Chapter 4

ANNIHILATOR DOMINATION

ANNIHILATOR DOMINATION

In this chapter, we define a new parameter on domination called the Annihilator dominating set and Annihilator dominating number.

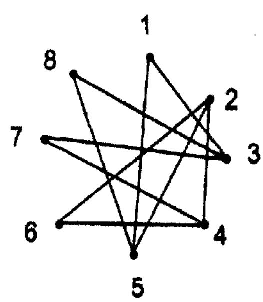
Definition : 4.1

A dominating set D of a graph G is said to be an annihilator dominating set, if its induced subgraph <V- D> is a graph containing only isolated vertices.

The annihilator domination number $\gamma_a(G)$ of G is the cardinality of an annihilator dominating set of the Smallest size

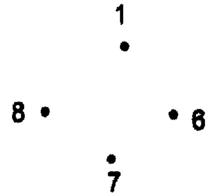
Illustration :

G :



Annihilator dominating set D = {2,3,4,5}

<V - D> :



The induced subgraph <V-D> of G is a graph with isolated vertices.

$\gamma_a(G) = 4$

Fig.4.1

This concept has very wide applications in real life situations. We list below three important applications among the several applications of annihilator dominating set.

APPLICATION- 1 :

In the field of agriculture, control of pests plays a major role. Several pests interact among themselves, resulting in the large scale production of pests causing wide spread damage to the agriculture products.

The one reasonable solution to this problem will be to isolate the pests, so that the interaction among the members of pest complex is prevented and it is easy for the agriculturists to eliminate the isolated types of the pests. We identified the network of the several types of pests as a graph with a vertex in the graph being a specific types of the pest and the other pests with which this type will interact denoting the adjacency of the graph.

The problem reduces to the following : We remove a set of vertices in the graph such that the induced sub graph $<V - D>$ contains only a set of independent vertices.
Which implies finding the annihilator dominating set of graph G.

APPLICATION 2

The Bacterial Virus of different varieties interact among themselves to produce diseases in an epidemic form causing devastating damage to human life. It is possible to eliminate this type of viruses by identifying certain specific viruses, elimination of which will result in isolation of the varieties of the viruses and once these viruses are isolated, it is easy to eliminate them by using the appropriate vaccination.

In terms of graph theory, this problem is that in a graph each vertex of which represent a virus variety and the edges adjacent with this vertex represent the other viruses with which this variety of viruses will interact. We have to identify an annihilator dominating set D in this graph so that the induced subgraph $\langle V-D \rangle$ is a set of isolated vertices.

APPLICATION 3

In requirements of the Department of Defence, it is observed that the strength of a unit in operation depends on the strength of the interaction of the several camps for the supply of essential armaments, weapons and essential commodities. The problem for the enemy side will be to eliminate certain posts or camps resulting in the collapse of the channels of interaction among the camps.

In Graph Theoretic model, the network of the camps of the unit represents a Graph with its vertex set as the set of camps and the channels with which each camp interacts is the adjancy of a vertex with the vertices in the graph.

The problem is to identify a set of vertices D in the graph, so that the removal of the vertex set D should help in the resulting graph of the induced sub graph <V-D> to be a graph with a set of isolated vertices only.

We now obtain several results on the annihilator dominating set and its relation with the other domination parameters.

From the definition, the following result is an immediate consequence.

Theorem 4.2 : In a graph G, $\gamma(G) \leq \gamma_{i}(G) \leq \gamma_{a}(G)$

We now obtain the annihilator domination number of some standard graphs.

Theorem 4.3 : If $K_{m,n}$ is a complete bipartite graph, with $2 \leq m \leq n$,

then $\gamma_a(K_{m,n}) = m$

Proof: If (X, Y) be the bi-partition of the graph $K_{m,n}$

with $|X| = m$ and $|Y| = n$

The removal of the m vertices in X will render the resulting graph <V-X> to be an independent set of n vertices only. It can be easily seen that the set X is an annihilator dominating set of minimum cardinality.

Thus $\gamma_a(K_{m,n}) = |X| = m$

Theorem 4.4 : If P_n is a path on n-vertices,

then $\gamma_a(P_n) = [n/2]$; where [X] represents the greatest integer less than or equal to X.

Proof : The removal of the alternate vertices in the path P_n will result in the induced sub graph of the remaining vertices to be a set of isolated vertices.

Hence $\gamma_a(P_n) = [n/2]$.

