CHAPTER 5

NEW MICROSLIP MODEL

5.1 INTRODUCTION

This Chapter deals with the improved microslip model set forth in the objective.

Scanlan and Swart (1968) pointed out for a quasistatic (dynamic) cable, the bending stiffness has spatial and temporal variations and ranges from about one-half to one-tenth of the theoretical maximum. The experimental findings of McConnell and Zemke (1980) indicate that the initial tangent bending stiffness is closer to that of loose wire assemblage and the value of (EI) found to scatter. But most of the theories yield higher values for initial tangent bending stiffness (EI). A new model is devised in this work that addresses this question.

For predicting the bending response of a helically wound pretensioned cable, two major hypotheses have been devised by earlier investigators for interwire slip during bending of a pretensioned strand, rope, cable etc., viz.

1. Coulomb friction stick-gross slip hypothesis which predicts a constant high initial tangent bending stiffness until slip initiates in a wire and a constant tangent low bending stiffness after all the wires in a strand cross section slip. The high initial stiffness is attributed to the sticking state when the wires contacting and clenching the core form a monolithic beam-like structure with no slip but rolling on
the core. The lower stiffness is due to the slipped state when all the wires having grossly slipped behave as an assemblage of individual wires.

2. Hertzian contact Coulomb friction microslip hypothesis which predicts a continuously increasing microslip (creep) and decreasing tangent stiffness right from the beginning of bending, finally resulting in gross slip and zero tangent stiffness (Mindlin 1949).

The initial stiffness predicted by both the hypotheses stated is found to be higher than that of experimental findings of various investigators. An attempt has been made here to devise a hypothesis and develop a new model that would narrow down the gap between the predicted and experimentally found responses of a strand under bending.

All assumptions earlier made (in Chapter 3) hold good here, except assumption 3 i.e. friction is limited. Further only one mode of slip in the wire axial direction is considered, since it is known to be the dominant one. The present work confines the study to the 'Resting-Lay' configuration of the stand cross section. The new model proposed rationally combines the two hypotheses as shown in Figure 4.1.


Hertzian contact Coulomb friction microslip hypothesis has been dealt by Hobbs and Raoof (1982, 1984) based on the theory introduced by Mindlin (1949).
Since the initial stiffness predicted by both the hypotheses stated are found to be higher than that of experimental findings of various investigators, the objective of the present work is to develop a model, based on the hypothesis discussed in Chapter 4, that would narrow down the gap between the predicted and experimentally found responses of a strand under bending.

5.2 NEW HYPOTHESIS

The present work confines the study to the 'Resting-Lay' configuration of the strand cross-section where helical wires have contact only with the core; not among themselves. (Figures 2.1 and 1.1b). The new model proposed rationally combines the two hypotheses as has been discussed in Chapter 4 (Figure 4.1).

The Coulomb friction stick-gross slip hypothesis is based on the wire axial displacement (normal to the wire cross section) \( \delta_1 \) as is obtained in a fiber in a beam. Since this has single body characteristics, only the modulus of elasticity \( E \) and the radius of cross section \( R \) of the wire together with the contact normal distributed force \( X_0 \) and the coefficient of friction \( \mu \) depict the stiffness of the wire. But the microslip hypothesis is based on the combined characteristics of two bodies in Hertzian contact where the relative shear displacement \( \delta_2 \) between the axes of the two bodies (wire and core) is caused by the application of a distributed force \( Z \) through their centre lines. Hence, the elastic moduli of the wire and core \( E \) and \( E_c \); the radii of the cross sections of the wire and the core \( R \) and \( R_c \) respectively; \( X_0, \mu \) and \( Z \), are all involved in depicting the stiffness. The displacements \( \delta_1 \) and \( \delta_2 \) considered here generally are not equal and the same.

