ESTIMATION OF O-D MATRIX THROUGH TRAFFIC VOLUME COUNTS - REVIEW OF LITERATURE
CHAPTER - 2
ESTIMATION OF O-D MATRIX THROUGH TRAFFIC VOLUME COUNTS
REVIEW OF LITERATURE

2.0 GENERAL

The Introduction has pointed out that there are six different approaches that have been suggested for estimation of Origin-Destination matrices. The aim of this chapter is to review the above techniques critically, so that suitable technique/techniques through indirect methods such as link volume modelling could be adopted for estimation of travel demand through Public transport system.

Unlike the travel by personalised modes of transport, the travel by Public transport system is conditioned by the availability of adequate service. Thus, supply accessibility becomes an important issue in the development of models for the estimation of travel demand by Public transport system.

Indirect simulation of travel demand has necessarily to take into account the available information on the distribution of homes, workplaces and other land uses. Thus, the description and appropriate specification of urban structure becomes vital. Such a specification would enable estimation of spatial travel demand more rigorously than a model without this.

Similarly, the activity system in an urban area undergoes pulsating changes in their trip attraction
characteristics during different periods of the day. A good temporal travel demand estimation necessarily calls for the specification regarding not only the urban structure, but also the urban activity sensitiveness during different hours of the day.

The objective of this chapter is to review the available literature on all the above counts with a view to improve the model specifications for indirect estimation of travel demand using link volume philosophy.

2.1 A STATE-OF-ART SUMMARY

The basic idea in the various link volume approaches is to update an initial seed matrix using a set of link volume counts in an iterative fashion so as to obtain an O-D matrix, which, when loaded on the network, match the ground counts more closely than any other O-D matrix combination.

Table 2.1 presents the comparison of the different approaches followed by many researchers starting from Low (1972) who uses gravity formulation to Gothe et al (1989) who recommends the use of multiobjective formulation. From the table, it can be seen that many of the models and approaches have remained in their formative stage without finding applications in real life.
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The fundamental equation concerning the travel demand and the link volumes can be represented as

\[ V_a = \sum_{ij} t_{ij} p_{ij}^a \]  \hspace{1cm} (2.1)

where,

- \( V_a \) is the traffic volume on link \( a \)
- \( t_{ij} \) is the trips at the cell element \( ij \)
- \( p_{ij}^a \) is the proportion of traffic between \( i \) and \( j \) using link \( a \).

In order to estimate \( T_{ij} \) matrix (\( nxn \)) uniquely, \( (n^2-n) \) such independent equations are necessary. But, it is difficult to have so many simultaneous synchronous traffic counts in practice, and hence, certain simplifying assumptions are made concerning \( T_{ij}'s \) and \( P_{ij}'s \) so that the problem is converted from one of simultaneous equations to parametric estimation. Based on these assumptions, the models reported so far can be grouped as shown in art 1.1.

2.1.1 Gravity Formulation approach

Most of the reported studies, wherein certain amount of physical validation of the estimation of O-D matrices through link volumes is available, can be found mostly following the Gravity Formulation Approach. This approach with proportional traffic assignment has been used extensively in urban and regional travel demand estimation in 1970's.
The $T_{ij}$'s in equation 2.1 can be written as

$$T_{ij} = K O_i O_j d_{ij}^{-m} \quad \ldots \quad (2.2)$$

where $O_i$ and $O_j$ are production and attraction proxies and $d_{ij}^{-m}$ is the travel time factor. One of the main criticisms of this approach is that by assuming a priori, that the traffic interactions follow Gravity Formulation, much of the information contained in the link volumes are not used in the estimation process.

2.1.2 Network Equilibrium approach

Congestion plays an important role in the urban situation. It may influence the route choice for personal modes of transport, but the influence may not be that apparent on Public transport systems, which are operated to accommodate very high load factors in developing countries. Hence, the network equilibrium approach through public transport does not have much significance.

2.1.3 Entropy Maximisation/Information Minimisation Approach

Entropy models are based on the philosophy that in a closed physical system, the elements tend to an arrangement which can be simulated in as many ways as possible, consistent with the constraints imposed on them. At least intuitively, it can be said that this arrangement is the one having minimum information. Van Vliet and Willumsen (1981) in their use of Entropy model (ME2) on the Reading data have the following suggestions for improving the performance of models:
i) Generation of better seeds either by the use of an out-of-date matrix or by a matrix in which interchanges are assumed to follow a gravity model.

ii) Inclusion of other information like trip length frequency distribution, total trip ends, etc.

iii) Generation of a rough estimate with some conventional technique and mixing the same with traffic counts of higher statistical reliability.

iv) Adoption of a simple all-or-nothing assignment without loss of much accuracy.