Theorem 4. 5 If S_n is a star on n-vertices, then $\gamma_a(S_n) = 1$.

Proof: If v is the vertex of degree > 1 in S_n.

Then $D_a = \{v\}$, will be the annihilator dominating set of S_n.

Also D_a is the annihilator dominating set of minimum cardinality.

Hence $\gamma_a(S_n) = 1$.

It is easily to deduce the following results.

Theorem 4.6 If W_n is a wheel on n-vertices,

then

$$\gamma_a(W_n) = \begin{cases} n/2 + 1 & ; \text{If } n \text{ is even} \\ \frac{(n+1)}{2} & ; \text{if } n \text{ is odd} \end{cases}$$

Now we have obtained the following result on a cycle.

Theorem 4.7: If C_n is a cycle on n-vertices,

then $\gamma_a(C_n) = \lceil n/2 \rceil$; where $\lceil x \rceil$ represents the smallest integer $\geq x$.

We have obtained the following result on a tree.

Theorem 4.8: If T is a tree on n-vertices with 'p' pendent vertices ($p \geq 3$), then $\gamma_a(T) \leq n - p$.

Proof : Consider the set D of all vertices v in T

Such that the degree (v) > 1 that is $D = \{ v \in T \ d(v) > 1 \}$.

Then the set D is certainly a dominating set of T and further it is also an annihilator dominating set. The induced sub graph < V – D> is a set of independent vertices, which is the set of the p pendent vertices of the tree T.

Thus $|D| = n - p$

Hence $\gamma_a(T) \leq n - p$.

Now we obtain an interesting expression for annihilator domination number of a graph G, in terms of split domination number and domination number.

Theorem 4.9 : For any graph G , $\gamma_a(G) \geq \gamma_s(G) + \sum_{i=1}^{t} \gamma(G_i)$,

where G_i's are the components of $<V-D_s>$, D_s being the split dominating set of G of minimum cardinality.

Proof: Let G be a graph with vertex set

$V(G) = \{v_1, v_2 \ldots\ldots\ldots\ldots v_n\}$.

If D_s is the split dominating set of minimum cardinality of G, then the removal of D_s vertices will result in the induced sub graph $<V - D_s>$ to be a disconnected graph with atleast two components.

Let $G_1, G_2 \ldots\ldots\ldots\ldots G_t$ be the nontrivial components with atleast two vertices in the $<V - D_s>$.

Case 1 :- If each G_i is a component with exactly two vertices

Then it can be seen that, the split dominating set D_s together with the dominating sets of each of these components $G_1, G_2 \ldots\ldots\ldots\ldots G_t$ will be the annihilator dominating set of minimum cardinality.

Hence in this case $\gamma_a(G) = \gamma_s(G) + \sum_{i=1}^{t} \gamma(G_i)$.

Case 2:- If some components "G_i" of $<V - D_s>$ have more than two vertices, then for the annihilator dominating set, we requre the vertices of split dominating set of G, dominating sets of each components G_i and some more vertices when ever necessary in each component G_i. Hence in this case $\gamma_a(G) \geq \gamma_s(G) + \sum_{i=1}^{t} \gamma(G_i)$.

Therefore combining the two caseswe conclude that

$$\gamma_a(G) \geq \gamma_s(G) + \sum^{t} \gamma(G_i).$$

Illustration :

G :

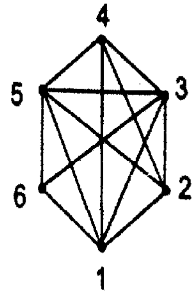


Fig. 4.2

$V(G) = \{1,2,3,4,5,6\}$

$D_t(G) = \{1,3,5\}$

$V\text{-}D_t = \{2,4,6\}$

let G_1 be the only non trivial component of $\langle V\text{-}D_t \rangle$ joining exactly two vertices 2,4

If $D_a = \{1,2,3,5\}$,

4.

6.

$\langle V\text{-}D_a \rangle$

Then D_a is an annihilitator dominating set of G.

