CHAPTER 4

HIERARCHICAL STATE ESTIMATION
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4.1 INTRODUCTION

Nowadays, state estimation in power systems has become a basic function. Its basic role is the generation of a reliable and a complete real time data base from information provided by signalling and measurements. The integrated state estimators (ISE) like FDSE are fast and efficient. They meet the on line requirements of large scale power systems.

The problem of on line state estimation for very large scale power systems containing thousand or more buses presents a formidable computational challenge. The difficulties to satisfy the conflicting requirements on speed, accuracy, low memory occupation and capability of detecting-identifying anomalous data increase with the system's size. The ISE algorithms are not suitable for such systems because of the following two main reasons:

* ISE algorithms ignore the natural division of the very large system into subsystems, complicating the whole computational procedure. For example, bad data may be present in one subsystem only and not in the whole system.
* Computational time for the overall estimation procedure is too long.

Many research papers [43-49] have been published on this topic. The basic idea is to reduce the complexity associated with the integrated high order problem by decomposing it into lower order sub-problems. Hierarchical state estimators are used to tackle the computational burden of ISE methods. A critical survey of hierarchical methods for state estimation of electric power systems is provided by Van Cutsem et al [47]. The survey lists the essential properties of HSE methods. The properties are briefly described below:

* Applicability Requirements: they may concern the decomposition scheme, the measurement configuration, redundancy etc.

* Robustness: aptitude to converge towards an acceptable solution under a wide range of circumstances (topology, load, measurement configuration, presence of large bad data).

* Accuracy: the final solution must be accurate enough for operating purposes if not optimal.

* Adaptability to the organisation schemes to be suited.
* Bad data analysis - to be good.
* Information exchange must be minimum.
* Gain in computing time must be attractive.
* Observability: adaptation to integrated observability methods must be good.

The HSE method of Van Cutsem et al [48] meets most of the properties listed above except that the method is sub-optimal and that measurements on short tie-lines must be provided to improve the accuracy of such tie-line estimates. In short, this HSE technique consists in performing a two-level calculation. In the lower (first) hierarchical level, a conventional basic WLS state estimation is carried out simultaneously but independently for all the subsystems. These local estimations are then coordinated at the upper (second) hierarchical level using again a basic WLS estimator. It is claimed in [49] that when this HSE algorithm is applied to a very large scale system, the size of the second level estimator may be much greater than that of the biggest subsystem estimator, with no further decomposition possible. Therefore, this algorithm does not seem to be tailored to very large systems.

To overcome the above deficiency a simple HSE method has been developed by Kurzyn [49]. But again his method has a disadvantage that it does not use the tie-line reactive flow
measurements. Moreover, the method requires a particular order in which individual subsystem estimators are to be coordinated to reduce the error in the estimation of subsystem's reference angles.

It is suggested in the discussion of [49] by Van Cutsem et al that to reduce the computational burden associated with the second level algorithm of his method, the well-known constant and/or decoupled gain matrix techniques along with sparsity programming can be exploited.

In this Chapter, the HSE method of Van Cutsem et al [48] is extended by making use of constant decoupled gain and Jacobian matrices along with sparsity-oriented bifactorization technique for the second level estimator as well as for the first level estimators. In the lower level the robust FDSEs are utilised. For the upper level, making use of the assumptions made in the FDSE, a modified fast decoupled state estimator is developed that is suitable for handling tie-line flow measurements and pseudomeasurements consisting of estimated states of boundary buses only. This particular choice of measurement set makes the algorithm simple and efficient.

The suitability and the main features of the overall procedure are illustrated on the basis of IEEE 14 bus system, divided into three subsystems for HSE. The results are compared with ISE.
4.2 CONCEPT OF THE HSE APPROACH

Definitions

Let the overall system be composed of $K$ non-overlapping areas (or subsystems) $A_1, A_2, \ldots, A_K$. Let $S_i$ denote the set of buses of the $i$th area and $S$ denote the set of buses of the overall system. It is obvious that

$$S_i \cap S_j = S_{\emptyset} \quad i \neq j, \quad j = 1, 2, \ldots, K$$

$$\bigcup_{i=1}^{K} S_i = S$$

where $S_{\emptyset}$ is the null set. The subsystems are interconnected by means of tie-lines which end at buses belonging to these subsystems. These buses are called the boundary nodes and are denoted by set $S_c$. This set $S_c$ defines a $(K+1)$th area, called the interconnection area. The subset $S_c^i$ will be the set of the boundary nodes belonging to the $i$th subsystem. It is obvious that

$$S_c^i = S_c \cap S_i$$

Now, for the $i$th subsystem there are

* $N_i$ nodes related to the set $S_i$
* $N_c^i$ boundary nodes related to the set $S_i \cap S_c$
In the \((K + 1)\)th interconnection area there are \(N_c\) nodes related by

\[
N_c = \sum_{i=1}^{K} N^i_c
\]

Fig. 4.1 illustrates the above definitions for the case of \(K = 3\).