Assuming \( \beta \) is small, wire and core are considered to be in line contact as if they are parallel. In order to understand the difference, a hypothetical stiffness could be defined for these hypotheses as,
\[ k_1 = \frac{dZ}{d\delta_1} \quad \text{(5.1)} \]
\[ k_2 = \frac{dZ}{d\delta_2} \quad \text{(5.2)} \]

Since \( \delta_1 \) and \( \delta_2 \) are caused by the action of one and the same force \( Z \) emanating from bending, the new hypothesis proposes a two springs-in-series concept so that the resultant relative displacement \( \delta \) could be expressed as a sum of \( \delta_1 \) and \( \delta_2 \). Hence, the effective stiffness will be given by

\[ k = \frac{dZ}{d(\delta_1 + \delta_2)} \quad \text{(5.3)} \]

Simplifying Equation (5.3) using Equations (5.1) and (5.2) leads to

\[ \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{(5.4)} \]

While the Equation (5.4) forms the basis for the new hypothesis, a further concept to relate \( \delta_1 \) and \( \delta_2 \) to the curvature of the strand is required. When a distributed wire axial force \( Z \) is developed due to bending, a curvature \( \kappa_1 \) is assumed to be proportional to \( \delta_1 \), another curvature \( \kappa_2 \) to be proportional to \( \delta_2 \) and the resultant curvature to be \( \kappa \) which is the sum of \( \kappa_1 \) and \( \kappa_2 \) depicting a higher curvature and a lower bending stiffness. This leads from Equation (5.4) to

\[ \kappa = \kappa_1 + \kappa_2 = \kappa_1 \left[ 1 + \frac{\kappa_2}{\kappa_1} \right] = \kappa_1 \left[ 1 + \frac{\delta_2}{\delta_1} \right] \quad \text{(5.5)} \]

5.3 ANALYSIS

The wire axial strain \( \varepsilon_w \) in the stick (partial slipped) region when the strand curvature is \( \kappa_1 \), is given by

\[ \varepsilon_w = \kappa_1 \, r \, \cos^2 \beta \, \cos \phi \quad \text{(5.6)} \]
and hence the wire axial force is

\[ T = EA \varepsilon_w = \kappa_1 EA r \cos^2 \beta \cos \phi \]  \hspace{1cm} (5.7)

The distributed force \( Z \) causing slip is given by

\[ Z = -\frac{dT}{dS} = \kappa_1 EA \cos^2 \beta \sin \beta \sin \phi \]  \hspace{1cm} (5.8)

where \( ds = r d\phi / \sin \beta \) is used. The displacement \( \delta_1 \) could be obtained by integrating the strain \( \varepsilon_w \) as

\[ \delta_1 = \int \varepsilon_w ds = \int \kappa_1 r \cos^2 \beta \cos \phi ds \]  \hspace{1cm} (5.9)

Applying the boundary condition \( u = 0 \) at \( \phi = 0 \), and from Equations (5.6) and (5.9)

\[ \delta_1 = \frac{\kappa_1 r^2 \cos^2 \beta}{\sin \beta} \sin \phi \]  \hspace{1cm} (5.10)

Eliminating \( \kappa \sin \phi \) from Equation (5.10) and introducing \( Z \) from Equation (5.8), the displacement \( \delta_1 \) is given by,

\[ \delta_1 = B Z \]  \hspace{1cm} (5.11)

where

\[ B = \frac{(r/R)^{1.3}}{E \pi \sin^2 \beta} \]  \hspace{1cm} (5.12)

which is same as that given by Hadj-Mimoune et al (1993) and Papailiou (1995, 1996) if the term \( (\mu^2 \sin^2 \beta) \) is ignored as negligibly small compared to unity.
In the microslip case, the shear displacement between the centrelines of two contacting cylinders has been considered. In the present strand a layer wire has continuous contact with the core wire and the lay angle $\beta$ is small. Hence the wire and core are approximated to be two parallel cylinders with line contact. Following Hobbs and Raoof (1982), the Hertzian normal displacement $\delta_n$ for two parallel cylinders as given in Roark and Young (1975) is,

$$\delta_n = \frac{C_E X_0}{\pi} \left[ \frac{2}{3} + \ln \left( \frac{4 D_1 D_2}{b^2} \right) \right]$$

(5.13)

where $b$, the width of rectangular contact area is,

$$b = 1.6 \left( X_0 K_D C_E \right)^{1/2}$$

(5.14)

In the above Equation (5.14),

$$C_E = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$$

(5.15)

and

$$K_D = \frac{D_1 D_2}{(D_1 + D_2)}$$

(5.16)

