Chapter 3

Edge Balance Index Set of Graphs

3.1 Introduction

Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. Let $f$ be a binary labeling from $E(G)$ to $Z_2$, where $Z_2 = \{0, 1\}$, which induces a vertex labeling $f^* : V(G) \longrightarrow Z_3$, where $Z_3 = \{0, 1, 2\}$, defined by $f^*(v) = 1(0)$ if the number of edges with label $1(0)$ is strictly greater than the number of edges with label $0(1)$ and $f^*(v) = 2$ if the number of edges with label $1$ is equal to number of edges with label $0$, which are incident on $v$. For $i \in Z_2$ we have $e_f(i) = e(i) = \text{card}\{e \in E(G) : f(e) = i\}$ and for $i \in Z_3$ we have $v_f(i) = v(i) = \text{card}\{v \in V(G) : f^*(v) = i\}$.

**Definition 3.1.1.** The graph $G$ is said to be edge-friendly if $|e(1) - e(0)| \leq 1$.

Reference [93] is based on this chapter.
Definition 3.1.2. The edge balance index set of a graph $G$, denoted $EBI(G)$, is defined as $\{|v_f(1) - v_f(0)| : \text{the edge labeling } f \text{ is edge-friendly}\}$.

Remark 3.1.3. For convenience, under the edge labeling $f$, edges labeled 1 are called 1-edges, and edges labeled 0 are called 0-edges. Likewise, a vertex is called 0 vertex if its induced vertex label is 0, 1-vertex if its induced vertex label is 1. The largest number in the $EBI(G)$ is denoted by $\max\{EBI(G)\}$.

Recently Wang, Zheng and Lee [92] obtained the edge balance index set of $C_n \times P_2$. Motivated by this, in this chapter we determine the edge balance index set of $C_n \times P_3$.

3.2 Main Results

Now we determine the edge-balance index set of $C_n \times P_3$ ($n \geq 3$) which consists of $3n$ vertices and $5n$ edges. We divide the problem of finding $EBI(C_n \times P_3)$ into six cases, viz,

$$n \equiv 0 \pmod{6}, \quad n \equiv 1 \pmod{6}, \quad n \equiv 2 \pmod{6},$$

$$n \equiv 3 \pmod{6}, \quad n \equiv 4 \pmod{6}, \quad n \equiv 5 \pmod{6}.$$
Lemma 3.2.1. In a graph $C_n \times P_3$ if $n \equiv 0 \pmod{6}$ i.e., $n = 6k (k \in \mathbb{N})$, then

$$\max\{EBI(C_n \times P_3)\} = 10k = \frac{5n}{3}.$$ 

Proof. Let $f$ be an edge friendly labeling on $C_n \times P_3$. Since the graph contains $5n = 30k$ edges we must have $e(0) = e(1) = 15k$. First we denote the vertices of $C_n \times P_3$ as shown in the Figure 3.2.1. Now we label the edges $u_{2q-1}u_{2q}$, $w_{2q-1}w_{2q}$ for $1 \leq q \leq k$ by 0, edges $v_{2q}v_{2q+1}$ for $1 \leq q \leq k - 1$ by 0, all the edges which are incident with vertex $v_q$ for $2k + 1 \leq q \leq 6k$ by 0 and we label all the remaining edges by 1. Then $v(0) = card\{v_{2k+1}, v_{2k+2}, \ldots, v_{6k}\} = 4k$ and $v(1) = card\{u_1, u_2, \ldots, u_{6k}, w_1, w_2, \ldots, w_{6k}, v_1, v_2, \ldots, v_{2k}\} = 14k$.

Here the number of edges incident with every vertex in a graph $C_n \times P_3$ is either 3 or 4. In our construction we have classified the vertices into 3 types, viz, $N_1, N_2$
and \( N_3 \) where, the vertex having two 1-edges and one 0-edge incident on it as \( N_1 \), the vertex having three 1-edges and one 0-edge incident on it as \( N_2 \) and the vertex having four 0-edges incident on it as \( N_3 \). If we interchange the labels of one 0-edge incident on the vertex of the type \( N_i \) and one 1-edge incident on the vertex of the type \( N_j \), for \( 1 \leq i, j \leq 3 \), the number of 1-vertex will always decrease where as the number of 0-vertex will either increase or remain constant. Therefore the value of \(|v(1) - v(0)|\) will always reduce. Hence if \( n \equiv 0 \pmod{6} \) i.e., \( n = 6k(k \in \mathbb{N}) \), then \( \max\{EBI(C_n \times P_3)\} = |v(1) - v(0)| = |14k - 4k| = 10k = \frac{5n}{3} \). \( \square \)