$\gamma_a(G) = \gamma_t(G) + \gamma(G_1) = 3 + 1 = 4$

Illustration :

G^{1}:

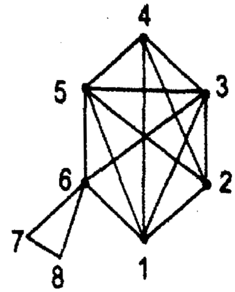


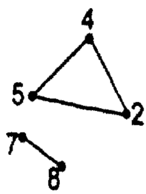
Fig. 4.2 a

$V(G) = \{1,2,3,4,5,6,7,8\}$

$D_t\ (G) = \{1,3,6\}$

$V\text{-}D_t = \{2,4,5,7,8\}$

$<V\text{-}D_t>$:



Let G_1 be the non trivial component of $<V\text{-}D_t>$ joining three vertices 2, 4, 5 and G_2 be the another non trivial component of $<V\text{-}D_t>$ joining exactly two vertices 7, 8.

Here $\gamma(G_1) = 1$ *and* $\gamma(G_2) = 1$. But for the annihilator dominating set, we have to remove one more vertex from the component of G_1 in addition to the dominating set. So that if $D_a = \{1,2,3,4,6,7\}$

$<V\text{-}D_a>$

Then D_a is an annihilitator dominating set of G'.

$\gamma_a(G') = 6 > \gamma_t(G') + \gamma(G_1) + \gamma(G_2) = 3 + 1 + 1 = 5$

§ ANNIHILATOR DOMINATION OF PRODUCT GRAPHS

In this chapter, we obtain the annihilator dominating sets and expressions for annihilator domination number of some product graphs defined earlier in chapter 2 that is from the definitions 2.2, 2.10 & 2.14.

The procedure to obtain the annihilator dominating set of a product graph is an extension of the procedure of finding a split domination set D_s of the product graph and then in the induced sub graph $<V-D_s>$, we should eliminate all the remaining edges and thus

$$\gamma_a (G_1 (k) G_2] = \gamma_s [G_1 (k) G_2] + \gamma [<V - D_s>]$$

For obtaining the annihilator domination number of a Kronecker product graph, we first prove the following lemma.

Lemma 4.10

If G_1 and G_2 are finite graphs without isolated vertices, then G_1 (k) G_2 is a finite graph without isolated vertices.

Prof : Since G_1 and G_2 are finite graphs, it follows that G_1 (k) G_2 is also a finite graph. By definition 2.2 since G_1, G_2 do not have isol ted vertices deg $_{G_1}(u_i) \neq 0$ for any i and also

deg $_{G_1}(v_j) \neq 0$ for any j .

Thus deg $_{G_1(k)G_2}(u_i, v_j) \neq 0$ for any i and j (by theorem 2.3).

So $G_1(k)G_2$ do not have any isolated vertices.

The following result is an immediate consequence.

Theorem 4.11 (i) $|V_{G_1(k)G_2}| = |V_{G_1}| \cdot |V_{G_2}|$

(ii) $|E_{G_1(k)G_2}| = 2 \cdot |E_{G_1}| \cdot |E_{G_2}|$

Now we obtain an upper bound for the annihilator domination number of $G_1(k)G_2$ in terms of the annihilator domination numbers of G_1 and G_2.

Theorem 4.12: If G_1 and G_2 are any two graphs without isolated vertices, then $\gamma_a[(G_1 (k) G_2)] \leq \min [\gamma_a (G_1) . |V_2| , |V_1|. \gamma_a(G_2)|]$

Proof : Let G_1 be a graph with p_1 vertices and G_2 be a graph with p_2 vertices.

Let $D_1 = \{ u_{d_1}, u_{d_2}, \ldots u_{d_r} \}$ be an annihilator dominating set of minimum cardinality of G_1 and $D_2 = \{ v_{d_1}, v_{d_2}, \ldots \ldots v_{d_s} \}$ be an annihilator dominating set of minimum cardinality of G_2.

Now in the product graph $G_1(k)\, G_2$ consider the set of vertices,

$D_a = \{ (u_{d_1}, v_1), (u_{d_1}, v_2), \ldots\ldots (u_{d_1}, v_{p_2}),$

$(u_{d_2}, v_1), (u_{d_2}, v_2), \ldots\ldots (u_{d_2}, v_{p_2}),$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$(u_{d_r}, v_1), (u_{d_r}, v_2), \ldots (u_{d_r}, v_{p_2})\}$

Since D_1 is an annihilator dominating set of G_1, the removal of D_1 vertices in G_1 will result in the induced sub graph $<V_1 - D_1>$ to be a set of independent vertices.

D_a is a dominating set of $G_1(k)G_2$.

For, if (u.v) is any vertex of $<V\text{-}D_a>$ in $G_1(k)G_2$, then u is adjacent with at least one vertex in $(u_{d_1}, u_{d_2} \ldots\ldots u_{d_r}\}$, as this is the dominating set of G_1 being its annihilator dominating set.