Substituting for $b^2$ from Equations (5.14)-(5.16), noting that $D_1 = 2R_c; D_2 = 2R; \nu_1 = \nu_2; E_1 = E_2$,

$$\delta_n = \frac{C_E X_0}{\pi} \left[ \frac{2}{3} + \ln \left( \frac{R + R_c}{0.32 X_0 C_E} \right) \right]$$

(5.17)
Multiplying both the numerator and denominator of the terms under logarithm by \( n \), and noting that \( 0.3271 \ll 1 \),

\[
\delta_n = \frac{C_E X_0}{\pi} \left[ \frac{2}{3} + \ln \left( \frac{(R + R_c) \pi}{X_0 C_E} \right) \right] \tag{5.18}
\]

\( S_{22} \) is defined as

\[
S_{22} = \frac{d\delta_n}{dX_0} \tag{5.19}
\]

Differentiating Equation (5.18) with respect to \( X_0 \),

\[
S_{22} = \frac{C_E}{\pi} \left[ -\frac{1}{3} + \ln \left( \frac{(R + R_c) \pi}{X_0 C_E} \right) \right] \tag{5.20}
\]

This coincides with the expression derived by Jolicoeur and Cardou (1996).

The tangential compliance along the wire axis for partial and initial slip cases of wire-core contact is derived in this paper using cue from Hobbs and Raoof (1982), where they confined this aspect only to two adjacent wires in contact in a layer. Assuming the same materials for the two bodies \( v_1 = v_2 = v \) and \( G_1 = G_2 = G \), though not necessary, the displacement \( \delta_2 \) is

\[
\delta_2 = \frac{3}{8} \frac{2 - v}{G} \frac{\mu X_0}{\mu X_0} \left[ 1 - \left( \frac{Z}{\mu X_0} \right)^{2/3} \right] \Phi \tag{5.21}
\]

Noting that \( S_{66} = \frac{d\delta_2}{dZ} \) and letting \( Z = 0 \) for initial compliance,

\[
S_{66} = \frac{2 - v}{4G} \Phi \tag{5.22}
\]
The relation between the initial compliances \((Z = 0)\) is given by Mindlin (1949) as

\[
S_{66} = S_{22} / (1 - v) \tag{5.23}
\]

Replacing \(\Phi\) in Equation (5.21) using Equations (5.22) and (5.23)

\[
\delta_2 = \frac{3}{2} \frac{\mu X_0 S_{22}}{1 - v} \left[ 1 - \left(1 - \frac{Z}{\mu X_0} \right)^{2/3} \right] \tag{5.24}
\]

which could be written as

\[
\delta_2 = D \left[ 1 - \left(1 - \frac{Z}{\mu X_0} \right)^{2/3} \right] \tag{5.25}
\]

where

\[
D = \frac{3}{2} \frac{\mu X_0 S_{22}}{1 - v} \tag{5.26}
\]

The relation between the distributed force \(Z\), bending curvature \(\kappa_1\) and the location of the wire in the strand cross section \(\phi\) is given in literature by Papailiou (1995, 1996) and Hadj-Mimoune et al (1993, 1994) to be

\[
Z = Z \kappa_1 \sin \phi \tag{5.27}
\]

where \(\overline{Z}\) is obtained from Equation (5.8) as

\[
\overline{Z} = E A \cos^2 \beta \sin \beta \tag{5.28}
\]

These expressions derived hold good for the location \(\phi\) of any wire which has not grossly slipped. Since \(Z = Z (\phi)\) in a wire of the strand bent with a given
curvature $\kappa$, it is necessary to average $\delta_1$ and $\delta_2$ over the appropriate region, as will be explained later. (Refer Equations (5.40), (5.41), (5.56) and (5.57). When gross slip has occurred,

$$Z = \mu X_0$$  \hspace{1cm} (5.29)

From Equations (5.27) and (5.29), at different wire locations $\phi$, where gross slip has occurred

$$\bar{Z}_K \sin \phi = \mu X_0$$ \hspace{1cm} (5.30)