The state vector of \(A_i\) \((i = 1, 2, \ldots, K)\) is determined by the composite vector

\[
X_i = [X^iT_c, X^iT_I]^T
\]

where the vector \(X^iT_c\) consists of the voltage magnitudes and phase angles at all its boundary nodes and the vector \(X^iT_I\) consists of the voltage magnitudes and phase angles at all its internal nodes.

In the \((K + 1)\)th interconnection area, the composite coordination state vector is defined as

\[
X_c = [X^1T_c, X^2T_c, \ldots, X^KT_c]^T
\]

and the \((K - 1)\) dimensional phase vector is defined as
\[ S_1 = (1,2,3,4,5) \]
\[ S_2 = (7,8,9,10) \]
\[ N_1 = 5, N_{L1} = 7 \]
\[ N_2 = 4, N_{L2} = 3 \]

\[ S_3 = (6,11,12,13,14) \]
\[ N_3 = 5, N_{L3} = 5 \]

\[ S_c = (4,5,6,7,9,10,11,14) \]
\[ N_c = 8, N_{L4} = 5 \]

Tie-line power flow measurements

FIGURE 4.1: Principle of System Aggregation
\[ u = [u_2, u_3, \ldots, u_K]^T \]

where \( u_i \) represents the phase angle of the slack-bus of the \( i \)th subsystem. The slack-bus of the first (A_1) subsystem is arbitrarily taken as the reference of the overall system (\( u_1 = 0 \)) as shown in Fig.4.1.

The overall system is represented by the integrated vector

\[ Z = f(X) + e \quad \ldots \quad (4.1) \]

where

- \( Z \) is the \( m \)-dimensional measurement vector
- \( e \) is the \( m \)-dimensional measurement error vector
- \( X \) is the \( n \)-dimensional state vector
- \( f(X) \) is the \( m \)-dimensional non-linear vector function

Within the framework of the HSE approach the measurement vector \( Z \) will be decomposed as follows:

\[ Z = [Z_1^T, Z_2^T, \ldots, Z_K^T, Z_u^T]^T \quad \ldots \quad (4.2) \]

where \( Z_i \) and \( Z_u \) denote the measurement vectors of the \( i \)th subsystem (\( i = 1, 2, \ldots, K \)) and of the interconnection area respectively.

The vector \( Z_i \) may comprise all kinds of measurements such as active and reactive power flows, injections and
voltage magnitudes available in the ith area.

The vector $Z_u$ consists of active and reactive power flows in the interconnection tie-lines exclusively. In principle, when decomposing $Z$ according to (4.2), it is implicitly assumed that each measurement belongs to only one area.

The above defined decomposition-aggregation rule is by no means restrictive. Any type of decomposition-aggregation, such as functional, geographical etc can be employed.

General Organization Scheme

A general scheme of the two-level state estimator is described in Fig.4.2. As can be seen, the state estimation of all $K$ areas is assumed to be carried out simultaneously and independently from each other. Independence between areas is in fact realized by neglecting some measurements. These are the components of $Z_u$ which cannot be treated at this stage. In order that each subsystem may run independently its own state estimation, it is necessary that each subsystem possesses its own slack-bus. Therefore, treating the $Z_u$ vector and estimating all the power flows in the interconnection tie-lines implies evaluating the vector $u$. These are the main tasks of the (coordination) upper hierarchical level.
FIGURE 4.2: Scheme of Two Level State Estimator

ESTIMATING $u$ and $X_c$

INTERCONNECTION
AREA

SECOND LEVEL

LOCAL ESTIMATOR

$X^{1}_c$

LOCAL ESTIMATOR

$X^{2}_c$

LOCAL ESTIMATOR

$X^K_c$

LEVEL

FIRST

AREA $A_1$

AREA $A_2$

AREA $A_K$

$Z_1$

$Z_2$

$Z^K$

$Z_u$
It should be noted that estimating the tie-line flows is of great importance because exchange between the subsystems present often financial interest and/or because some of the tie-lines may transport part of the total system power. It should also be emphasized that the proposed two-level procedure implies only few data communication between lower and upper level computers.