2.1.4 Bayesian Statistical Approach

From the standpoint of Bayesian inference, the Entropy model solution is just one extreme case from a continuum of possibilities. Maher (1983) proposes a method allowing much flexibility in the degree of prior beliefs. It also allows for different degrees of belief in different parts of the prior. The prior beliefs are modified by observations to produce posterior beliefs; the stronger the prior beliefs the less influence the observations will have in determining the posterior beliefs. The posterior beliefs, then, are a weighted average of the prior beliefs and the observations. The assumption of proportional assignment and multivariate normality of the counts gives rise to simple updating scheme, thus making the method computationally attractive.
2.1.5 Multiobjective Formulation

In Multiobjective programming formulations for estimating O-D matrices, there are two objectives. While the first one is to satisfy the traffic count constraints, the other objective is to search for a solution as close to the target matrix as possible. Depending on the degree of belief in the available information, the planner can choose the weights to be given to different objectives.

The confidence in the available link counts and the priori O-D matrix are combined into a single objective model and the parameters of such a model are estimated so that the solution is pareto optimal. The model formulation appears as:

\[
\begin{align*}
\min \quad & \sum_{j=1}^{N} t_j \left( \log \frac{t_j}{\bar{t}_j} - 1 \right) \\
\text{s.t.} \quad & \sum_{j=1}^{N} P_{ij} t_j - V_i = 0, \quad i = 1, 2, \ldots M \\
& t_j \geq 0, \quad j = 1, 2, \ldots N
\end{align*}
\]

These objectives are combined into a single objective model as:

\[
\begin{align*}
\min \quad & r_1 \sum_{j=1}^{N} t_j \left( \log \frac{t_j}{\bar{t}_j} - 1 \right) + r_2 \sum_{i=1}^{M} V_i \left( \log \frac{V_i}{\bar{V}_i} - 1 \right)
\end{align*}
\]
\[
\text{s.t. } \sum_{j=1}^{N} P_{ij} t_j - V_i = 0, \ i = 1, \ldots, M \quad \ldots (2.7)
\]
\[
t_j \geq 0, \ j = 1, \ldots, N \quad \ldots (2.8)
\]

The models in equations (2.6) to (2.8) are identical to the Willumsen model, if \( r_1 \) is made equal to one and \( r_2 \) is set to a value which expresses the relative belief in traffic counts compared to that of the target matrix.

2.1.6 Selection of technique/techniques for travel demand estimation

The Information Minimisation, Entropy Maximisation and Bayesian approaches belong to the same family as pointed out by Wilson (1970). The interchangeability among the three approaches is demonstrated by Cascetta and Nguyen (1988), which serves as a framework for all future works in the estimation of O-D matrices using link counts.

Taking into account the need for quality data for the application of optimisation technique such as multi-objective formulation, it is felt that either Information minimisation or Entropy maximisation models with one point data is the method which could be adopted for spatial demand estimation. However, models like Bayesian methods could still be used for the temporal demand estimation within different time periods of a day. In view of its versatility to create confidence bands instead of point estimates, the algorithm for Bayesian approach as discussed by Maher (1983) is presented further.
2.2 ALGORITHM FOR THE ESTIMATION OF O-D MATRIX USING BAYESIAN APPROACH

Unlike in EM/1M approaches, in this method, prior beliefs will not be expressed in a single value for each variable, but in the form of a prior distribution with a specified mean and variance. Again, it will be assumed that there is, in general, random errors in the observations and so there will be a dispersion matrix associated with these observations. Main assumption made in this approach is that true distribution of counts and the O-D cells, both of them, follow multi-variate normal distribution. Based on the above assumptions, the updating equations for the mean vector and dispersion matrix are derived. These are given by

\[
\begin{align*}
t &= \bar{t} + UH^T (W + HUH^T)^{-1}(g - H\bar{t}) \\
V &= U - UH^T(W+HUH^T)HU
\end{align*}
\]  \hspace{1cm} (2.9) \hspace{1cm} (2.10)