**Lemma 3.2.2.** In a graph \( C_n \times P_3 \) if \( n \equiv 0 \pmod{6} \) i.e., \( n = 6k(k \in \mathbb{N}) \), then

\[
\{0, 1, 2, 3, \ldots, 10k - 1\} \subset EBI(C_n \times P_3).
\]

**Proof.** If \( n \equiv 0 \pmod{6} \), i.e \( n = 6k(k \in \mathbb{N}) \), then by Lemma 3.2.1 \( \max\{EBI(C_n \times P_3)\} = 10k \). Now we interchange some edge labels in the construction for \( \max\{EBI(C_n \times P_3)\} \) to get the remaining members of the edge balance index set. For example by interchanging the labels of edges \( v_1w_1 \) and \( w_1w_2 \) we have \( |v(1) - v(0)| \) will reduce by one.

Therefore by interchanging the labels of edges \( v_1w_1 \) and \( w_1w_2 \) we obtain, \( |v(1) - v(0)| = 10k - 1 \),

\( v_2v_3 \) and \( v_3u_3 \) we obtain, \( |v(1) - v(0)| = 10k - 2 \),

\( v_1w_1 \) and \( w_1w_2, v_2v_3 \) and \( v_3u_3 \) we obtain, \( |v(1) - v(0)| = 10k - 3 \),

\( v_2v_3 \) and \( v_3u_3, v_4v_5 \) and \( v_5u_5 \) we obtain, \( |v(1) - v(0)| = 10k - 4 \),

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$v_1w_1$ and $w_1w_2$, $v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$ we obtain, $|v(1) - v(0)| = 10k - 5$, 

$v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$, ..., $v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$ we obtain, $|v(1) - v(0)| = 8k + 2$, 

$v_1w_1$ and $w_1w_2$, $v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$, ..., $v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$ we obtain, $|v(1) - v(0)| = 8k + 1$, 

$v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$, ..., $v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$, $u_{2k}u_{2k+1}$ and $u_{2k+1}v_{2k+1}$ we obtain, $|v(1) - v(0)| = 8k$, 

$v_1w_1$ and $w_1w_2$, $v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$, ..., $v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$, $u_{2k}u_{2k+1}$ and $u_{2k+1}v_{2k+1}$ we obtain, $|v(1) - v(0)| = 8k - 1$, 

$v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$, ..., $v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$, $u_{2k}u_{2k+1}$ and $u_{2k+1}v_{2k+1}$, $u_{2k+1}u_{2k+2}$ and $u_{2k+2}v_{2k+2}$ we obtain, $|v(1) - v(0)| = 8k - 2$, 

$v_1w_1$ and $w_1w_2$, $v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$, ..., $v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$, $u_{2k}u_{2k+1}$ and $u_{2k+1}v_{2k+1}$, $u_{2k+1}u_{2k+2}$ and $u_{2k+2}v_{2k+2}$ we obtain, $|v(1) - v(0)| = 8k - 3$, 

$v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$, ..., $v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$, $u_{2k}u_{2k+1}$ and $u_{2k+1}v_{2k+1}$, $u_{2k+1}u_{2k+2}$ and $u_{2k+2}v_{2k+2}$ we obtain, $|v(1) - v(0)| = 2$, 

$v_1w_1$ and $w_1w_2$, $v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$, ..., $v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$, $u_{2k}u_{2k+1}$ and $u_{2k+1}v_{2k+1}$, $u_{2k+1}u_{2k+2}$ and $u_{2k+2}v_{2k+2}$ ... $u_{6k-1}u_{6k}$ and $u_{6k}v_{6k}$ we obtain, $|v(1) - v(0)| = 1$, 

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we obtain, \( |v(1) - v(0)| = 0 \).

