Chapter 3

SPLIT DOMINATION - ARITHMETIC GRAPHS

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In most of the researches in Graph theory, the investigators are content with establishing the existence of a graph with a given graphical parameter. For example, given domination number as n does there exist a graph with this as the domination number? Similarly does there exist a graph with given bondage number or with given domatic number? These problems have been investigated successfully. However in the matter of applications of these results to real life situations it becomes necessary to evolve the method of constructing such a graph with a given parameter. Construction of a graph with a given Graph theoretic parameter is generally difficult by the usual graph theoretic methods. In many applications of domination number, bondage number, or domatic number, it becomes necessary to construct a graph with as few vertices and/or edges as possible with a given domination number or bondage number or domatic number. It is in this context the usage of elementary number theoretic principles will help in the constructions of such graphs. In Vasumathi and Vangipuram [32], the construction of a graph with a given domination number has been given, using such a method. It is also amazing to observe how such a graph with a given domination number can be enlarged to include more vertices and edges in a methodical, simple manner without affecting the domination number. A similar method of construction using again elementary principles of number theory helped in the construction of a graph with a graceful degree sequence by Vijayasaradhi and Vangipuram [33].

In this chapter, we have developed a method of construction of graph with a given number as the split domination number of the graph. For this purpose we make use of an arithmetic graph V_m with its vertex set as the set of all divisors of m (except 1) and defining the adjancey property of the arithmetic graph suitably.

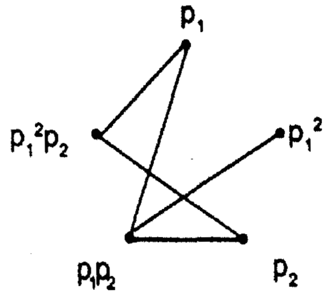
Definition 3.1

The Arithmetic graph V_m is defined as a graph with its vertex set as the set of all divisors of m (excluding 1) where m > 1 and $m = p_1^{a_1} p_2^{a_2} \ldots p_r^{a_r}$, a canonical representation of m, where p_i's are distinct primes and a_i's ≥ 1 and two distinct vertices a , b are adjacent in this graph if $(a, b) = p_i$, for $1 \leq i \leq r$ and they are not of the same parity.

The vertices a and b are said to be of the same parity if both a and b are the pairs of the same prime.

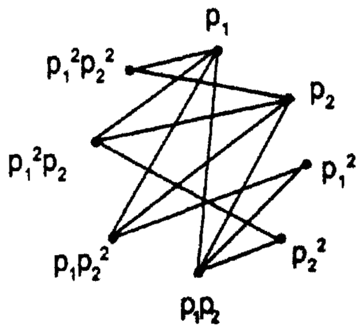
Ex : $a = p^2$; $b = p^5$

Illustration :



The graph V_m with $m = p_1^2p_2$, where p_1, p_2 are any two primes

Fig.3.1



The graph V_m with $m = p_1^2p_2^2$, where p_1, p_2 are any two primes

Fig. 3.2

The split domination of these arithmetic graphs have been studied as it enables us to construct graphs with a given split domination number in a very simple way.

We have obtained that the split domination number of the V_m graph is r+1, where m is a positive integer and $m = p_1^{a_1} p_2^{a_2} \ldots p_r^{a_r}$ is the canonical representation. $p_1, p_2, \ldots, p_r$ are distinct primes and a_i's $\geqslant 1$.

Theorem 3.2

If $m = p_1^{a_1} p_2^{a_2} \ldots\ldots p_r^{a_r}$, where a_i's $\geqslant 1$, for $i = 1,2,\ldots\ldots r$;

then $\gamma_s(V_m) \leq r + 1$.

Where r is the number of distinct prime factors of m.

Proof:

Let $m = p_1^{a_1} p_2^{a_2} \ldots\ldots p_r^{a_r}$ where $a_i \geqslant 1$, for $1 \leq i \leq r$

and $p_1, p_2, \ldots\ldots, p_r$ are distinct primes.

The set of vertices

$D = \{ p_1, p_2, \ldots\ldots, p_r, p_1 p_2 \ldots\ldots p_r \}$ is a split dominating

set

For , If v is any vertex in $V - D$, then v is of the form

$p_1^{b_1}\ p_2^{b_2} \ldots\ p_r^{b_r}$, where $0 \leq b_i \leq a_i$ not all b_i's are '0' at the same time.

Then the vertex $p_1^{b_1}\ p_2^{b_2} \ldots\ p_r^{b_r}$ is adjacent with p_i in D. Thus D is dominating set of V_m.

Further this is also a split dominating set.