Where

- \(t\) - the posterior mean vector of O-D cells, of length \(n\).
- \(\bar{t}\) - the prior mean vector of O-D cells, of length \(n\)
- \(V\) - posterior dispersion matrix of O-D cells, of size \(n \times n\)
- \(U\) - prior dispersion matrix of O-D cells, of size \(n \times n\)
- \(H\) - \(h_{ij}\) proportion of trips in O-D pair \(j\) using link \(i\) of size \(m \times n\), where \(m\) is the number of counted links.
- \(g\) - mean vector of link counts, of length \(n\)
- \(W\) - dispersion matrix of link volume counts, of size \(m \times m\)
While, the above equation system is quite general, there can be many simplifying assumptions made, so that, the mathematics are simplified. Some of these assumptions are:

i) Link volumes are independent (matrix $W$ becomes diagonal but need not be error-free).

ii) Link volumes are independent and free from errors ($W$ becomes $0$ matrix)

iii) Link volumes need not be independent but error-free

From Eq. (2.9) and (2.10), Maher (1983) developed three methods for the estimation of O-D matrices based on different assumptions. They are:

i) Independent Observation method

ii) Observation without error

iii) Least informative prior

As it is not possible to ensure error-free link volume count observations in the field, it is difficult to use second and third methods. Hence development of computer program and experimentation are confined to the first method only, the details of which are described in Appendix 2.1.

2.3 GENERATION OF REALISTIC SEED MATRICES

The basic idea of the link volume modelling philosophy, as referred in art. 2.1, requires the recurrent modification of an initial trip matrix so that the model flows are made equal to the observed ones.
Maher (1983) argues that Entropy maximisation approach places greater emphasis on the variability in link volume counts compared to that of the variability in the priori O-D entries, and the demand obtained using the above is placed at one end of the extreme of the possible range of solutions. But, while finding travel demand with either priori O-D matrix or link volumes, whose variabilities are unknown, it is but logical that priori guess matrix must, as far as possible, be the one which is nearer to the reality, reflecting the travel characteristics of the people of the study area.

Van Vliet and Willumsen (1981) have observed that the formulation of O-D matrix from link volume counts by this approach is only an extension of Kruithof/Furness balancing factor algorithm for biproportional problems. Then, sophistication can at best be achieved at the stage of introducing the realistic seed matrix.

Nagaraj and Chari (1986) have reported the first successful application of link volume philosophy in India using Entropy Maximisation to regional situation. Their experiments were confined to the estimation of the regional bus passenger flow. Seed matrices were generated by them through gravitational approach and they tried to solve the problem of sparse matrix in a region by introducing certain concepts from central place theory and a relationship which limits the sphere of influence of many of the growth centres. As the heirarchical scheme used by them may not be
applicable to urban areas, the ZIPTAC - ZODIAC package proposed by them may have limited application in the estimation of urban travel demand.

2.4 SPECIFICATION OF URBAN SPATIAL STRUCTURE

Traffic is a joint consequence of the interaction between activity levels and that of the transportation system provisions.

Sherratt (1960) has used the distribution of homes as a unit of urban travel model. One of the parameters of this distribution has been used by him to describe the 'spread of homes' for many cities.

It has been well recognized in the literature that good specification of spatial structure invariably provides better travel demand models. Attempts to integrate land use and transportation models were begun in the early 1970's based on Lowry's model (1964). The need for better theoretical basis for travel demand led to the entropy based models of Wilson (1970, 1974).

2.4.1 Definition of urban structure

Hutchinson (1974) defines urban structure as particular articulation of adapted spaces, or land in different uses, that might exist in an urban area. Human activities housed by the adapted spaces interact within the context of various activity systems. The transport and communication networks facilitate the activity system interactions.
Thomson (1977) argues that to understand remedies to urban transport problems, it is necessary to understand the meaning of city structure. The city structure refers to the size and shape of the city and the spatial distribution of homes, jobs and other land use activities within its geographical area. He has classified the cities and explained the land use and transport strategic solutions for each class, highlighting the role of the city centre and the other major sub-centres.

2.4.2 Quantification of urban spatial structure

Black and Katakos (1986) proposed a quantitative method for classifying the city structure based on the classical Transportation Problem of Operations Research.

The study of urban spatial traffic patterns by Vaughan (1987) is a classical work in which the specification of urban structure is provided by the mathematical description of the way in which homes, work places and the transportation network are distributed over an urban area, primarily, in terms of distances to the city centre.

The transport, communication and urban form cover an overlapping area of rising academic and practical concern. Wigan (1988) traces several of the many themes brought together under different professional banners, and shows how a confluence of interest is emerging. Mahmassani et al (1988) reports that the spatial density function of population provides a useful characterization of urban
FIG. 2.1 FUNCTIONAL FORMS OF TRAVEL FACTOR FUNCTION
(HUTCHINSON 1974)
structure. When taken at different time intervals, they indicate the development pattern of an urban area which describes the evolution of urban structure in terms of the spatial patterns of residential location, automobile ownership and other variables.