Suppose v is adjacent with some vertex v_k in G_2 (certainly there exists at least one vertex v_k in G_2, Since G_2 has no isolated vertices), then (u, v) is adjacent with (u_{d_j}, v_k) in D_a.

Thus D_a is a dominating set.

Further this is also an annihilator dominating set.

This can be established as follows:

We know that D_1 is an annihilator dominating set of G_1 and the induced sub graph $<V_1 - D_1>$ is an empty graph.

Let u_i, u_j be any two vertices in $<V_1-D_1>$. They are isolated vertices of $<V_1 - D_1>$.

If (u_1, v_1) and (u_2, v_2) are any two vertices in $<V-D_a>$, then $u_1, u_2 \neq u_{d_i}$, for any i.

As all the vertices of G_1 (k) G_2 whose first coordinates are vertices in D_1 are removed by virtue of the removal of D_a.

It follows that u_1, u_2 are not adjacent and consequently (u_1,v_1) and (u_2, v_2) are not adjacent in $<V - D_a>$.

Thus D_a is an annihilator dominating set of G_1 (k) G_2. similarly, by the same argument we can prove that the set of vertices,

$D_a' = \{ (u_1, v_{d_1}), (u_2, v_{d_1}) \ldots\ldots\ldots (u_{p_1}, v_{d_1}),$

$(u_1, v_{d_2}) (u_2, v_{d_2}) \ldots\ldots\ldots\ldots (u_{p_1}, v_{d_2}),$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$(u_1, v_{d_s}), (u_2, v_{d_s}) \ldots\ldots (u_{p_1}, v_{d_s})\}$

is also an annihilator dominating set of G_1 (k) G_2.

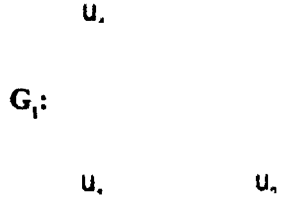
Thus it follows that

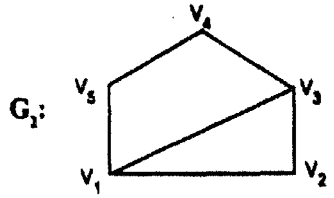
$$\gamma_a [G_1(k)G_2] \leq \min [|D_a|, |D_a'|]$$

$$= \min [\gamma_a(G_1). |V_2|, |V_1|. \gamma_a(G_2)]$$

Hence $\gamma_a [G_1(k)G_2] \leq \min [\gamma_a(G_1). |V_2|, |V_1|. \gamma_a(G_2)]$

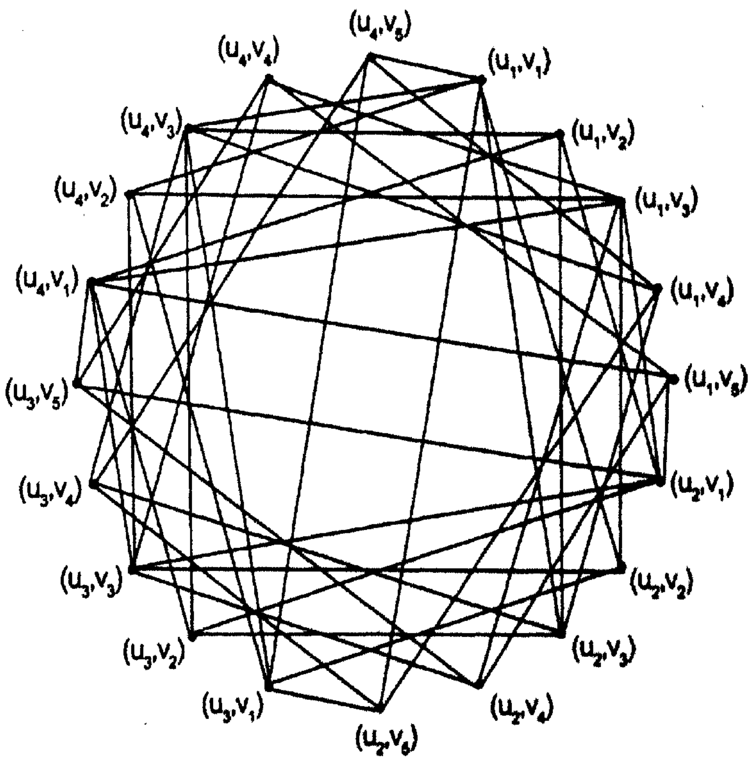
Illustration :