4.3 FIRST-LEVEL STATE ESTIMATION (FSE)

The choice of the state estimation algorithm for each of the K subsystems is arbitrary and may differ from one another. In this paper the robust FDSE developed based on sparsity oriented bifactorization technique is utilised for each subsystem.

Adopting the general expression (4.1) to the ith subsystem, the relation between $Z_i$ and $X_i$ becomes

$$Z_i = f_i(X_i) + e_i$$  \hspace{1cm} \ldots (4.3)

The best estimate $\hat{X}_i$ of $X_i$ is defined as the value of $X_i$ which minimises the WLS criterion

$$J_i(X_i) = [Z_i - f_i(X_i)]^T W_i^{-1} [Z_i - f_i(X_i)]$$  \hspace{1cm} \ldots (4.4)

where $W_i = E [e_i e_i^T]$
As shown in Chapter 2, the final equations of the robust FDSE is arrived at after setting $\frac{\partial J_i}{\partial X_i} = 0$, linearising and utilising the assumption made in developing the FDSE, for the $i$th subsystem.

It should be noted that each subsystem uses solely its own data and measurements, centralized to its own dispatching centre. It is again noted that the phase angles of all the buses belonging to an area are computed with respect to the angle of its own slack-bus.

The result of the above FSE is the estimate $\hat{X}_i$, $i = 1,2, ..., K$. The part of the estimate relative to the boundary nodes, i.e. the subvector $\hat{X}_c^i$, are then sent to the coordination (second) level.

4.4 SECOND LEVEL STATE ESTIMATION (SSE)

As already mentioned, each subsystem state vector $X_i = [X_i^{1T}, X_i^{2T}]^T$ $i = 1,2, ..., K$ has been estimated in its own subsystem, which possesses its own slack-bus. Estimating the overall system's state vector and therefore the interconnection power flows necessitates estimating the vector $u$ and (re)evaluating the vector $X_c$ which is given by

$$X_c = [X_c^{1T}, X_c^{2T}, \ldots, X_c^{KT}]^T$$

...(4.5)
where $X_c^i$ consists of voltage magnitudes and phase angles of the ith subsystem's boundary nodes. That is

$$X_c^i = [b_c^i, v_c^i]^T$$ ...(4.6)

The composite second level state vector is

$$X_s = [u^T, X_c^T]^T$$ ...(4.7)

As far as the second level measurement vector, it will be defined by the composite vector

$$Z_s = [Z_u^T, \hat{X_c}^T]^T$$ ...(4.8)

That is, $Z_s$ will consist of the following subvectors:

- $Z_u$ the $m_u$-dimensional measurement vector consisting of real and reactive power flows in the inconnection tie-lines

- $\hat{X_c}$ the value of vector (4.5) resulting from the FSE of all subsystems. Because of lack of coordination at the first level, the component of $\hat{X_c}$ may be considered as measurements affected by errors. In the sequel, they may be referred as pseudodomeasurements to emphasize the fact that they are not standard measurements.

Making use of the general relationship (4.1), $Z_s$ can be written as
\[ Z_s = f_s(X_s) + e_s \] ...(4.9)

The above equation (4.9) can be written as given below utilizing (4.7) and (4.8).

\[
\begin{bmatrix}
Z_u \\
X_c
\end{bmatrix} =
\begin{bmatrix}
f_u(u, X_c) \\
X_c
\end{bmatrix} +
\begin{bmatrix}
e_u \\
e_c
\end{bmatrix} \quad \text{...(4.10)}
\]

where

\[ E[e_u] = 0 \]
\[ E(e_u e_u^T) = W_u \] ...(4.11)

and

\[ E[e_c] = 0 \]
\[ E[e_c e_c^T] = W_c \] ...(4.12)

The value of \( W_c \) can be calculated by computing the estimation error variances for the states of the boundary buses of each subsystem, at the end of the FSE. These variances may be calculated in the off-line mode. Thus, the second level covariance matrix is

\[
W_s =
\begin{bmatrix}
W_u & 0 \\
0 & W_c
\end{bmatrix} \quad \text{...(4.13)}
\]
**Modified Algorithm of the SSE**

By minimizing the quadratic criterion

\[ J_s(X_s) = [Z_s - f_s(X_s)]^T W_s^{-1} [Z_s - f_s(X_s)] \]  \( \ldots (4.14) \)

the optimality condition becomes

\[-2H_s W_s^{-1} [Z_s - f_s(X_s)] = 0 \]  \( \ldots (4.15) \)

where

\[ H_s = \delta f_s / \delta X_s \]

The equation (4.15) after linearisation yields

\[ [H_s^T(X_s^i) W_s^{-1} H_s(X_s^i)] [X_s^{i+1} - X_s^i] = H_s^T(X_s^i) W_s^{-1} [Z_s - f_s(X_s^i)] \]  \( \ldots (4.16) \)

The above algorithm given by (4.16) may impose large computational burden in case the size of the interconnection area becomes large. The decoupling between the active and reactive quantities is utilized to develop an efficient algorithm. Further, the assumptions made in developing the robust FDSE are also utilized. Since the measurements for the second level consist of line flows and \( \hat{X}_C \) only the algorithm becomes simple.