**Theorem 3.2.3.** In a graph \( C_n \times P_3 \), if \( n \equiv 0 \pmod{6} \) i.e., \( n = 6k (k \in \mathbb{N}) \), where \( n \geq 3 \), then

\[
EBI(C_n \times P_3) = \{0, 1, 2, \ldots, \frac{5n}{3}\}.
\]

**Lemma 3.2.4.** In a graph \( C_n \times P_3 \), if \( n \equiv 1 \pmod{6} \) i.e., \( n = 6k + 1 (k \in \mathbb{N}) \), then

\[
\max\{EBI(C_n \times P_3)\} = 10k + 1 = \frac{5n-2}{3}.
\]

**Proof.** Let \( f \) be an edge friendly labeling on \( C_n \times P_3 \). Since the graph contains 5\( n = 5(6k + 1) = 30k + 5 \) edges, we have two possibilities: (i) \( e(0) = 15k + 3 \), \( e(1) = 15k + 2 \) and (ii) \( e(0) = 15k + 2, e(1) = 15k + 3 \). Now we consider the first case namely \( e(0) = 15k + 3 \) and \( e(1) = 15k + 2 \). First we denote the vertices of \( C_n \times P_3 \) as shown in the Figure 3.2.1. We Label the edges \( u_{2q-1}u_{2q}, w_{2q-1}w_{2q} \) for \( 1 \leq q \leq k \) by 0, edges \( v_{2q}v_{2q+1} \) for \( 1 \leq q \leq k - 1 \) by 0, all the edges which are incident with vertex \( v_q \) for \( 2k + 1 \leq q \leq 6k + 1 \) by 0 and all the remaining edges by 1. Then \( v(0) = \text{card}\{v_{2k+1}, v_{2k+2}, \ldots, v_{6k+1}\} = 4k + 1 \) and \( v(1) = \text{card}\{u_1, u_2, \ldots, u_{6k+1}, w_1, w_2, \ldots, w_{6k+1}, v_1, v_2, \ldots, v_{2k}\} = 14k + 2 \).

Here the number of edges incident with every vertex in a graph \( C_n \times P_3 \) is either 3 or 4. In our construction we have classified the vertices into 3 types,
viz, $N_1, N_2$ and $N_3$ where, the vertex having two 1-edges and one 0-edge incident on it as $N_1$, the vertex having three 1-edges and one 0-edge incident on it as $N_2$ and the vertex having four 0-edges incident on it as $N_3$. If we interchange the labels of one 0-edge incident on the vertex of the type $N_i$ and one 1-edge incident on the vertex of the type $N_j$, for $1 \leq i, j \leq 3$, the number of 1-vertex will always decrease where as the number of 0-vertex will either increase or remain constant. Therefore the value of $|v(1) - v(0)|$ will always reduce. Hence if $n \equiv 1(\text{mod } 6)$ i.e., $n = 6k + 1 (k \in \mathbb{N})$, then $\max\{EBI(C_n \times P_3)\} = |v(1) - v(0)| = |14k + 2 - (4k + 1)| = 10k + 1 = \frac{5n-2}{3}$.

\begin{lemma}
In a graph $C_n \times P_3$ if $n \equiv 1(\text{mod } 6)$ i.e., $n = 6k + 1 (k \in \mathbb{N})$ then 
\begin{align*}
\{0, 1, 2, 3, \ldots, 10k\} \subset EBI(C_n \times P_3).
\end{align*}
\end{lemma}

\begin{proof}
If $n \equiv 1(\text{mod } 6)$, i.e., $n = 6k + 1 (k \in \mathbb{N})$, then by Lemma 3.2.4, $\max\{EBI(C_n \times P_3)\} = 10k + 1$. Now we interchange some edge labels in the construction for $\max\{EBI(C_n \times P_3)\}$ to get the remaining members of the edge balance index set. For example by interchanging the labels of edges $v_1w_1$ and $w_1w_2$ we have $|v(1) - v(0)|$ will reduce by one.