Annihilator dominating set of $G_1=\{u_1,u_3\}$ and $\gamma_a(G_1)=2$

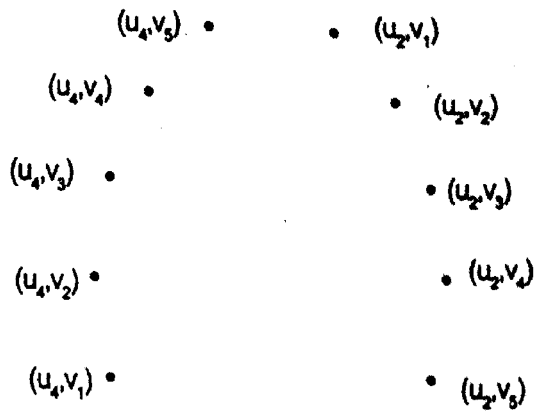
Annihilator dominating set of $G_2=\{v_1,v_3,v_5\}$ and $\gamma_a(G_2)=3$



$G_1(K)\,G_2$

$D_a = \{u_1, u_3\} \times \{v_1, v_2, v_3, v_4, v_5\}$

$= \{(u_1,v_1), (u_1,v_2), (u_1,v_3), (u_1,v_4), (u_1,v_5), (u_3,v_1), (u_3,v_2), (u_3,v_3),$
$(u_3,v_4), (u_3,v_5)\}$



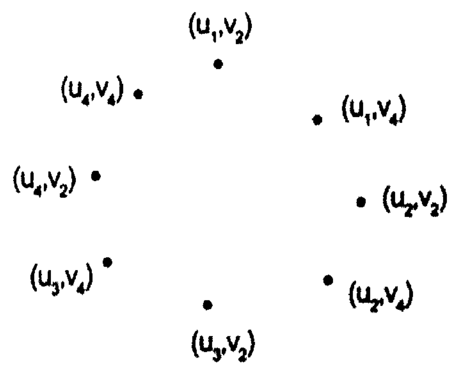
$< V\text{-}D_a >$

D_a is an annihilator dominating set, since <V-Da> is a set of isolated vertices.

$|D_a| = \gamma_a(G_1) \cdot |V_2| = 2.5 = 10$

$D_a' = \{u_1, u_2, u_3, u_4\} \times \{v_1, v_3, v_5\}$

$= \{(u_1,v_1), (u_1,v_3), (u_1,v_5), (u_2,v_1), (u_2,v_3), (u_2,v_5), (u_3,v_1), (u_3,v_3), (u_3,v_5), (u_4,v_1), (u_4,v_3), (u_4,v_5)\}$



$< V\text{-}D_a' >$

Da' is also an annihilator dominating set.

$|D_a'| = |V_1| \cdot \gamma_a(G_2) = 4 \cdot 3 = 12$

$\gamma_a[G_1 (K) G_2] \leq \min[\,|D_a|, |D_a'|\,]$

$= \min[10, 12] = 10$

$\gamma_a[G_1 (K) G_2] \leq 10$

Fig. 4.3

We now obtain an upper bound for an annihilator dominating set of $G_1(L)G_2$

Theorem 4.13: If G_1 and G_2 are any two graphs without isolated vertices,

Then $\gamma_a[G_1(L)G_2] \leq \gamma_a(G_1) \cdot |V_2| + |V_1| \cdot \gamma_a(G_2) - \gamma_a(G_1) \cdot \gamma_a(G_2)$

Proof: Let G_1 be a graph with p_1 vertices and G_2 be a graph with p_2 vertices

$$V(G_1) = \{u_1, u_2, \ldots\ldots\ldots\ldots u_{p_1}\} = V_1 \text{ and}$$

$$V(G_2) = \{v_1, v_2 \ldots\ldots\ldots\ldots v_{p_2}\} = V_2 \text{ say}$$

Let $D_1 = \{u_{d_1}, u_{d_2} \ldots\ldots u_{d_r}\}$ be an annihilator dominating set of G_1 and $D_2 = \{v_{d_1}, v_{d_2} \ldots\ldots\ldots v_{d_s}\}$ be an annihilator dominating set of G_2

Now to obtain an annihilator dominating set of G_1 (L) G_2 we proceed as follows :

Consider the set of vertices

$$D = \{(u_{d_1}, v_1), (u_{d_1}, v_2) \ldots\ldots\ldots (u_{d_1}, v_{p_2}),$$

$$(u_{d_2}, v_1), (u_{d_2}, v_2) \ldots\ldots\ldots (u_{d_2}, v_{p_2}),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$(u_{d_r}, v_1), (u_{d_r}, v_2) \ldots\ldots\ldots (u_{d_r}, v_{p_2})\}.$$

D is a dominating set of G_1 (L) G_2.