Let the subscripts \( p \) and \( q \) represent the active and reactive quantities respectively. Then
\[
Z_s = \begin{bmatrix} Z_{sp} \\ Z_{sq} \end{bmatrix} = \begin{bmatrix} f_{sp} (X_{sp}, X_{sq}) + e_{sp} \\ f_{sq} (X_{sp}, X_{sq}) + e_{sq} \end{bmatrix} \quad \cdots(4.17)
\]

where

\[
Z_{sp} = \begin{bmatrix} P_{Fs} \\ \hat{\theta}_c \end{bmatrix} \quad \text{real power tie-line flows in the inter-connection area}
\]

\[
Z_{sq} = \begin{bmatrix} Q_{Fs} \\ \hat{V}_c \end{bmatrix} \quad \text{reactive power tie-line flows in the inter-connection area}
\]

\[
x_s = [u^T, x_c^T]^T = [u^T, 6_{c}^T, v_c^T]^T = [x_{sp}^T, x_{sq}^T] \quad \cdots(4.18)
\]

where

\[
x_{sp} = [u^T, 6_{c}^T]^T \quad \cdots(4.19)
\]

\[
x_{sq} = [v_c^T]^T \quad \cdots(4.20)
\]

\[
f_{sp} = \begin{bmatrix} P_{Fs} (X_{sp}, X_{sq}) \\ 6_{c} (6_{c}) \end{bmatrix} \quad \cdots(4.21)
\]

\[
f_{sq} = \begin{bmatrix} Q_{Fs} (X_{sp}, X_{sq}) \\ V_c (V_c) \end{bmatrix} \quad \cdots(4.22)
\]
Using the decoupling principle, $H_s$ and $G_s$ become

$$H_s = \frac{\delta f_s(X_s)}{\delta X_s} = \begin{bmatrix} \frac{\delta f_{sp}}{\delta X_{sp}} & 0 \\ 0 & \frac{\delta f_{sq}}{\delta X_{sq}} \end{bmatrix} = \begin{bmatrix} H_{sp} & 0 \\ 0 & H_{sq} \end{bmatrix}$$

...(4.23)

$$G_s = H_s^T W_s^{-1} H_s = \begin{bmatrix} G_{sp} & 0 \\ 0 & G_{sq} \end{bmatrix}$$

...(4.24)

where

$$G_{sp} = H_{sp}^T W_{sp}^{-1} H_{sp}$$

...(4.25)

$$G_{sq} = H_{sq}^T W_{sq}^{-1} H_{sq}$$

...(4.26)

The following simplifying assumptions are further made as listed in the development of the robust FDSE:

* Use a flat voltage profile ($V_c = 1$, $6_c = 0$, $u = 0$) while computing the gain and Jacobian matrices.

* Neglect the series resistance when computing $H_{sp}$ and $G_{sp}$.

* Transform the real and reactive power flow measurements by dividing by the corresponding voltage magnitude.
The decoupled equations becomes
\[
G_{sp} [x_{sp}^{i+1} - x_{sp}^{i}] = H_{sp}^T W_{sp}^{-1} [Z_{sp} - f_{sp}(x_{s})] \tag{4.27}
\]
\[
G_{sq} [x_{sq}^{i+1} - x_{sq}^{i}] = H_{sq}^T W_{sq}^{-1} [Z_{sq} - f_{sq}(x_{s})] \tag{4.28}
\]

The above normal equations (4.27) and (4.28) are solved in a sequential manner using sparsity-oriented bi-factorization technique and scheme two ordering, until the solution converges. The above fast decoupled algorithm is simple and efficient. This algorithm overcomes the limitation of Van Cutsem et al method.