The curvature at the initiation of gross slip $\kappa_0$ could be derived from Equation (5.27) and (5.30) when $\phi = 90^\circ$, as

$$\bar{Z}_{\kappa_0} = \mu X_0$$ \hspace{1cm} (5.31)

and hence

$$\kappa_0 = \frac{\mu X_0}{Z}$$ \hspace{1cm} (5.32)

There are $m$ wires in a strand cross section at discrete locations $\phi$. The problem needs the study of the states of all the wires in a cross section. If there is a large number of wires in a layer of the strand under constant curvature bending, a simpler method could be used. In that the state of a single wire could be considered along its length over a continuous range of $0 \leq \phi \leq \pi/2$, averaged over $\pi/2$ and multiplied by the number of wires $m$ to get reasonably accurate results. While the condition [Equation (5.29)] i.e. $Z = \mu X_0$ determines the initiation of slip, the gross slip boundary is given by the integrated values of Equation (5.29), as

$$\int Z \, dS = \int \mu X_0 \, dS$$ \hspace{1cm} (5.33)
Since $dS = r \, d\phi / \sin \beta$, Equation (5.33) yields,

$$\frac{r}{\sin \beta} \int Z\, d\phi = \frac{r}{\sin \beta} \mu X_0 \, d\phi$$  \hspace{1cm} (5.34)

In view of Equation (5.27), the left side of Equation (5.34) gives the wire axial force $T$ at $\phi$ and the right side gives the corresponding friction force at $\phi$. Thus, at the slip boundary denoted by $\kappa_b$

$$Z_{\kappa_b} \cos \phi_b = \left(\frac{\pi}{2} - \phi_b\right) \mu X_0$$  \hspace{1cm} (5.35)

From Equations (5.32) and (5.35), the corresponding curvature is

$$\kappa_b = \frac{\kappa_0}{\cos \phi_b}$$  \hspace{1cm} (5.36)

The above equation agrees with Papailiou (1995, 1996) for small values of $(\mu \sin \beta)$. At initiation of gross slip, $\phi_b = \pi/2$ in the limit in Equation (5.36) and hence $\kappa_b = \kappa_0$. For full gross slip $\phi_b = \phi_f = 0$ when $\kappa_1 = \kappa_b = \kappa_f$ and hence from Equation (5.36)

$$\kappa_f = (\pi/2) \, \kappa_0$$  \hspace{1cm} (5.37)

The states of the wire at different levels of bending are identified for the complete solution of the bending problem.

Stage 1: No gross slip: $\kappa_1 \leq \kappa_0$

Stage 2: Gross slip starts and progresses: $\pi/2 \geq \phi_b \geq 0$; $\kappa_a \leq \kappa_1 = \kappa_b \leq \kappa_f$

Stage 3: Gross slip completed and thereafter: $\phi_b = 0$; $\kappa_1 = \kappa_b \geq \kappa_f$
When stage 3 is reached, contributions from the wire axial force to the tangent bending stiffness is zero and only a constant tangent stiffness due to the wire couples of the loose wire assemblage would be available thereafter.

5.3.1 **Stage 1:** $\kappa_1 \leq \kappa_0$

Wire axial force is given by Equation (5.7) and with $\kappa = \kappa_1$,

$$T = EAr \kappa_1 \cos^2 \beta \cos \phi$$  \hspace{1cm} (5.38)

The layer bending moment due to $T$ is,

$$M_T = m \int_0^{\pi} rT \cos \beta \cos \phi \, d\phi$$  \hspace{1cm} (5.39)

Using Equation (5.38) in Equation (5.39)

$$M_T = \frac{m}{2} EAr^2 \kappa_1 \cos^3 \beta$$  \hspace{1cm} (5.40)

which is in agreement with Papailiou (1995, 1996).