Therefore by interchanging the labels of edges $v_1w_1$ and $w_1w_2$ we obtain, $|v(1) - v(0)| = 10k,$

$v_2v_3$ and $v_3u_3$ we obtain, $|v(1) - v(0)| = 10k - 1,$

$v_1w_1$ and $w_1w_2$, $v_2v_3$ and $v_3u_3$ we obtain, $|v(1) - v(0)| = 10k - 2,$

$v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5$ we obtain, $|v(1) - v(0)| = 10k - 3,$

\end{proof}
\[v_1w_1 \text{ and } w_1w_2, \ v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5 \text{ we obtain, } |v(1) - v(0)| = 10k - 4, \]

\[v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1} \text{ we obtain, } |v(1) - v(0)| = 8k + 3, \]

\[v_1w_1 \text{ and } w_1w_2, \ v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1} \text{ we obtain, } |v(1) - v(0)| = 8k + 2, \]

\[v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1}, \ u_{2k}u_{2k+1} \text{ and } u_{2k+1}v_{2k+1} \text{ we obtain, } |v(1) - v(0)| = 8k + 1, \]

\[v_1w_1 \text{ and } w_1w_2, \ v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1}, \ u_{2k}u_{2k+1} \text{ and } u_{2k+1}v_{2k+1} \text{ we obtain, } |v(1) - v(0)| = 8k, \]

\[v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1}, \ u_{2k}u_{2k+1} \text{ and } u_{2k+1}v_{2k+1}, \ u_{2k+1}u_{2k+2} \text{ and } u_{2k+2}v_{2k+2} \text{ we obtain, } |v(1) - v(0)| = 8k - 1, \]

\[v_1w_1 \text{ and } w_1w_2, \ v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1}, \ u_{2k}u_{2k+1} \text{ and } u_{2k+1}v_{2k+1}, \ u_{2k+1}u_{2k+2} \text{ and } u_{2k+2}v_{2k+2} \text{ we obtain, } |v(1) - v(0)| = 8k - 2, \]

\[v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1}, \ u_{2k}u_{2k+1} \text{ and } u_{2k+1}v_{2k+1}, \ u_{2k+1}u_{2k+2} \text{ and } u_{2k+2}v_{2k+2} \text{ we obtain, } |v(1) - v(0)| = 3, \]

\[v_1w_1 \text{ and } w_1w_2, \ v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1}, \ u_{2k}u_{2k+1} \text{ and } u_{2k+1}v_{2k+1}, \ u_{2k+1}u_{2k+2} \text{ and } u_{2k+2}v_{2k+2}, \ldots, u_{6k-1}u_{6k} \text{ and } u_{6k}v_{6k} \text{ we obtain, } |v(1) - v(0)| = 2, \]
\[ v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1}, \ u_{2k}u_{2k+1} \text{ and } u_{2k+1}v_{2k+1}, \ u_{2k+1}u_{2k+2} \text{ and } u_{2k+2}v_{2k+2}, \ldots, u_{6k}u_{6k+1} \text{ and } u_{6k+1}v_{6k+1} \text{ we obtain,} \]
\[ |v(1) - v(0)| = 1, \]
\[ v_1w_1 \text{ and } w_1w_2, \ v_2v_3 \text{ and } v_3u_3, \ v_4v_5 \text{ and } v_5u_5, \ldots, v_{2k-2}v_{2k-1} \text{ and } v_{2k-1}u_{2k-1}, \ u_{2k}u_{2k+1} \text{ and } u_{2k+1}v_{2k+1}, \ u_{2k+1}u_{2k+2} \text{ and } u_{2k+2}v_{2k+2}, \ldots, u_{6k}u_{6k+1} \text{ and } u_{6k+1}v_{6k+1} \text{ we obtain,} \]
\[ |v(1) - v(0)| = 0. \]

**Theorem 3.2.6.** In a graph \( C_n \times P_3 \), if \( n \equiv 1(\text{mod}6) \) i.e., \( n = 6k + 1(k \in \mathbb{N}) \), then
\[ EBI(C_n \times P_3) = \{0, 1, 2, \ldots, \frac{5n-2}{3}\}. \]

**Lemma 3.2.7.** In a graph \( C_n \times P_3 \), if \( n \equiv 2(\text{mod}6) \) i.e., \( n = 6k + 2(k \in \mathbb{N}) \), then
\[ \max\{EBI(C_n \times P_3)\} = 10k + 2 = \frac{5n-4}{3}. \]