For, if (u,v) is a vertex of $G_1(L)G_2$, then u is adjacent with some vertex in $D_1 = \{ u_{d_1}, u_{d_2}, \ldots u_{d_r}\}$, since D_1 is the annihilator dominating set of G_1.

For the sake of definiteness, let u be adjacent with u_{d_i}, for some i.

Now v in G_2 is adjacent with some vertex v_j in G_2, as G_2 is a simple graph without isolated vertices

Thus the vertex (u, v) in $G_1(L)G_2$ is adjacent with (u_{d_i}, v_j).

However, the removal of this dominating set from $G_1(L)\,G_2$ will give us the set of vertices V – D , where V is vertex set of $G_1(L)\,G_2$.

The induced sub graph <V–D> is not an independent set of vertices. For, the removal of set of vertices of D enables only the elimination of the adjacency in the product graph, obtained by the use of the first part of the definition (2.14) viz., $w_1, w_2 \in G_1\,(L)\,G_2$ are adjacent if $u_1 u_2 \in E(G_1)$.

To eliminate the adjacency of the product graph in the remaining graph, we note that the adjacency will be due to the second part of the definition (2.14) viz., $w_1, w_2 \in G_1\,(L)\,G_2$ are adjacent if $u_1 = u_2$ & $v_1 v_2 \in E\,(G_1)$.

For this purpose, we consider the set of vertices

$$D' = \{ u_1, v_{d_1}), (u_2, v_{d_1}) \ldots\ldots (u_{p_1}, v_{d_1}),$$

$$(u_1, v_{d_2}), (u_2, v_{d_2}) \ldots\ldots (u_{p_1}, v_{d_2}),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$(u_1, v_{d_s}), (u_2, v_{d_s}) \ldots\ldots (u_{p_1}, v_{d_s})\}.$$

Proceeding in the same process as above

we can prove that D' is also a dominating set of $G_1(L)G_2$

Thus if $D_a = D \cup D'$, then since both D and D' are dominating sets of G_1 (L) G_2, D_a is also a dominating set of $G_1(L)G_2$.

More over D_a is an annihilator dominating set, because it eliminates the adjacency an account of 2.14 (a) as well as 2.14 (b) between any two vertices in the induced sub graph $< V - \{D \cup D^1\} >$

Thus $\gamma_a[G_1(L)G_2] \leq |D_a|$

$$= | D \cup D^1 |$$

But $| D \cup D^1| = | D | + | D^1| - | D \cap D^1|$

$$= | D | + | D^1| - | D_1 \times D_2|$$

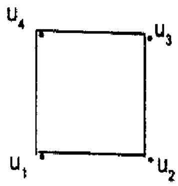
where $| D | = \gamma_a(G_1) . p_2 = \gamma_a (G_1) . |V_2|$

$| D^1| = p_1 . \gamma_a(G_2) = |V_1| . \gamma_a(G_2)$

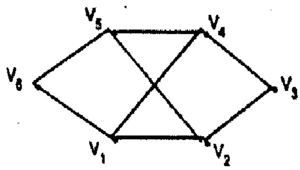
$| D_1 \times D_2| = rs = \gamma_a(G_1) . \gamma_a(G_2)$

Hence $\gamma_a [G_1(L)G_2] \leq \gamma_a (G_1) . |V_2| + |V_1| . \gamma_a(G_2) - \gamma_a (G_1) . \gamma_a (G_2)$

Illustration :