The averaged value of $\delta_i$ is considered as explained following the Equation (5.32)

$$\bar{\delta}_i = \frac{1}{\pi} \int_0^{\pi} \delta_i \, d\phi$$  \hspace{1cm} (5.41)

Using Equations (5.11) and (5.27) in Equation (5.41)

$$\bar{\delta}_i = \frac{2}{\pi} B \bar{Z} \kappa_1$$  \hspace{1cm} (5.42)
Similarly, the averaged value of $\delta_2$ is,

$$\bar{\delta}_2 = \left( \frac{2}{\pi} \right) \int_0^{\pi} \delta_2 \, d\phi$$  \hfill (5.43)

Using Equations (5.25)-(5.27) in Equation (5.43)

$$\bar{\delta}_2 = D \left( \frac{2}{\pi} \right)^2 \left[ 1 - \left( 1 - \frac{Z \kappa_1 \sin \phi}{\mu X_0} \right)^2 \right] \, d\phi$$ \hfill (5.44)

Hence from Equation (5.5), replacing $\delta$ by $\bar{\delta}$,

$$\kappa_2 = \kappa_1 \left( \frac{\bar{\delta}_2}{\bar{\delta}_1} \right)$$ \hfill (5.45)

$$\kappa = \kappa_1 + \kappa_2 = \kappa_1 \left[ 1 + \left( \frac{\bar{\delta}_2}{\bar{\delta}_1} \right) \right]$$ \hfill (5.46)

5.3.2 Stage 2: $\pi/2 \geq \phi_b \geq 0$ and $\kappa_0 \leq \kappa_1 \leq \kappa_f$

Denoting fully slipped case by subscript $s$ and partially slipped case by $p$, the wire axial force in the fully slipped region ($\pi/2 \geq \phi \geq \phi_b$) is $T_s$ and that in the partial slipped region ($\phi_b \geq \phi \geq 0$) is $T_p$. The boundary $\phi_b$ is given by Equation (5.36). From RHS of Equation (5.34)

$$T_s = \mu X_0 \left( \frac{\pi}{2} - \phi \right) \frac{r}{\sin \beta}$$ \hfill (5.47)

and the corresponding layer bending moment
Using Equation (5.47) in Equation (5.48)

\[ M_{T_0} = m \left( \frac{2}{\pi} \right) \int_0^\pi \rho \, T_r \, \cos \beta \cos \phi \, d\phi \]  

(5.48)

Similarly, in the partial slip region from Equation (5.38)

\[ T_p = E \pi \kappa_1 \cos^2 \beta \cos \phi \]  

(5.50)

\[ M_{T_p} = m \left( \frac{2}{\pi} \right) \int_0^\pi T_p \cos \beta \cos \phi \, d\phi \]  

(5.51)

Using Equation (5.50) in Equation (5.51)

\[ M_{T_p} = \frac{m}{2} E \pi \kappa_1 \cos^2 \beta \kappa_1 \left( \frac{2}{\pi} \right) \left[ \phi_b + \frac{\sin 2\phi_b}{2} \right] \]  

(5.52)

\[ M_T = M_{T_s} + M_{T_p} \]  

(5.53)

For start of gross slip, \( \phi_b = \pi/2 \) and \( \kappa_1 = \kappa_0 \). Hence Equation (5.49) yields zero but Equation (5.52) yields,

\[ M_{T_0} = \frac{m}{2} E \pi \cos^2 \beta \kappa_0 \]  

(5.54)

For full gross slip, \( \phi_b = 0, \kappa_1 = \kappa_f = (\pi/2) \kappa_0 \) and Equation (5.52) yields zero but Equation (5.49) yields,
To find $\kappa, \delta_1, \text{ and } \delta_2$ are required.

\[
\delta_1 = \left( \frac{2}{\pi} \right) \left[ \frac{N}{\mu X_0} \int_{\phi_0}^{\phi_b} \delta_{1t} \, d\phi + \int_{\phi_0}^{\phi_b} \delta_{1p} \, d\phi \right] \quad (5.56)
\]

\[
\delta_2 = \left( \frac{2}{\pi} \right) \left[ \frac{N}{\mu X_0} \int_{\phi_0}^{\phi_b} \delta_{2t} \, d\phi + \int_{\phi_0}^{\phi_b} \delta_{2p} \, d\phi \right] \quad (5.57)
\]

where for fully slipped region from Equations (5.11), (5.27) and (5.30) and partly slipped region from Equations (5.25), (5.27) and (5.30),

\[
\delta_{1s} = B \mu X_0 \quad (5.58)
\]

\[
\delta_{1p} = B Z \kappa_1 \sin \phi \quad (5.59)
\]

\[
\delta_{2s} = D \quad (5.60)
\]

\[
\delta_{2p} = D \left[ 1 - \left( 1 - \frac{\kappa_1}{\kappa_0} \sin \phi \right)^2 \right] \quad (5.61)
\]