**Proof.** Let \( f \) be an edge friendly labeling on \( C_n \times P_3 \). Since the graph contains \( 5n = 5(6k + 2) = 30k + 10 \) edges we must have \( e(0) = e(1) = 15k + 5 \). First we denote the vertices of \( C_n \times P_3 \) as shown in the Figure 3.2.1. Now we label the edges \( u_{2q-1}u_{2q}, \ w_{2q-1}w_{2q} \) for \( 1 \leq q \leq k \) by 0, edges \( v_{2q}v_{2q+1} \) for \( 1 \leq q \leq k - 1 \) by 0, all the edges which are incident with vertex \( v_q \) for \( 2k + 1 \leq q \leq 6k + 2 \) by 0 and all the remaining edges by 1. Then \( v(0) = \text{card}\{v_{2k+1}, v_{2k+2}, \ldots, v_{6k+2}\} = 4k + 2 \) and \( v(1) = \text{card}\{u_1, u_2, \ldots, u_{6k+2}, w_1, w_2, \ldots, w_{6k+2}, v_1, v_2, \ldots, v_{2k}\} = 14 + 4k. \)

Here the number of edges incident with every vertex in a graph \( C_n \times P_3 \) is either 3 or 4. In our construction we have classified the vertices into 4 types, viz,
$N_1, N_2, N_3$ and $N_4$ where, the vertex having two 1-edges and one 0-edge incident on it as $N_1$, the vertex having three 1-edges and one 0-edge incident on it as $N_2$, the vertex having four 0-edges incident on it as $N_3$ and the vertex having three 1-edges incident on it as $N_4$. If we interchange the labels of one 0-edge incident on the vertex of the type $N_i$ and one 1-edge incident on the vertex of the type $N_j$, for $1 \leq i, j \leq 4$, then the number of 1-vertex will either decrease or remain constant and the number of 0-vertex will either increase or remain constant. Therefore the value of $|v(1) - v(0)|$ will reduce or remain constant. Hence if $n \equiv 2 \pmod{6}$ i.e., $n = 6k + 2 (k \in \mathbb{N})$, then $max\{EBI(C_n \times P_3)\} = |v(1) - v(0)| = |14k + 4 - (4k + 2)| = 10k + 2 = \frac{5n-4}{3}$. \hfill \Box

**Lemma 3.2.8.** In a graph $C_n \times P_3$ if $n \equiv 2 \pmod{6}$ i.e., $n = 6k + 2 (k \in \mathbb{N})$ then 
\[ \{0, 1, 2, 3, \ldots, 10k + 1\} \subset EBI(C_n \times P_3). \]

**Proof.** If $n \equiv 2 \pmod{6}$, i.e., $n = 6k + 2 (k \in \mathbb{N})$, then by Lemma 3.2.7 $max\{EBI(C_n \times P_3)\} = 10k + 2$. Now we interchange some edge labels in the construction for $max\{EBI(C_n \times P_3)\}$ to get the remaining members of the edge balance index set of a graph. For example by interchanging the labels of edges $v_1w_1$ and $v_{6k+2}w_{6k+2}$ we have $|v(1) - v(0)|$ will reduce by one.

Therefore by interchanging the labels of edges $v_1w_1$ and $v_{6k+2}w_{6k+2}$ we obtain, $|v(1) - v(0)| = 10k + 1$,

$v_2v_3$ and $v_3u_3$ we obtain, $|v(1) - v(0)| = 10k$,

$v_1w_1$ and $v_{6k+2}w_{6k+2}$, $v_2v_3$ and $v_3u_3$ we obtain, $|v(1) - v(0)| = 10k - 1$,
\(v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5\) we obtain, \(|v(1) - v(0)| = 10k - 2,\)

\(v_1w_1\) and \(v_6k+2w_{6k+2}, v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5\) we obtain, \(|v(1) - v(0)| = 10k - 3,\)

\(:\)

\(v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5, \ldots, v_{2k-2}v_{2k-1}\) and \(v_{2k-1}u_{2k-1}\) we obtain, \(|v(1) - v(0)| = 8k + 4,\)

\(v_1w_1\) and \(v_6k+2w_{6k+2}, v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5, \ldots, v_{2k-2}v_{2k-1}\) and \(v_{2k-1}u_{2k-1}\)

we obtain, \(|v(1) - v(0)| = 8k + 3,\)

\(v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5, \ldots, v_{2k-2}v_{2k-1}\) and \(v_{2k-1}u_{2k-1}\)

