CHAPTER 5
SYSTEM MODELLING FOR ALPHA BIO-FEEDBACK

Based on the experimental studies reported in the previous two chapters, a model based on the feed-back control system is proposed and explained in this chapter.

5.1 SYSTEM MODEL FOR ALPHA BIO-FEEDBACK

A typical alpha EEG wave form is shown in Fig.5.1. On visual examination it resembles an amplitude and frequency modulated carrier. It may be represented by [John G. Okyere (1986)]

\[
f(t) = A_c \left[ 1 + k_a m_a(t) \right] \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m_f(t) \, dt \right]
\]

(5.1)

where

- \(A_c\) is the unmodulated carrier amplitude
- \(f_c\) is the frequency of the unmodulated carrier wave
- \(k_f\) is frequency modulation index
- \(k_a\) is amplitude modulation index
- \(m_a(t)\) is a modulating signal for the AM process
- \(m_f(t)\) is a modulating signal for the FM process

A control system based model for alpha bio-feedback incorporating AM/FM alpha EEG source is shown in Fig.5.2. The EEG source is connected to a suitable amplifier in the forward path. In the feedback path a suitable
FIG. 5.1 ALPHA EEG RHYTHM FROM A NORMAL SUBJECT

FIG. 5.2 CONTROL SYSTEM MODEL FOR ALPHA BIO-FEEDBACK
alpha split band filter, a phase shifting network and a visual activator are connected as shown in Fig.5.2.

5.2 EFFECT OF PHASE SHIFT ON ALPHA EEG SOURCE

The experimental results on alpha bio-feedback reported in the previous chapter 4 indicate that an introduction of phase shift to alpha band II signals changes the spectral distribution of power content. One possible explanation for the change in spectral power content is another amplitude modulating factor which is phase shift dependent. Hence the EEG source earlier represented in Equation 5.1 with phase-shifted visual feedback activation may be represented by

\[
X(t) = A_v \left[ 1 + k_{m}(t) \right] \left[ f(o) \right] \frac{1}{\pi} \cos \left( 2\pi f t + 2\pi k_r \right) \int_0^t m_i(t) \, dt \quad (5.2)
\]

where \( f(o) \) is the new amplitude modulating factor due to phase shift.

We may represent \( f(o) \) as a power series

\[
f(o) = A_0 + A_1 o + A_2 o^2 + A_3 o^3 + \ldots \quad (5.3)
\]

5.2.1 Estimation of \( f(o) \)

We shall now attempt to estimate \( f(o) \), based on the experimental results reported in chapter 4.

From Table 4.5 we extract the following readings.

<table>
<thead>
<tr>
<th>Phase Angle</th>
<th>0°</th>
<th>120° (2.07 rad)</th>
<th>240° (4.14 rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>% power in alpha</td>
<td>10.00</td>
<td>15.10</td>
<td>12.6</td>
</tr>
<tr>
<td>% power in alpha normalised with 0°</td>
<td>1</td>
<td>1.51</td>
<td>1.26</td>
</tr>
</tbody>
</table>
Substituting these values in Equation 5.3, we get

\[ A_0 = 1 \]

\[ A_0 + 2.07 A_1 + 4.386 A_2 = 1.51 \]

\[ A_0 + 4.14 A_1 + 17.54 A_2 = 1.24 \]

By solving the above set of equation, we get

\[ A_0 = 1, \ A_1 = 7.8 \text{ and } A_2 = -1.75 \]

Hence \( f(\phi) = 1 + 7.8 \phi -1.75 \phi^2 \) (5.4)

5.3 VALIDATION OF THE PHASE-SHIFT MODEL EQUATION

From the experimental results reported in chapter 4, we have seen that the maximum alpha band power is reached with a phase shift of 115°. By using Equation 5.4 and making \( df(\phi)/d\phi = 0 \), we get the maximum power at 126° phase shift. This indicates an error of 10% between the model prediction and the experimental results. This may be attributed to the use of truncated power series expansion for \( f(\phi) \), limited to only 3 terms.