The elements of the Jacobian matrices are derived as follows:

Consider a tie-line connected between two boundary buses i and j belonging to two different areas, say $K_1$ and $K_2$ respectively. Let $u_{K_1}$ be the reference angle of the slack bus of subsystem $K_1$ and $u_{K_2}$ be the reference angle of the slack bus of subsystem $K_2$. Let $\delta_{ci}$ and $\delta_{cj}$ represent the phase angles of the boundary buses i and j with respect to $u_{K_1}$ and $u_{K_2}$ respectively. Then $\delta_{cij} = \delta_{ci} - \delta_{cj}$ and $u_{K_{12}} = u_{K_1} - u_{K_2}$. Similarly $V_{ci}$ and $V_{cj}$ are voltage magnitudes of buses i and j. Let the tie-line admittance be $y_{ij} = g_{ij} + j\omega_{bij}$. The equations for real and reactive flows become
\[
\begin{align*}
P_{Fi j} &= -V_{ci}V_{cj} \left[ g_{ij} \cos(\delta_{cij} + u) + b_{ij} \sin(\delta_{cij} + u) \right] + g_{ij}v_1^2 \\
Q_{Fi j} &= -V_{ci}V_{cj} \left[ g_{ij} \sin(\delta_{cij} + u) - b_{ij} \cos(\delta_{cij} + u) \right] - (b_{ij} + b_{cij})v_1^2
\end{align*}
\]  
\(\text{(4.29)}\)  
\(\text{(4.30)}\)

where \(b_{cij}\) = half line charging susceptance of line i-j

**Elements of the Jacobian Matrix \(H_{sp}\)**

Differentiating with respect to \(i\) and evaluating the derivative at \(V_c = 1, \delta_c = 0, u = 0\)

\[
\frac{\delta P_{Fij}}{\delta \delta_{ci}} = -b_{ij}
\]

when resistance is neglected,

\[
\frac{\delta P_{Fij}}{\delta \delta_{cj}} = -\frac{1}{x_{ij}}\quad \text{...(4.31)}
\]

where \(x_{ij}\) is the reactance of the tie-line i-j

Similarly, the other elements evaluated with \(V_c = 1, \delta_c = 0, u = 0\) and the resistance of the tie-line \(r_{ij} = 0\), are

\[
\frac{\delta P_{Fij}}{\delta \delta_{cj}} = \frac{1}{x_{ij}}\quad \text{...(4.32)}
\]
\[ \frac{\delta P_F_{ij}}{\delta u_{K_1}} = -1/x_{ij} \quad \ldots (4.33) \]

\[ \frac{\delta P_F_{ij}}{\delta u_{K_2}} = 1/x_{ij} \quad \ldots (4.34) \]

The derivatives with respect to \( u_{K_1} \) becomes zero if \( u_{K_1} = u_1 \), where \( u_1 \) is the reference for the whole system. The elements of the pseudomeasurements \( \hat{\xi}_c \) become

\[ \frac{\delta \hat{\xi}_c}{\delta \hat{\xi}_c} = 1.0 \quad \ldots (4.35) \]

and

\[ \frac{\delta \hat{\xi}_c}{\delta u} = 0 \quad \ldots (4.36) \]

Elements of the Jacobian Matrix \( H_{sq} \)

The elements of the reactive Jacobian matrix \( H_{sq} \) with

\( V_c = 1, \hat{\xi}_c = 0 \) and \( u = 0 \) are

\[ \frac{\delta Q_F_{ij}}{\delta V_{ci}} = -b_{ij} - 2b_{cij} = -b_{ij}^* \quad \ldots (4.37) \]

\[ \frac{\delta Q_F_{ij}}{\delta V_{cj}} = b_{ij} \quad \ldots (4.38) \]

and

\[ \frac{\delta V_c}{\delta V_c} = 1 \quad \ldots (4.39) \]
The matrices $H_{sp}$ and $H_{sq}$ are constant, real and sparse. They need be computed only at the beginning of the study. The matrices $G_{sp}$ and $G_{sq}$ are constant, real and symmetric. These matrices $G_{sp}$ and $G_{sq}$ are factorised only at the beginning of the programme. These constant matrices reduce the computation time. Since the gain matrix $G_s$ and the Jacobian matrix $H_s$ are decoupled the storage requirement is also reduced. Sparsity-oriented bifactorization technique further increases the efficiency of the algorithm. Moreover, the algorithm is designed to handle only tie-line flows and $\hat{X}_c$, making algorithm simple. The method developed is fast and meets the requirements of large scale system.