\(u_{2k+1}v_{2k+1}\) we obtain, \(|v(1) - v(0)| = 8k + 2,\)

\(v_1w_1\) and \(v_6k+2w_{6k+2}, v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5, \ldots, v_{2k-2}v_{2k-1}\) and \(v_{2k-1}u_{2k-1}, u_{2k}u_{2k+1}\)

\(u_{2k}u_{2k+1}\) and \(u_{2k+1}v_{2k+1}\) we obtain, \(|v(1) - v(0)| = 8k + 1,\)

\(v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5, \ldots, v_{2k-2}v_{2k-1}\) and \(v_{2k-1}u_{2k-1}, u_{2k}u_{2k+1}\)

\(u_{2k+1}v_{2k+1}, u_{2k+1}u_{2k+2}\) and \(u_{2k+2}v_{2k+2}\) we obtain, \(|v(1) - v(0)| = 8k,\)

\(v_1w_1\) and \(v_{6k+2}w_{6k+2}, v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5, \ldots, v_{2k-2}v_{2k-1}\)

\(v_{2k-1}u_{2k-1}, u_{2k}u_{2k+1}\) and \(u_{2k+1}v_{2k+1}, u_{2k+1}u_{2k+2}\) and \(u_{2k+2}v_{2k+2}\) we obtain, \(|v(1) - v(0)| = 8k - 1,\)

\(:\)

\(v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5, \ldots, v_{2k-2}v_{2k-1}\) and \(v_{2k-1}u_{2k-1}, u_{2k}u_{2k+1}\)

\(u_{2k+1}v_{2k+1}, u_{2k+1}u_{2k+2}\) and \(u_{2k+2}v_{2k+2}, \ldots, u_{6k}u_{6k+1}\) and \(u_{6k+1}v_{6k+1}\) we obtain, \(|v(1) - v(0)| = 2,\)

\(v_1w_1\) and \(v_{6k+2}w_{6k+2}, v_2v_3\) and \(v_3u_3, v_4v_5\) and \(v_5u_5, \ldots, v_{2k-2}v_{2k-1}\) and \(v_{2k-1}u_{2k-1}, u_{2k}u_{2k+1}\)
we obtain, $|v(1) - v(0)| = 1$,

$v_2v_3$ and $v_3u_3$, $v_4v_5$ and $v_5u_5, \ldots, v_{2k-2}v_{2k-1}$ and $v_{2k-1}u_{2k-1}$, $u_{2k}u_{2k+1}$ and $u_{2k+1}v_{2k+1}$, $u_{2k+1}u_{2k+2}$ and $u_{2k+1}v_{2k+2}, \ldots, u_{6k+1}u_{6k+2}$ and $u_{6k+2}v_{6k+2}$, we obtain,

$|v(1) - v(0)| = 0$. \hfill \square

**Theorem 3.2.9.** In a graph $C_n \times P_3$, if $n \equiv 2 \pmod{6}$ i.e., $n = 6k + 2 (k \in \mathbb{N})$, where $(n \geq 3)$, then

$$EBI(C_n \times P_3) = \{0, 1, 2, \ldots, \frac{5n-4}{3}\}.$$  

Similarly one may prove the following theorems.

**Theorem 3.2.10.** In a graph $C_n \times P_3$, if $n \equiv 3 \pmod{6}$ i.e., $n = 6k + 3 (k = 0, 1, 2, \ldots)$, where $(n \geq 3)$, then

$$EBI(C_n \times P_3) = \{0, 1, 2, \ldots, \frac{5n}{3}\}.$$  

**Theorem 3.2.11.** In a graph $C_n \times P_3$, if $n \equiv 4 \pmod{6}$ i.e., $n = 6k + 4 (k = 0, 1, 2, \ldots)$, where $(n \geq 3)$, then

$$EBI(C_n \times P_3) = \{0, 1, 2, \ldots, \frac{5n-2}{3}\}.$$  

**Theorem 3.2.12.** In a graph $C_n \times P_3$, if $n \equiv 5 \pmod{6}$ i.e., $n = 6k + 5 (k = 0, 1, 2, \ldots)$, where $(n \geq 3)$, then

$$EBI(C_n \times P_3) = \{0, 1, 2, \ldots, \frac{5n+2}{3}\}.$$