2.4.3 Trip length frequency distribution

Transportation planners use trip length frequency distribution from activity centres such as CBD as a parameter to describe the gradient of activity linkages in relation to the space. Fig. 2.1 represents some of the theoretical functions as discussed by Hutchinson (1974). While some of the distribution functions are skewed over wider trip lengths, some of them show abrupt cut-off. A knowledge about these gradients will be helpful to describe the intensity of the activity system in a spatial sense.

Thus it appears that better specification of urban structure in the form of population and employment distributions and their integration in the urban environment through transportation system characteristics will enable better travel demand models to be simulated. However, travel itself is conditioned further by the transportation system accessibility. Hence, it becomes necessary to explore further on the above so that better model specifications are possible to be provided.
2.5 ACCESSIBILITY

Accessibility is the concept that links functionally, the spatial location of land use activities with the service provided by the transport system (Blunden and Black, 1984).

Though accessibility indicators have been defined and proposed by Hansen as early as in 1959, there is still no general agreement among researchers and planners on their precise value and significance. It has been interpreted quite diversely among transportation planners, and different measures have been proposed. While Neuburger (1971) proposes consumer surplus approach for deriving accessibility indicators, Koenig (1975), Cochrane (1975) and Williams and Senior (1978) use behavioural models.

As defined by Dalvi (1978) the accessibility denotes the ease with which any land use activity can be reached from a location, using a particular transport system. Usage of accessibility in transportation planning has been elaborated by Polus and Kumove (1979).

According to Koenig (1980), the accessibility indicators provide a sound tool for evaluating transport policies, especially at disaggregate level and as derived from behavioural models, they are intrinsically able to consider both the motivations and resistance factor of trip making.
2.5.1 Accessibility as a tool for estimation of travel demand

Transit accessibility is the dominant factor in determining the percentage transit usage by Hsu and Surti (1975).

According to them

\[ S_i = \sum_{K=1}^{K} A_{ki} \sqrt{B_k} \left( \sum_{n} A_{kn} D_{in} / P_i \right) \]...(2.11)

Where

- \( S_i \) - Accessibility Index for district i
- \( A_{ki} \) - Service coverage of route K in district i
- \( B_k \) - Service frequency in the unit of number of service per hour during the peak period
- \( D_{in} \) - number of working trips between i and n
- \( P_i \) - total number of working trips in i

As per equation 2.11, the accessibility itself is a function of transit usage and hence not directly applicable for travel demand estimation.

However, the quality of service provided by the transportation system, as reflected in the frequency of service between any zone and other zones, the service coverage of routes, number of bus stops etc. are better indicators of the degree of accessibility, a particular traffic zone enjoys in its spatial set up.
2.6 TEMPORAL TRAVEL DEMAND

The specific spatial and temporal patterns of travel demand that are likely to occur in an urban area are a function of the properties of the area. However, some similarities in land development and travel demand have been observed in cities throughout the world (Hutchinson 1974).

Time-wise and space-wise systems effects and conjoint time-space effects have been discussed by Diandas (1986). Shunk et al (1968), Maejima (1979) and Litinas and Ben Akiva (1982) argue that trip to work typifies other journey types also. They contend that peak hour demand for travel can be made use of to reproduce the demand on highway system at other times as well. Thus, there is a positive scope for exploring the use of link volume modelling procedure for getting off-peak hour O-D matrix. This is an area where much work has not been carried out.

2.7 IDENTIFICATION OF THE PROBLEM

The subject matter of this thesis has been introduced in chapter 1. The literature review has suggested that following are some of the directions in which the research could progress.

i) The use of link volume procedure for estimation of travel demand has not been sufficiently explored for realistic situation in urban areas, eventhough, the theoretical aspects of the problem have already been well-documented.
ii) The problem of generation of a good seed matrix for the purpose of updating has not been resolved comprehensively. The use of seed matrices using data from secondary sources is aimed at reduction of resources required in terms of time and money.

iii) The city structure influences the travel demand. A good description of the city structure may improve the travel demand estimation process.

iv) Trip productions, trip attractions, trip length frequency distribution etc., if available, can be used as additional information in formulating or updating a guess seed. These data are possible to be compiled from the entries in the conductors' statistical records.

v) Demand and supply variables simultaneously affect the observed ground counts. Hence, there is a need for estimating travel demands which are conditioned by supply variables, so that the effect of changes of the latter could be studied.

vi) It is essential that link volume philosophy be extended to travel by public transport system which is the most predominant mode of travel in many developing countries.

vii) It is necessary to experiment the estimation of temporal travel demands at many time periods in the day, knowing the travel demands at any one time interval and link volumes at other time-period.
2.8 CONCLUSIONS

This chapter, while reviewing the various methods that are available for estimation of O-D matrix using link volume counts has identified that Entropy maximisation and Bayesian statistical approach probably would be ideal for the conduct of experiments in the estimation of spatial and temporal travel demand.