5.3.3 Stage 3: $\kappa_1 \geq \kappa_f$

For stage 3, the moment $M_{r_f}$ given by Equation (5.55) will remain constant and hence there is no additional tangent stiffness thereafter. Only the bending stiffness due to the wire couples $H$, $G$ and $G'$ will be present.
5.4 STRAND BENDING MOMENT

When the effects of wire shear forces $N$ and $N'$ are ignored (as they are small compared to the predominant wire axial force) the total moment due to layer is,

$$M_W = M_T + M_R$$  \hspace{1cm} (5.62)

where $M_R$ the moment due to rotation of the wires is

$$M_R = M_H + M_G + M_{G'}$$  \hspace{1cm} (5.63)

$$M_H = -m \left( \frac{2}{\pi} \right)^2 \int_0^\pi GJ \omega_{30} \sin \beta \cos \phi \, d\phi$$

$$= \frac{m}{2} GJ \sin^2 \beta \cos \beta \kappa_1$$  \hspace{1cm} (5.64)

$$M_G = \left( \frac{2}{\pi} \right)^2 \int_0^\pi EI \omega_{10} \sin \phi \, d\phi$$

$$= \frac{m}{2} EI \cos \beta \kappa_1$$  \hspace{1cm} (5.65)

$$M_{G'} = m \left( \frac{2}{\pi} \right)^2 \int_0^\pi EI \omega_{20} \cos \beta \cos \phi \, d\phi$$

$$= \frac{m}{2} EI \cos^2 \beta \kappa_1$$  \hspace{1cm} (5.66)

As shown by LeClair and Costello (1988) and Sathikh et al (2000)

$$\omega_{10} = \cos \beta \sin \phi \kappa_1$$  \hspace{1cm} (5.67)
The strand bending moment $M_b$ is given by

$$M_b = M_w + M_e \quad M_e = E_e I_0 \kappa$$  \hspace{1cm} (5.68)

### 5.5 BENDING STIFFNESS

$M_b$ and $\kappa$ give the strand moment curvature relation. The 
**effective secant bending stiffness of the strand** is given by

$$B = \frac{M_b}{\kappa}$$  \hspace{1cm} (5.69)

But what is of interest to the present study is the effective tangent bending stiffness. This requires the computation of slope as a function of strand curvature $\kappa$, from the $M_b-\kappa$ curve. This is carried out in the next Chapter.

### 5.6 RESULTS AND DISCUSSION

The numerical results are worked out for the parameters given in Table 5.1.

The wire initial distributed contact force $X_0$ is computed from the well-known Love's (1944) equation for axial tension of the strand without rotation and could be obtained from the expressions derived in literature, e.g. Sathikh, Moorthy et al (1996) for the given strand initial strain $\varepsilon_0$. The computations of $K_1$, $K_2$, $\kappa$ and bending moments are straightforward from the expressions given in the earlier sections. Figure 5.2 depicts the variation in the tangent bending stiffness with curvature for stages 1 and 2. The tangent bending stiffness is determined dividing $\Delta M_b$ by $\Delta \kappa$. This value is plotted at the midpoint of the two curvatures considered for $\Delta \kappa$. The equation of best fit is also shown in the figure for the two stages.
Table 5.1 Parameters used in the Numerical Example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity for the wire and core materials</td>
<td>$E, E_c$</td>
</tr>
<tr>
<td>Radius of the wire</td>
<td>$R$</td>
</tr>
<tr>
<td>Radius of the core</td>
<td>$R_c$</td>
</tr>
<tr>
<td>Lay angle</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Number of wires</td>
<td>$m$</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>$\nu$</td>
</tr>
<tr>
<td>Coefficient of friction</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Wire axial Strain</td>
<td>$\varepsilon_0$</td>
</tr>
</tbody>
</table>

* R > $R_c$ do not conform to 'resting-lay' configuration. Yet for comparison of the hypotheses they are chosen deliberately from Hadj-Mimoune et al (1993).

The moment-curvature at different stages of bending is given in Table 5.2 and Figure 5.1.