The SSE algorithm requires the acquisition of pseudo-measurement vector $\hat{X}_c$ and the tie-line measurements $Z_u$. The effectiveness of the SSE operation depends on the second-level redundancy. The following redundancies may be defined at this level:

* The global second-level redundancy, expressed by
  \[ \eta_s = \frac{(2N_c+m_u)}{2N_c + (K-1)} \]  \ ...(4.40)

* The redundancy with respect to u vector expressed by
  \[ \eta_u = \frac{m_{up}}{(K-1)} \]  \ ...(4.41)
where

\[ m_{\text{up}} = \text{number of active power flow measurements in the tie-lines.} \]

This latter redundancy takes generally quite large values. For example, in the case of Fig.4.1, \( \eta_u = 10/2 = 5 \).

4.5 NUMERICAL EXAMPLE AND DISCUSSIONS

General Description and Data

The modified SSE algorithm and the overall HSE procedure are tested and compared with the ISE method in which the robust FDSE is used. IEEE 14 bus system is considered to illustrate the applicability of the method. This network is decomposed into three subsystems. The interconnection (4th) area comprises 5 tie-lines and 8 boundary nodes. Table 4.1 and Fig.4.1 describe this network and its various characteristics. The bus and line data for the 14 bus system are given in Appendix A. Its power flow results are provided in Appendix B. The static state estimation results for the integrated system are furnished in Chapter 2. The details of the measurements for each subsystem are furnished in Table 4.2. The redundancies used are also listed.
<table>
<thead>
<tr>
<th>Area</th>
<th>Set of Buses</th>
<th>Number of Buses</th>
<th>Number of lines</th>
<th>Set of Buses</th>
<th>Number of Buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1,2,3,4,5</td>
<td>5</td>
<td>7</td>
<td>4,5</td>
<td>2</td>
</tr>
<tr>
<td>A2</td>
<td>7,8,9,10</td>
<td>4</td>
<td>3</td>
<td>7,9,10</td>
<td>3</td>
</tr>
<tr>
<td>A3</td>
<td>6,11,12,13,14</td>
<td>5</td>
<td>5</td>
<td>6,11,14</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
<td>4,5,6,7,9,10,11,14</td>
<td>8</td>
<td>5</td>
<td>4,5,6,7,9,10,11,14</td>
<td>8</td>
</tr>
</tbody>
</table>
### TABLE 4.2: NUMBER OF MEASUREMENTS

<table>
<thead>
<tr>
<th>Area</th>
<th>Active Power Injection Flows</th>
<th>Reactive Measurements Injection Flows Voltage magnitude</th>
<th>Total</th>
<th>Redundancy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1$</td>
<td>2</td>
<td>6</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>$A_3$</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>$A_4$</td>
<td>-</td>
<td>10</td>
<td>-</td>
<td>*36</td>
</tr>
</tbody>
</table>

* The interconnection area $A_4$ has 8 phase angles and 8 voltage magnitudes from the boundary buses as pseudomeasurements in addition to 20 measurements of tie-line flows, leading to the maximum global second level redundancy

$$\eta_s \max = \frac{(2 \times 8 + 20)}{(2 \times 8 + (3-1))} = 2.0$$
Description of Simulation Procedure

The exact state and the corresponding line power flows are calculated using a standard power flow programme. The results of power flow are furnished in Appendix B. The noisy measurements are created by adding to the exact power flow values a Gaussian random component with zero mean and variance $\sigma$ chosen as follows:

* for power measurements

$$\sigma = 0.01 |Z_t| + 0.002 \text{ FS}$$

where $FS = \text{full scale of the meters, assumed to be 1 p.u.}$

$Z_t = \text{true meter readings}$

* for the voltage magnitude measurements

$$\sigma = 0.001 |Z_t|$$

The errors are added only to the actual measurements.

Convergence Criteria

The convergence of the ISE and HSE estimators is tested using the following criteria:

$$\max |\Delta V| \leq 0.0001 \text{ p.u.}$$

$$\max |\Delta \delta| \leq 0.0001 \text{ radian}$$

$$\max |\Delta u| \leq 0.0001 \text{ radian}$$
Discussion of the Results

The states estimated from the ISE and HSE algorithms are presented in Tables 4.3 and 4.4. In Table 4.3, the voltage magnitudes obtained from the ISE and HSE are tabulated whereas Table 4.4 gives the phase angle estimates. The phase angles are referred to the same absolute reference slack-bus of area $A_1$. For the HSE procedure, this implies that for each phase angle, the corresponding component of the $u$ vector is added.

A comparison of the accuracies of the states obtained from the hierarchical and integrated estimators shows that the deviation of state vector is larger for HSE than for the ISE. But the errors are within the acceptable limits.

The performance of the modified second level state estimation algorithm with respect to the accuracy of the reference slack-bus angle estimates is reported in Table 4.5. The algorithm converges in 3-1/2 iterations as can be seen from the Table 4.6. The convergence is fast and reliable.

