SYNOPSIS

Flow through porous medium is of great importance in the fields of chemical, engineering, petroleum technology, agricultural engineering, studies on underground water resources, seepage of water in river beds, movements of natural gas, oil and water through oil resources. The study of buoyancy driven convection flows through porous media has been stimulated by its applications in several geophysical and geothermal problems [5-10, 14,15,19,23,25]. Interest in understanding the connective transportation in porous material is increasing owing to the development of geothermal energy technology, high performance insulation for buildings and cold storage's, drying technology and many other areas [13,20,24,36]. Most of the works use Darcy's law for the flow through porous medium which is valid for highly dense porous medium as well as for low Reynolds number flow. However, when the medium is sparsely packed, the Darcy law is not applicable, for in this case the medium one has to take into account the viscous shear, however small may be, in addition to Darcy resistance. In problems where permeability is small near the boundaries i.e. the particles are loosely packed so that there exists a boundary layer thickness very near the surfaces. This boundary layer type equation for flow through porous medium was proposed by Brinkman (4) based on the drag theory of particles constituting the porous media. In this model the particles of the fluid are taken as spheres and they are placed in position by external forces. Suppose that the flow is steady and incompressible, and inertial terms are negligible, the forces acting on volume elements of fluid containing the particles are considered. Let $F_i$ dv be the force on the surface of the volume element due to the normal and shearing stresses.

In view of the Navier–Stokes equations, we have

$$\vec{F} = \nabla p + \mu \nabla^2 \vec{V}$$

(1)

Where $p$ is the pressure, $\vec{V}$ is the velocity vector, $\mu$ is the viscosity and $\nabla^2$ is the laplacian operator.
Applying the theory of statistical mechanics the mean flow velocity $\bar{q}$ is given by

$$\bar{q} = \frac{1}{V_0} \int_0 V \, \mathrm{d}V$$

by where $V$ is the total volume, $V_0 (< V)$ is the volume under consideration.

Let $F_2 \, \mathrm{d}V$ be the force contributed by the particles on the velocity of the flow of the fluid. That is

$$F_2 = -\left(\frac{\mu}{K}\right) q$$

Where $K$ is a constant dependent on the particle density.

If we assume that only viscous forces are acting on the fluid, then

$$F_1 + F_2 = 0$$

Submitting (1) and (3) in (4) we obtain

$$-\nabla p + \mu \nabla^2 q - \frac{\mu}{K} \bar{q} = 0$$

We observe that if the particle densities are high, then the term $\mu \nabla^2 q/K$. Hence Darcy's law is a limiting case of the equation (5) for low permeabilities. Brinkman's model is proposed to account for large distortions of velocity near the boundary. This model is applicable for porous media both for low and high permeability.

Another noteworthy development in this theory of porous medium flow is due to Beavers and Joseph [1] who proposed a physically plausible empirical slip condition, for the tangential component of velocity, at the porous interface. This slip criterion had a great bearing on the subsequent investigations, mostly in the last decade, involving both plane [3,12,18,29,342,33,37] and curved [16,35] porous boundaries. Later, Saffman's [30] work showed that the empirical condition [1] may perhaps be best regarded as an approximate condition valid for small values of $k$, the permeability of the medium [16].
A detailed study of the boundary value problems in flow through porous media has been performed by Kotto and Masuoka [17], Masuoka [32] and Masuoka et al., [22] (discussed the heat transfer in porous layers) gave a conclusive criterion for the onset of convection currents in porous layers. Beavers and Joseph (1)m Saffman [30], Yamoto and Yashida [28], Yamamoto and Iwamura [38], Pattabhiramacharyulu [39] and many others have examined a variety of flow phenomenon in porous media.

I now briefly discuss the methodology adopted in the thesis. The finite element method is a powerful numerical technique devised to evaluate complex physical processes.

The finite element method is a technique in which a given domain is represented as a collection of simple domains, called finite elements, so that it is possible to systematically construct the approximation functions needed in a variation or weighted residual approximation of the solution of a problem over each element. The finite element method differs from the traditional Rayleigh – Ritz, Galerkin, least squares, collocation, and other weighted residual methods in the manner in which the approximation functions are constructed. However this difference is responsible for the following three basis features of the finite element method:

1. Division of the whole into parts, which allows representation of geometrically complex domains as collections of geometrically simple domains that enable a systematic derivation of the approximation functions.