For, the vertex $p_1^{a_1}\ p_2^{a_2} \ldots\ p_r^{a_r}$ is not adjacent with any vertex in <V – D>

Hence D is a split dominating set.

Further this is also a minimal split dominating set.

For, If we remove any vertex v from D, then v is of form either p_i, for $1 \leq i \leq r$ or $p_1 \cdot p_2 \ldots p_r$.

If v is of the form p_i, say for the sake of definiteness, $v = p_j$

If we remove the vertex p_j , then the vertex $p_1^{a_1}\ p_2^{a_2} \ldots\ p_r^{a_r}$ is adjacent with p_j. So $D - \{ v \}$ is not a split dominating set.

On the other hand if we remove $p_1\ p_2 \ldots\ p_r$ from D , the vertices $p_1^2, p_1^3, \ldots p_1^{a_1}, p_2^2, p_2^3, \ldots\ p_2^{a_2}$ etc., are not adjacent with any vertex in $D - \{v\}$.

This means that $D - \{ p_1\ p_2 \ldots\ p_r \}$ is not a dominating set.

Hence D is a minimal split dominating set.

There fore, $\gamma_s [V_m] \leq r+1$, where r is the core of 'm' .

§ ***Construction of a graph whose split domination number is atmost t:***

With the help of the above theorem we will now construct a graph with the given split domination number.

These constructions are quite useful in the applications of domination theory in real life situations.

If we are required to construct a graph with a given split domination number 't', we proceed as follows :

Choose $m = p_1^{a_1} p_2^{a_2} \ldots\ldots p_{t-1}^{a_{t-1}}$ where p_i's are distinct primes and a_i's >1.

Consider an arithmetic graph V_m.

By Theorem 3.2, the split domination number of V_m is t and the split dominating set is $\{ p_1, p_2, \ldots\ldots p_{t-1}, p_1 p_2 \ldots\ldots p_{t-1} \}$.

Illustration :

The construction of a graph with a given split domination number as 3 :

(i) Given $t = 3$; we have $t-1 = 2$, choose any two primes p_1, p_2 and let $m = p_1^2 p_2^2$

The vertices of V_m are the devisors of m (except 1) :

$p_1, p_2, p_1^2, p_2^2, p_1p_2, p_1p_2^2, p_1^2p_2, p_1^2p_2^2$

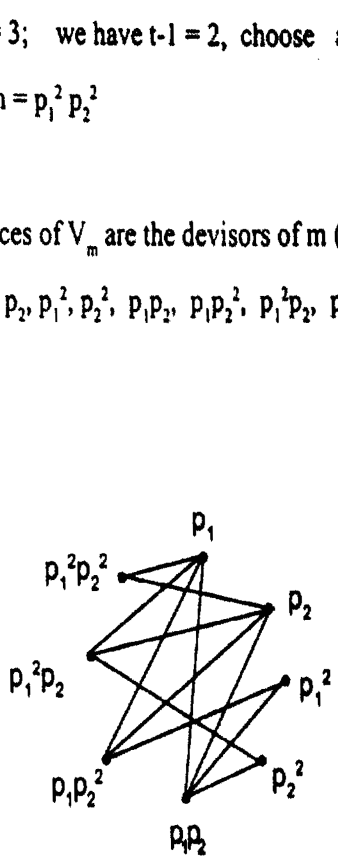


Fig. 3.3

The V_m graph with $m = p_1^2 p_2^2$

$\{p_1, p_2, p_1 p_2\}$ is the minimum split dominating set.

$\gamma_s(V_m) \leq 3$

(ii) Given $t = 3$; we have $t-1 = 2$, choose any two primes p_1, p_2 and let $m = p_1^2 p_2^3$

The vertices of V_m are the devisors of m (except 1) :

$p_1, p_2, p_1^2, p_2^2, p_2^3, p_1p_2, p_1p_2^2, p_1p_2^3, p_1^2p_2, p_1^2p_2^2, p_1^2p_2^3$

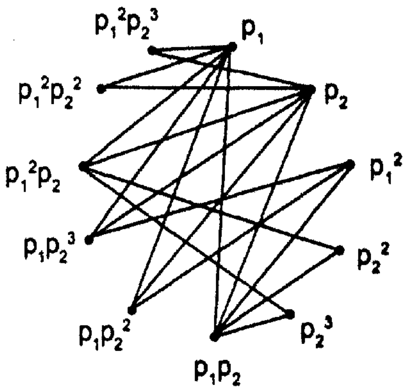


Fig. 3.4

The V_m graph with $m = p_1^2 p_2^3$

$\{p_1, p_2, p_1 p_2\}$ is the minimum split dominating set.

$\gamma_s(V_m) \leq 3$