The performance indices for the subsystems, interconnection area and the integrated system are tabulated in Table 4.7. As can be seen from the Table the ratio $J_E/J_M$ for all the system is less than one, indicating the good filtering performance of the FSE, SSE and ISE.
### TABLE 4.3: COMPARISON OF ISE AND HSE RESULTS—VOLTAGE MAGNITUDES

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>$V^t$ from power flow</th>
<th>ISE</th>
<th>HSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\widehat{V}$</td>
<td>$C_v$ (%)</td>
</tr>
<tr>
<td>1</td>
<td>1.0600</td>
<td>1.0604</td>
<td>0.0411</td>
</tr>
<tr>
<td>2</td>
<td>1.0724</td>
<td>1.0729</td>
<td>0.0421</td>
</tr>
<tr>
<td>3</td>
<td>1.0871</td>
<td>1.0876</td>
<td>0.0428</td>
</tr>
<tr>
<td>4</td>
<td>1.0839</td>
<td>1.0841</td>
<td>0.0165</td>
</tr>
<tr>
<td>5</td>
<td>1.0822</td>
<td>1.0825</td>
<td>0.0244</td>
</tr>
<tr>
<td>6</td>
<td>1.1545</td>
<td>1.1552</td>
<td>0.0619</td>
</tr>
<tr>
<td>7</td>
<td>1.1331</td>
<td>1.1337</td>
<td>0.0537</td>
</tr>
<tr>
<td>8</td>
<td>1.1840</td>
<td>1.1846</td>
<td>0.0545</td>
</tr>
<tr>
<td>9</td>
<td>1.1282</td>
<td>1.1287</td>
<td>0.0476</td>
</tr>
<tr>
<td>10</td>
<td>1.1259</td>
<td>1.1265</td>
<td>0.0566</td>
</tr>
<tr>
<td>11</td>
<td>1.1368</td>
<td>1.1374</td>
<td>0.0485</td>
</tr>
<tr>
<td>12</td>
<td>1.1398</td>
<td>1.1403</td>
<td>0.0443</td>
</tr>
<tr>
<td>13</td>
<td>1.1344</td>
<td>1.1352</td>
<td>0.0682</td>
</tr>
<tr>
<td>14</td>
<td>1.1242</td>
<td>1.1156</td>
<td>0.1286</td>
</tr>
</tbody>
</table>

**Summary of Table 4.3**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ISE</th>
<th>HSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_v(%)$ ave</td>
<td>0.0522</td>
<td>0.0940</td>
</tr>
<tr>
<td>$C_v(%)$ max</td>
<td>0.1286</td>
<td>0.1763</td>
</tr>
</tbody>
</table>

Voltage magnitudes are expressed in per unit.
<table>
<thead>
<tr>
<th>Bus</th>
<th>True Value</th>
<th>ISE</th>
<th>HSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-6°</td>
<td>C6</td>
<td>-6°</td>
</tr>
<tr>
<td>1</td>
<td>00.0000</td>
<td>00.0000</td>
<td>00.0000</td>
</tr>
<tr>
<td>2</td>
<td>5.2982</td>
<td>5.3279</td>
<td>0.5607</td>
</tr>
<tr>
<td>3</td>
<td>12.9765</td>
<td>13.0277</td>
<td>0.3948</td>
</tr>
<tr>
<td>4</td>
<td>10.7133</td>
<td>10.7781</td>
<td>0.6051</td>
</tr>
<tr>
<td>5</td>
<td>9.2883</td>
<td>9.3564</td>
<td>0.7336</td>
</tr>
<tr>
<td>6</td>
<td>14.4430</td>
<td>14.4765</td>
<td>0.2323</td>
</tr>
<tr>
<td>7</td>
<td>13.4403</td>
<td>13.5037</td>
<td>0.4714</td>
</tr>
<tr>
<td>8</td>
<td>13.4403</td>
<td>13.5096</td>
<td>0.5154</td>
</tr>
<tr>
<td>9</td>
<td>14.8181</td>
<td>14.8930</td>
<td>0.5053</td>
</tr>
<tr>
<td>10</td>
<td>14.9934</td>
<td>15.0660</td>
<td>0.4839</td>
</tr>
<tr>
<td>11</td>
<td>14.8233</td>
<td>14.8723</td>
<td>0.3306</td>
</tr>
<tr>
<td>12</td>
<td>15.1677</td>
<td>15.2175</td>
<td>0.3284</td>
</tr>
<tr>
<td>13</td>
<td>15.2085</td>
<td>15.2551</td>
<td>0.3067</td>
</tr>
<tr>
<td>14</td>
<td>15.8524</td>
<td>15.9455</td>
<td>0.5876</td>
</tr>
</tbody>
</table>

