1968) are applicable to the silk fabrics.