Table 5.2 Strand Curvatures and Bending Moments

<table>
<thead>
<tr>
<th>Curvature x 10^5 (mm⁻¹)</th>
<th>Bending Moment (N mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{Tp}$</td>
</tr>
<tr>
<td>- 1.00</td>
<td>2436.5</td>
</tr>
<tr>
<td>- 2.00</td>
<td>4872.9</td>
</tr>
<tr>
<td>- 3.00</td>
<td>7309.4</td>
</tr>
<tr>
<td>90°</td>
<td>3.39</td>
</tr>
<tr>
<td>60°</td>
<td>3.56</td>
</tr>
<tr>
<td>45°</td>
<td>3.77</td>
</tr>
<tr>
<td>30°</td>
<td>4.11</td>
</tr>
<tr>
<td>20°</td>
<td>4.42</td>
</tr>
<tr>
<td>10°</td>
<td>4.82</td>
</tr>
<tr>
<td>0°</td>
<td>5.34</td>
</tr>
</tbody>
</table>

$M_F = M_{Tp} + M_{Ty}$; $M_R = M_H + M_C + M_G$; $M_{a} = M_F + M_R$; $M_C = E_c L_c$; $M_{p} = M_{a} + M_C$
Curves 1 and 2 qualitatively represent the bending moments due to the earlier hypotheses 1 and 2.

Comparison of $B_{\text{min}}$ and $B_{\text{max}}$ for some selected models is given in Table 5.3

Hadj-Mimoune et al (1993, 1994) used the EPRI (1979) expressions for the upper and lower bounds of bending stiffness as given by

$$B_{\text{max}} = B_{\text{min}} + \left(\frac{m}{2}\right) EA r^2$$  \hspace{1cm} (5.74)

$$B_{\text{min}} = m (EI) + E_c I_c$$  \hspace{1cm} (5.75)
For $B_{\text{min}}$ and $B_{\text{max}}$ Papailiou (1996) used the following expressions:

\begin{align*}
B_{\text{max}} &= B_{\text{min}} + \left( \frac{m}{2} \right) EA r^2 \cos^3 \beta \\
B_{\text{min}} &= m \left( EI \right) \cos \beta + Ec I_c
\end{align*} \tag{5.76} \tag{5.77}

If $\beta$ is taken as zero (i.e. $\cos \beta = 1$), these reduce to that of EPRI. A more general model for the Preslip response of strand bending has been proposed and evolved in Chapter 3. The Equation 3.44, of Model 8 (reproduced for completeness) is

\begin{align*}
B_{\text{max}} &= \left( \frac{m}{2} \right) \left[ EA r^2 \cos^3 \beta - \sin^2 \beta \cos^3 \beta \left( EI \cos^2 \beta + GJ \sin^2 \beta \right) + GJ \sin^2 \beta \cos^3 \beta \\
&\quad + EI \cos \beta \left( 1 + \sin^2 \beta \right) + EI \cos^5 \beta \right] + Ec I_c \tag{5.78}
\end{align*}

and neglecting the effect of $N'$

\begin{align*}
B_{\text{min}} &= \left( \frac{m}{2} \right) \left[ GJ \sin^2 \beta \cos^3 \beta + EI \cos \beta \left( 1 + \sin^2 \beta \right) + EI \cos^5 \beta \right] + Ec I_c \tag{5.79}
\end{align*}

For the new microslip model, $B_{\text{max}}$ is given by, the initial tangent bending stiffness ($k=0$)

\begin{align*}
B_{\text{max}} &= B_1 / \left( 1 + k_1 / k_{o2} \right) + B_{\text{min}} \tag{5.80}
\end{align*}

where $B_1$ is obtained from Equation (5.40) by dividing $M_T$ by $\kappa_1$; and $k_1$ and $k_{o2}$ from Equations (5.1), (5.2), (5.11), (5.12), (5.25) and (5.26), (where $k_{o2}$ corresponds to $k_2$ at $Z=0$). $B_{\text{min}}$ is given by

\begin{align*}
B_{\text{min}} &= \left( dM_R / d\kappa \right) + Ec I_c \tag{5.81}
\end{align*}

where $M_R$ and $\kappa$ are given by Equation (5.63) and (5.46) respectively.
The numerical values for the input parameters given in Table 5.1 are:

\[ B_1 = 2.4365 \times 10^8 \text{ N mm}^2; \quad k_1 = 21038 \text{ N/mm}; \]
\[ k_{02} = 30703 \text{ N/mm}; \quad B_{\text{min}} = 0.4211 \times 10^8 \text{ N mm}^2 \text{ and} \]
\[ B_{\text{max}} = 1.867 \times 10^8 \text{ N mm}^2 \]