Summary of Table 4.4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ISE</th>
<th>HSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C6 (%) ave</td>
<td>0.4325</td>
<td>0.4869</td>
</tr>
<tr>
<td>C6 (%) max</td>
<td>0.7336</td>
<td>0.9051</td>
</tr>
</tbody>
</table>

The phase angles are expressed in degrees.
### TABLE 4.5: ANALYSIS OF REFERENCE PHASE ANGLES

<table>
<thead>
<tr>
<th>Area</th>
<th>True Values -u°</th>
<th>Estimated Values -u°</th>
<th>Percentage Error C_u(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>00.0000</td>
<td>00.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>A_2</td>
<td>13.4403</td>
<td>13.4493</td>
<td>0.4387</td>
</tr>
<tr>
<td>A_3</td>
<td>14.4430</td>
<td>14.4494</td>
<td>0.3559</td>
</tr>
</tbody>
</table>
TABLE 4.6: CONVERGENCE PROPERTIES OF THE SSE PROCEDURE

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$-u_2$ deg</th>
<th>$-u_3$ deg</th>
<th>$J_s$</th>
<th>$J_{sp}$</th>
<th>$J_{sq}$</th>
<th>max $\Delta V_c$</th>
<th>max $\Delta \delta_c$</th>
<th>max $\Delta u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Value</td>
<td>0.0000</td>
<td>0.0000</td>
<td>95037</td>
<td>78312</td>
<td>16725</td>
<td>.1577</td>
<td>-.1915</td>
<td>-.2690</td>
</tr>
<tr>
<td>After iteration 1</td>
<td>14.4588</td>
<td>15.4146</td>
<td>1650</td>
<td>1439</td>
<td>210</td>
<td>.0062</td>
<td>.0046</td>
<td>.0183</td>
</tr>
<tr>
<td>After iteration 2</td>
<td>13.4081</td>
<td>14.4278</td>
<td>21.08</td>
<td>19.30</td>
<td>1.78</td>
<td>.0002</td>
<td>-.0004</td>
<td>-.0018</td>
</tr>
<tr>
<td>After iteration 3</td>
<td>13.5107</td>
<td>14.5020</td>
<td>4.65</td>
<td>2.85</td>
<td>1.79</td>
<td>.00002</td>
<td>.0000</td>
<td>.00002</td>
</tr>
<tr>
<td>After iteration 3.5</td>
<td>13.4993</td>
<td>14.4944</td>
<td>4.41</td>
<td>2.61</td>
<td>1.79</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* per unit  ** radians
<table>
<thead>
<tr>
<th>System</th>
<th>$J_M$</th>
<th>$J_E$</th>
<th>$R_{ave}$</th>
<th>$R_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.5097</td>
<td>0.2998</td>
<td>0.4239</td>
<td>1.3806</td>
</tr>
<tr>
<td>A2</td>
<td>0.5291</td>
<td>0.1878</td>
<td>0.3943</td>
<td>0.6757</td>
</tr>
<tr>
<td>A3</td>
<td>1.1666</td>
<td>0.6899</td>
<td>0.6393</td>
<td>2.0192</td>
</tr>
<tr>
<td>A4</td>
<td>0.3376</td>
<td>0.2192</td>
<td>0.3473</td>
<td>1.0715</td>
</tr>
<tr>
<td>ISE</td>
<td>0.7338</td>
<td>0.3015</td>
<td>0.4301</td>
<td>1.6173</td>
</tr>
</tbody>
</table>
4.6 CONCLUSIONS

A hierarchical concept is used to solve the static
state estimation problem for very large scale composite
power systems. The solution is obtained by performing a
two-level calculation. In the lower level, a robust FDSE
is used and the estimation is carried out simultaneously
for all subsystems. The coordination of these local estima-
tors is realised in the upper level.

A modified algorithm is developed for the upper
level. The developed algorithm is based on the principle
of decoupling between the active and reactive quantities.
The algorithm is designed to handle the tie-line flows and
the estimated states from the first-level estimators. The
active/reactive decoupling makes the gain and Jacobian mat-
rices constant. These matrices need be computed only once.
Sparsity-oriented bifactorization technique with scheme two
ordering is used to solve the normal equations of the algo-
rumh. The algorithm is fast and suitable for on line app-
llication for large scale power systems.