2. Derivation of approximation functions over each element; the approximation functions are often algebraic polynomials that are derived using interpolation theory.

3. Assembly of elements, which is based on continuity of the solution and balancing of internal fluxes; the assemblages of elements reprints a discrete analogue of the original domain, and associated system of algebraic equations represents a numerical analogue of the mathematical model of the problem being analysed.
These three features, which constitute three major steps of the finite element formulation, are closely related. The finite element method not only overcomes the shortcomings of the traditional variational methods, but it is also endowed with the features of effective computational technique.

A traditional engineering interpretation of finite element method is given by Zienkiewicz [40] and its applications of fluid mechanics are treated by Shames [31] and Becker [32]. A mathematical perspective of this method is provided by Strong and Fix [34], Oden and Reddy [26] and Fletcher [11]. Keeping this in view, we propose to adopt finite element techniques in investigating the proposed problems which gives a satisfactory analysis of the complex physical solution.

In this thesis an attempt is made to discuss the flow in immiscible fluids through channels bounded by porous beds. The analytical and computational solutions are obtained by using finite element analysis with both linear and quadratic elements. The porous medium is sparsely packed so that the Brinkman’s model is used in the governing linear momentum equation. The thesis is comprised of five chapters.

Chapter 1: Flow of immiscible fluids through a channel bounded by porous beds.

This chapter aims in investigating the flow of two immiscible fluids bounded by porous beds making use of the finite element analysis with quadratic elements. The geometry of the interface of the immiscible fluids as well as the interface between each fluid and the adjacent porous bed are assumed to be known for the purpose of analysis. The clean fluid regions are governed by Navier-Stokes equations while the Brinkman model is used for the fluid through porous beds. The Ritz finite element analysis with line elements is used to obtain quadratic polynomial approximation solutions for the governing equations. The flow region is divided into four zones and the global matrix solution is obtained making use of interelement continuity and boundary conditions. Two cases are discussed viz. Firstly, when the flow is under the influence of a transverse magnetic field and secondly the non-magnetic case. The velocity and shear stresses are evaluated at different levels parallel to the boundary walls and their behaviour is discussed computationally for variations in the governing parameters. The advantage of
finite element analysis over the routine analytical investigation is that the former predicts
the intricacies of the flow behaviour with in the clean fluid region as well as the porous
bed region at different horizontal levels where as the later gives the overall behaviour of
the flow.

Chapter-2 : Heat transfer in immiscible fluids through a channel with porous beds
bounded by differentially heated rigid plates.

In this chapter, heat transfer in the flow of two viscous incompressible immiscible
fluids in a channel with porous beds bounded by differentially heated rigid plates, is
discussed. Solutions of the governing equations have been obtained by dividing the flow
region into four zones applying appropriate matching conditions. The velocity,
temperature and the shear stresses are evaluated using the finite element analysis and
their behaviour is discussed for variations in the governing parameters.

Chapter-3 : Forced convection in a channel bounded by porous beds.

In this chapter firstly we obtain the exact analytical solution of the flow and heat
transfer equations under the usual unidirectional flow assumption. This exact solution is
computationally analyzed to investigate the flow behaviour for variations in the
governing parameters. Secondly, the finite element analysis is made with eight nodded
serendipity elements taken across the boundaries, which felicitates in discussing the flow
behaviour in between parallel planes at unit distance apart normal to the flow direction.
The assembled global matrices for the unknown global nodes related to the velocity and
temperature are solved to obtain the velocity, temperature and the Nusselt number. Their
behaviour is discussed computationally for variations in the governing parameters.

Chapter-4 : Hagen – Poiseuille flow in circular duct bounded by porous bed

In this chapter, the Hagen-Poiseuille flow in circular duct bounded by a porous
lining is discussed using the finite element analysis. The Brinkman equation is used in the
porous bed while the Navier-Stokes equation governs the flow in the core region. The
velocity, shear stresses are evaluated and their behaviour is investigated for different
values of the governing parameters.
Chapter-5: Magnetohydrodynamic flow in circular duct bounded by a porous bed.

In this chapter, the influence of a radial magnetic field on the flow through a duct is discussed. The analytical and numerical solutions are obtained using the finite element method. The behaviour of the velocity, shear stresses under the influence of a magnetic field is discussed, making use of computational analysis.

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References

CHAPTER – I

MHD FLOW OF IMMISCIBLE FLUIDS THROUGH CHANNEL BOUNDED BY POROUS BEDS