CHAPTER 4

MICROSLIP HYPOTHESIS FOR WIRE-CORE SLIP

4.1 INTRODUCTION

For all integral solid cross sections, the stiffness is defined as a combination of the material and geometrical properties of the particular integral body. In the case of a wire and a core of a strand in contact there are two bodies under normal and axial-shearing forces (Figure 4.1).

Figure 4.1 Wire and core as parallel cylinders in contact under axial distributed force, Z.
Hence, there are three additional parameters viz. the normal distributed contact force $X_0$, the coefficient of friction $\mu$ (both representing the limiting shearing force $\mu X_0$) and the tangential distributed force $Z$. Thus the equivalent modulus of elasticity for this system is a function of the elastic moduli $E$ and $E_C$, and radii $R$ and $R_C$, respectively of the wire and the core and the forces $X_0$ and $Z$ and the coefficient of friction $\mu$. Based on Mindlin's (1949) theory, Hobbs and Raoof (1982) developed an expression for the tangential compliance for two parallel cylinders in contact subjected to a set of two shearing forces $Z$ applied along the centre lines of the core and the wire. In bending, this force is normally computed by a constitutive equation based on the wire axial (normal) strain $\varepsilon_w$, which is related to the axial deformation (compliance) of the wire cross section caused by the wire fibre stress. Hadj Mimoune et al (1993) adopted a hypothesis, where the maximum of this axial deformation of the wire centre at the point of gross slip due to Coulomb friction was (mistakenly) equated to the maximum shear deformation of the wire centre due to microslip derived by Hobbs and Raoof (1982) to obtain the bending stiffness. The various experimental studies have inferred that the strand bending stiffness is much lower right from the start of bending of the strand. This needs a different hypothesis to predict this lower value of stiffness.

According to the new microslip hypothesis, these two deformations are considered as two different and independent deformations of the centre line as a result of one and the same set of forces $Z$ applied along the centre lines of the core and wire. Hence, a system of two-elastic-springs- in-series is adopted and an effective lower elastic modulus $E_{\text{eff}}$ is derived.

### 4.2 Analysis

For adoption to bending of the wire in a strand, Hadj Mimoune et al (1993) used a shear displacement $\delta$ between the lines of forces passing through the centre of the wire and the surface of contact on the core in the preslip Coulomb stick friction
stage. Since Mindlin's (1949) formulation represents the displacement between the
two lines of forces passing through the centres of the wire and the core, a new \( \delta_1 \) is
defined in this case considering the distance between the lines of forces, as equal to
the distance \( r' \) between the centre lines. **This is the first difference in the present
work.** Thus the wire axial displacement \( \delta_1 \) modified from Hadj Mimoune (1993) and
ignoring the term \( (\mu^2\sin^2\beta) \) as negligibly small compared to unity, is

\[
\delta_1 = \frac{Z (r / R)^2}{E \pi \sin^2 \beta}
\]

where \( \beta = \) lay angle and \( r = R + R_C \) (approximately) since only contact between wire
and core and no contact between adjacent wires in the layer is assumed. (Detailed
derivation is given in Chapter 5). Under no slip condition a stiffness \( k_1 \), is derived
from equation (4.1) as,

\[
k_1 = \frac{dZ}{d\delta_1} = \pi E (R/r)^2 \sin^2 \beta
\]

which is a constant slope as long as \( \delta_1 \) is not large enough to cause slip and also is the
initial tangent stiffness.

In the microslip case, the shear displacement between two parallel
cylinders has been considered by Hobbs and Raoof (1982) after Mindlin (1949). In the
present case with a single layer of wires continuously in contact with a single core
wire the parallel cylinder concept is adopted as a good approximation since \( \beta \) is very
small normally. This displacement is

\[
\delta_2 = \frac{3}{8} \frac{(2-v)}{G} \mu X_0 \left[ 1 - \left( \frac{Z}{\mu X_0} \right)^2 \right]^\frac{3}{2}
\]
where the shear modulus is

\[ G = \frac{E}{2(1 + \nu)} \]  

(4.4)

\[ \phi = \frac{4G}{(1 - \nu)(2 - \nu)} S_{22} \]  

(4.5)

\[ v = \text{Poisson's ratio} \]

and the initial normal compliance is

\[ S_{22} = \frac{2(1 - \nu^2)}{\pi E} \left\{ -\frac{1}{3} + \ln \frac{1}{0.64(1 - \nu^2)} + \ln \left[ \frac{(R + R_c)}{X_o} \right] \right\} \]  

(4.6)

where \( E_c = E \) has been assumed for simplicity though not necessary. It may be noted that the formulation of Equations (4.4)-(4.6) rests on the Hertzian contact mechanics adopted by Mindlin (1949) extending it for Coulomb friction microslip case. Mindlin (1949) has derived the relation between the normal compliance to the tangential compliance only. Hobbs and Raoof (1982) used this only for wire-wire contacts in the layer and derived \( S_{22} \). While for the wire-core contact, normal and transverse compliances have been adopted by Jolicoeur and Cardou (1996), they did not extend the same for the tangential compliance along the wire axis. This aspect is considered in the present work. This is the second difference. A stiffness \( k_2 \) is derived from equation (4.4) as

\[ k_2 = \frac{dZ}{d\delta_2} = k_{02} \left( 1 - \frac{Z}{\mu X_o} \right)^{\frac{1}{3}} \]  

(4.7)

where \( k_{02} \) corresponding to \( k_2 \) at \( Z = 0 \) depicting the initial tangent stiffness and used for simplifying the terms, is given by
The effective initial stiffness $k_{\text{eff}}$ for two-springs-in-series is given by

$$k_{\text{eff}} = \frac{1}{1 + (k_1/k_2)}$$

In bending, the wire axial force is related to $\varepsilon_{\text{w}}, E$ and $A$ and $(EA)$ is involved in the basic constitutive equations, representing the wire stiffness in the Coulomb stick friction case. Therefore, $k_1$ is taken as a reference to be proportional to $E$. Similarly, the effective modulus of elasticity $E_{\text{eff}}$ is taken proportional to $k_{\text{eff}}$. Hence, it may be inferred from equation (4.9) that

$$\left(\frac{E_{\text{eff}}}{E}\right) = \left(\frac{G_{\text{eff}}}{G}\right) = \left(\frac{k_{\text{eff}}}{k}\right) = \frac{1}{1 + (k_1/k_2)}$$

This forms the basis for the new microslip bending model developed in the next Chapter.