G_1



G_2

Annihilator dominating set of

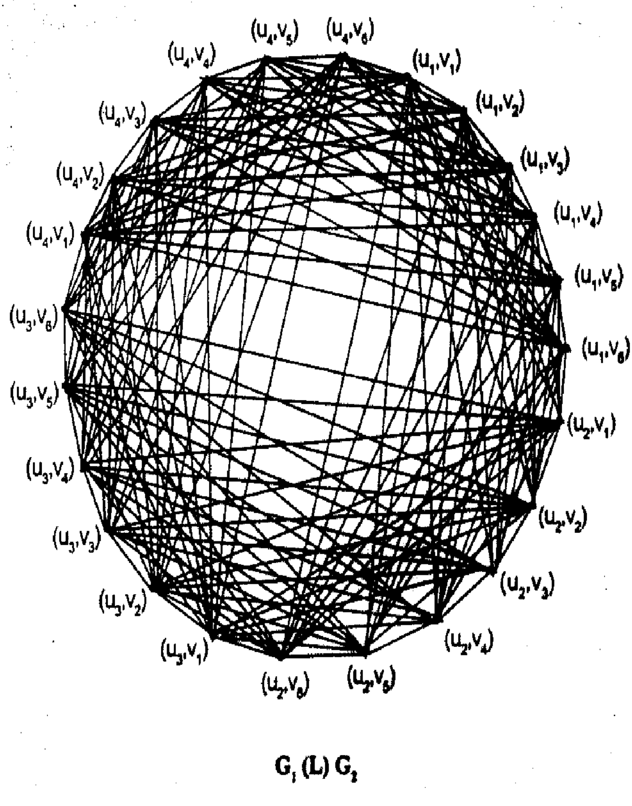
$G_1 = \{u_1, u_3\} = D_1$

$\gamma_a(G_1) = 2 = |D_1|$

Annihilator dominating set of

$G_2 = \{v_1, v_3, v_6\} = D_2$

$\gamma_a(G_2) = 3 = |D_2|$



G_1 (L) G_2

$D = \{u_1,u_3\} \times \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$= \{(u_1,v_1), (u_1,v_2), (u_1,v_3), (u_1,v_4), (u_1,v_5), (u_1,v_6), (u_3,v_1), (u_3,v_2), (u_3,v_3), (u_3,v_4), (u_3,v_5), (u_3,v_6)\}$

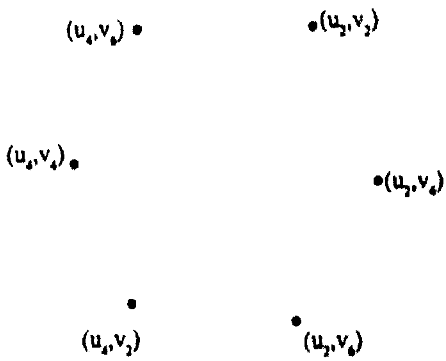
$|D| = \gamma_a(G_1) \cdot |V_2| = 2.6 = 12$

$D' = \{u_1,u_2,u_3,u_4\} \times \{v_1, v_3, v_5\}$

$= \{(u_1,v_1), (u_1,v_3), (u_1,v_5), (u_2,v_1), (u_2,v_3), (u_2,v_5), (u_3,v_1), (u_3,v_3), (u_3,v_5), (u_4,v_1), (u_4,v_3), (u_4,v_5)\}$

$|D'| = |V_1| \cdot \gamma_a(G_2) = 4.3 = 12$

$D_a = D \cup D' = \{(u_1,v_1), (u_1,v_2), (u_1,v_3), (u_1,v_4), (u_1,v_5), (u_1,v_6), (u_2,v_1), (u_2,v_3), (u_2,v_5), (u_3,v_1), (u_3,v_2), (u_3,v_3), (u_3,v_4), (u_3,v_5), (u_3,v_6), (u_4,v_1), (u_4,v_3), (u_4,v_5)\}$



<V-D∪D'>

D_a is an annihilator dominating set.

$$\begin{aligned}\gamma_a[G_1(L)G_2] &\le |D_a| = |D \cup D'| \\ &= |D| + |D'| - |D \cap D'| \\ &= |D| + |D'| - |D_1 \times D_2| \\ &= 12 + 12 - 6 \\ &= 18\end{aligned}$$

$$\gamma_a[G_1(L)G_2] \le 18$$

Fig . 4.4

We now obtain an upperbound for the Lexicograph product graph.

We observe that from the definitions (2.10) & (2.14) that G_1 (C)G_2 is a sub graph of G_1(L) G_2. We have the following result as an immediate consequence.

Theorem 4.14

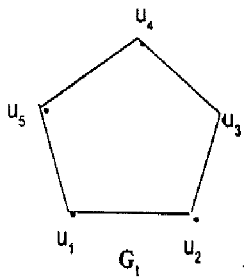
If G_1 and G_2 are any two graphs without isolated vertices,

then $\gamma_a[G_1(C)G_2] \leq \gamma_a(G_1).|V_2| + |V_1|.\gamma_a(G_2) - \gamma_a(G_1).\gamma_a(G_2)$.