While in almost all investigations, the difference in \( B_{\text{min}} \) is only marginal as can be seen from Table 5.3, there is wide variation in \( B_{\text{max}} \). Hadj-Mimoune et al (1993, 1994) have chosen the simplest EPRI model for which \( B_{\text{max}} \) is the highest \( (3.387 \times 10^8 \text{ N mm}^2) \). The general model (hypothesis 1), discussed in Chapter 4, that includes the effect of lay angle \( \beta \), yields \( 2.9149 \times 10^8 \text{ N mm}^2 \) for \( B_{\text{max}} \). This \( B_{\text{max}} \) is about 86% that of EPRI. The \( B_{\text{max}} \) for Papailiou’s (1996) model is \( 2.8627 \times 10^8 \text{ N mm}^2 \) and is closer to that of the preslip model developed and discussed in Chapter 3.

### Table 5.3 Comparison of \( B_{\text{min}} \) and \( B_{\text{max}} \) (N mm\(^2\))

<table>
<thead>
<tr>
<th>Investigator</th>
<th>( B_{\text{min}} )</th>
<th>( B_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hadj-Mimoune et al (1993)</td>
<td>( 0.4507 \times 10^8 )</td>
<td>( 3.3870 \times 10^8 )</td>
</tr>
<tr>
<td>Papailiou (1996)</td>
<td>( 0.4262 \times 10^8 )</td>
<td>( 2.8627 \times 10^8 )</td>
</tr>
<tr>
<td>Preslip Model (Present) [Chapter 3]</td>
<td>( 0.4217 \times 10^8 )</td>
<td>( 2.9149 \times 10^8 )</td>
</tr>
<tr>
<td>Present (Microslip Model)</td>
<td>( 0.4211 \times 10^8 )</td>
<td>( 1.8670 \times 10^8 )</td>
</tr>
</tbody>
</table>

The present microslip model, which uses the two-springs-in-series concept, predicts a lower \( B_{\text{max}} \) as \( 1.867 \times 10^8 \text{ N mm}^2 \), as expected. The modified initial effective bending stiffness of Hadj-Mimoune et al (1993, 1994) for the layer-core slip (as read from their graph) is \( 1.827 \times 10^8 \text{ N mm}^2 \) which differs from the present with wire-core slip only by 2%, though subsequent values of \( B \) are different. The lower stiffness found
experimentally by Scanlan and Swart (1968) and McConnell and Zemke (1980) justify the low $B_{\text{max}}$ of the present model. Figure 5.2 indicates the slip path ($B - \kappa$ relation) for the present microslip model. This forms the basis for further development to study variable bending curvature cable structure.

![Figure 5.2 Schematic diagram indicating the Slip path](image)

### 5.7 CONCLUSION

A new hypothesis with two-springs-in-series concept has been developed for predicting the bending stiffness and hysteresis loss of a resting-lay strand. The familiar Coulomb friction stick-gross slip concept has been combined with the Mindlin's (1949) Coulomb friction microslip concept, adopted by Hobbs and Raoof (1982) in developing this new hypothesis. The soundness of this model is justified by
its low stiffness, as experimentally found by other investigators. In the present paper the new hypothesis is confined to the strand under constant strand curvature bending. Hypothesis 1 has been applied earlier by Sathikh (1989), Hardy (1990), Hadj-Mimoune et al (1994) and Papailiou (1995, 1996) for the study of bending stiffness in a vibrating cable where strand curvature varies along the span.

This completes the work set forth to achieve in the objective (aim) at the beginning of this thesis report, in that an improved microslip model to predict the bending response of the cables both in preslip and post slip conditions.

However, a physical model and experimental study is described in the following chapters.