The review also has suggested that, urban structure specification in the form of population and employment distributions, transportation distribution functions etc. are being used for the improvement of demand estimation. The transit accessibility model, as presented by Hsu and Surti (1975) offers another scope for taking into account the influence of transportation supply on travel demand. While there are no specific models that have been attempted for temporal travel demand estimation, researchers like Maejima (1979) and Litinas and Ben Akiva (1982) and others have expressed a confidence that peak hour travel data itself may be possible to be used for simulating travels at other time periods.

Though, theoretically this literature review has been partly successful in the selection of appropriate model procedures for spatial and temporal travel demand estimations, it is felt that greater insight into the modelling process could be achieved by a pilot study. The presentation of the results of such a pilot study conducted, is the subject matter of chapter 3.
APPENDIX - 2.1
DIFFERENT METHODS FOR THE ESTIMATION OF
O-D MATRIX USING BAYESIAN APPROACH

Based on Eq.(2.9) and Eq.(2.10) Maher (1983) developed three methods for estimation of O-D matrices based on different assumptions, which are presented as:

i) INDEPENDENT OBSERVATION

Main assumption made in this method is, that the link volumes are independent i.e., the dispersion matrix will consist of non-zero elements on the main diagonal only. Consider any single observation at a time. The dispersion matrix, \( W \), reduces to a single element say, \( c^2 \). The matrix \( H \) becomes a row vector say, \( h^T \). Therefore, \( W + HUH^T \) becomes a scalar \( c^2 + h^TUh \). The updating equations Eq. (2.9) and Eq. (2.10) become:

\[
\begin{align*}
t & = \bar{t} + \frac{Uh}{c^2 + h^TUh} (g - h^T\bar{t}) \quad \ldots \quad (2.1.1) \\
V & = U - \frac{Uhh^TU}{c^2 + h^TUh} \quad \ldots \quad (2.1.2)
\end{align*}
\]

The calculations in the special case of a single observation are therefore very simple. Now, if the \( m \) observations are independent, the matrix \( W \) will consist of non-zero elements on the main diagonal only. Because of the sequential nature of Bayes' Theorem for independent observations, the method in Eq. 2.1.1 and 2.1.2 can be applied sequentially to the individual observations (in any...
order); the posterior from one becomes the prior for the next, and so on.

ii) OBSERVATION WITHOUT ERROR

If the observations may be made over a long time, the resulting average flow estimates are virtually free of random error; i.e., \( c^2 = 0 \). Then the Eq.(2.1.1) and Eq.(2.1.2) become:

\[
\begin{align*}
\tilde{t} &= \tilde{t} + \frac{U_h}{h^T U_h} (g - h^T \tilde{t}) \\
V &= U - \frac{U h^T U}{h^T U h}
\end{align*}
\]  

...(2.1.3) (2.1.4)

iii) LEAST INFORMATIVE PRIOR

In this method, the dispersion matrices \( U \) and \( W \) are scaled by factors \( \bar{b} \) and \( \bar{d} \) so that the trace of the matrices \( H^T U H \) and \( W \) is \( m \). The resultant scaled matrices are \( \bar{b} U_o \) and \( \bar{d} F \). The equation (2.9) becomes

\[
\tilde{t} = \tilde{t} + \bar{b} U_o H^T (\bar{d} F + \bar{d} H U_o H^T)^{-1} (g - H \tilde{t})
\]  

...(2.1.5)

If \( \bar{b} \) is allowed to become infinity and \( \bar{d} \) remains fixed, the prior distribution becomes least informative i.e. minimal confidence is on the prior. Then Eq.(2.9) and Eq.(2.10) becomes:

\[
\begin{align*}
t^* &= \tilde{t} + U_o H^T (H U_o H^T)^{-1} (g - H \tilde{t}) \\
v^* &= U - U_o H^T (H U_o H^T)^{-1} H U_o
\end{align*}
\]  

...(2.1.6) (2.1.7)

Each observation can be introduced individually and independently by considering the covariance terms in \( F \) as zero.