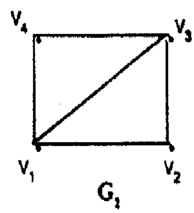
Proof : To get an annihilator dominating set of the product graph G_1(C)G_2, we proceed along the same lines as in the previous theorem (4.13) and it can be easily seen that the set D_a as defined in the previous theorem is an annihilator dominating set of G_1 (C)G_2.

Hence $\gamma_a[G_1(C)G_2] \leq \gamma_a(G_1).|V_2| + |V_1|.\gamma_a(G_2) - \gamma_a(G_1).\gamma_a(G_2)$

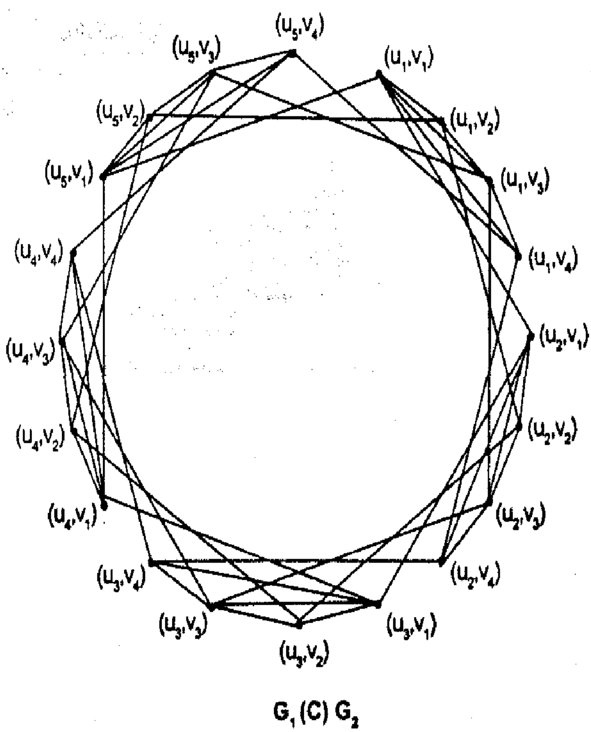
Illustration :



Annihilator dominating set of
$G_1 = \{u_1, u_3, u_5\} = D_1$
$\gamma_a(G_1) = 3 = |D_1|$



Annihilator dominating set of
$G_2 = \{v_1, v_3\} = D_2$
$\gamma_a(G_2) = 2 = |D_2|$



$G_1 (C) G_2$

$D = \{u_1,u_3,u_5\} \times \{v_1, v_2, v_3, v_4\}$

$= \{(u_1,v_1), (u_1,v_2), (u_1,v_3), (u_1,v_4), (u_3,v_1), (u_3,v_2), (u_3,v_3), (u_3,v_4), (u_5,v_1), (u_5,v_2), (u_5,v_3), (u_5,v_4)\}$

$|D| = \gamma_a(G_1) \,.\, |V_2| = 3.4 = 12$

$D' = \{u_1,u_2,u_3, u_4,u_5\} \times \{v_1, v_3\}$

$= \{(u_1,v_1), (u_1,v_3), (u_2,v_1), (u_2,v_3), (u_3,v_1), (u_3,v_3), (u_4,v_1), (u_4,v_3), (u_5,v_1), (u_5,v_3)\}$

$|D'| = |V_1| \,.\, \gamma_a(G_2) = 5.2 = 10$

$D_a = D \cup D' = \{(u_1,v_1), (u_1,v_2), (u_1,v_3), (u_1,v_4), (u_2,v_1), (u_2,v_3), (u_3,v_1), (u_3,v_2), (u_3,v_3), (u_3,v_4), (u_4,v_1), (u_4,v_3), (u_5,v_1), (u_5,v_2), (u_5,v_3), (u_5,v_4)\}$

< V-D∪D'>

D_a is an annihilator dominating set.

$$
\begin{aligned}
\gamma_a[G_1(C)G2] &\le |D_a| = |D \cup D'| \\
&= |D| + |D'| - |D \cap D'| \\
&= |D| + |D'| - |D_1 \times D_2| \\
&= 12 + 10 - 6 \\
&= 16
\end{aligned}
$$

$\gamma_a[G_1(C)G2] \le 16$

Fig. 4.5