CHAPTER - II

REVIEW OF LITERATURE
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Bulk - Transportation Problem:

The ‘Classical Transportation Problem’ is given by

(i) A set of sources \{ i, i = 1, 2, ..., m \} and their capacities \( b(i) \)

(ii) A set of destinations \{ j, j = 1, 2, ..., n \} and their requirements \( r(j) \)

(iii) The transportation cost \( c(i,j) \) of one unit of a commodity from source \( i \) to
destination \( j \).

It is required that the destinations should be supplied with their requirements from the sources such that the total cost of the transportation is least. The objective function is a linear function. \( \sum \sum C(i,j) \times (i,j) \), where \( x(i,j) \) is the amount supplied from source \( i \) to the destination \( j \).

The transportation problem can be generalised in many directions. The cost \( c(i,j) \) given above, is a unit cost. In some situations the transportation cost may depend only on \( (i,j) \) the source and the destination, but not on the actual amount transported. This cost is called the 'Bulk-Transportation cost'. This leads to one more generalisation of the transportation problem. The objective function in this case is \( \sum \sum C(i,j) \times H(x(i,j)) \), where \( H \) is a step function with \( H(\alpha) = 1 \) if \( \alpha > 0 \) and \( H(\alpha) = 0 \) if \( \alpha = 0 \), and \( C(i,j) \) is the Bulk-Transportation cost. This problem with the additional restriction that a destination should get its supply from one source only is studied in another Bulk-Transportation (Sundara Murthy - 1976).

Another generalisation of the transportation problem arises in the following situation.

The set of destinations are known and fixed. But there are no sources which are existing, only a set of potential places where sources can be located are known. This set may include some destinations. The problem is to locate some sources in the potential places and supply to the destinations, such that the total cost of transportation is a minimum.
This problem with the bulk-transportation costs and with the restrictions (i) that a destination should get its supply from one source only and (ii) the number of sources to be located should not be more than a specified number was studied by Sundara Murthy - 1976. Here it is assumed that the sources have an unlimited capacity and this makes the specification of the requirement of the destinations and also the location of more than the one source in a place irrelevant.

The following restricted version of the above problem was studied earlier (EL-Shaieb-1973, Curry and Skeith-1969, Pevelle and Swin - 1970).

The destinations themselves are the only set of potential places for locating a source. If a source is located in a destination then it should get its supply from that source only. The number of sources to be located are fixed in number. A destination should get its supply from one source only.

In this case the problem reduces to the case of partitioning the destination set into two sub-sets in which one is called 'source set' and the other is called 'destination set'. The sources are assigned to the destinations which will supply their requirements. The cost of supplying to a destination from the source located at it is taken as zero, and the costs of supply from the other sources to this are taken as non-negative. Curry and Skeith - 1969 formulated this problem and used Dynamic programming to solve it, but EL - Shaieb - 1973 observes that it was "without much of computational success" Revelle and Sain - 1970 formulated this problem as a linear programming which will not guarantee an integer optimal solution. EL-Shaieb - 1973 presents a 'Branch and Bound' algorithm to solve this problem. For this algorithm at each node (of the solution tree) the lower bound requires a series of calculations and rearrangements of the values. This algorithm does not implicitly consider all the branches from a node. A rule for the choice of a branch form a node is given, but theoretically it is not justified that it leads to an optimal feasible solution.

In the sequel we will develop a lexi-search algorithm based on 'Pattern recognition technique' for the general problem stated earlier

Let $N = \{ 1, 2, \ldots, n \}$ be the set of $n$ destinations

$S = \{ 1, 2, \ldots, m \}$ be the set of $m$ potential places for the location of a source
$M_o$ be the maximum number of sources allowed 

$$D = \{ d(i, j) \} \ i \in S, j \in N,$$ 

where $d(i, j)$ is the bulk transportation cost supplying the quantity from source $i$ to the destination $j$.

It can be noted that once a source is located in a potential place then it is identified by that place. $d(i, j)$'s can be taken as either costs or distances. If $d(i, j)$ is not the shortest distance (or least cost), the shortest distance can be calculated and that can be taken as $d(i, j)$ (Pollack and Wirebenson - 1960, Pandit - 1963 and Dantzig - 1966). There is no restriction on the $d(i, j)$ values. The problem can be stated as follows.

Locate a maximum of $M_o$ Sources among the $m$ places of the set $S$ and assign these sources to the destinations of the set $N$ (where a destination gets its supply from the source assigned to it), such that the total bulk transportation cost is a minimum, satisfying the condition that a destination should get its supply from one source only.

Another 'Bulk-Transportation' problem (Sundara Murthy - 1976) with the following Characteristics.

(i) The source are fixed and the capacity of each source is given
(ii) The requirements of the destinations are given
(iii) A destination should get its supply from one source only, but a source can supply to any number of destination subject to its capacity.
(iv) The bulk transportation cost from a source to a destination is given

The objective is to supply the requirements of the destinations with a minimum cost satisfying the above conditions.

This problem can be formulated as 'transportation problem' with the additional restriction that $x(i, j) = 0$ or $1$, and solve it as a Integer programming problem. This also can be formulated as a 0-1 programming problem and can be solved (Balas - 1965 and Glover - 1965). But none of these methods will take the advantage of the combinatorial structure of the problem which is very close to that of the assignment problem. Demai and Roveda - 1971 first formulated this problem and they have developed a 'branch and bound algorithm' to solve it. They also gave a practical situation where this problem arises. Later, Srinivasan and Thompson - 1973 developed a branch and bound algorithm for the same problems. They have formulated a modified transportation problem for
which the optimal solution of the above problem will be a basic feasible solution. They believe that this algorithm is better than the one presented by Demaio and Roveda - 1971 because their algorithm utilises the structure of the transportation problem. The main drawback of Demaio and Roveda's algorithm is that method a lot of calculations is required for checking the feasibility of a solution, and several solutions have to be recorded till the optimal solution is identified. The latter's algorithm ignores the simple structure of the problem which is very close to that of assignment problem in finding an optimal solution. In this algorithm a series of transportation problems are formulated and solved. Integer solutions are expected at each stage.

Latter a lexi-search algorithm which takes out the drawbacks of Demaio and Roveda algorithm and takes the advantage of the simple structure of the problem to get an optimal feasible solutions was studied by (a) Pandit and Sundara Murthy - 1975 and (b) Sundara Murthy - 1976). They presented the problem as following and developed a lexi search algorithm.

Let \( D = (1, 2, \ldots, n) \) be the set of \( n \) destinations

\[ S = \{ 1, 2, \ldots, m \} \] be the set of \( m \) sources

\[ SC = SC (i), \text{ie} S \] be an array where \( SC (i) \) is the availability at source \( i \)

\[ DR = DR (j), \text{ja} D \] be an array, where \( DR (j) \) is the requirement of the destination \( j \)

\[ C = \{ C(i,j) \}, i \in S, j \in D, \text{where} \ C (i, j) \text{, is the bulk transportation cost of supplying from source } i \text{ to destination } j \]

The problem is to find a supply - schedule with a least transportation cost in which all the destinations get their requirements with the condition that each destination should get its requirement from one source only.

**Single Travelling Salesman Problem**

The travelling salesman problem (TSP) is one of the most widely studied combinatorial programming problems in the literature of Operations Research. Many researchers have developed different algorithms for the solution of TSP so far. Here, we shall present an integrated overview of some of the exact and approximate algorithms developed for the solution.
The methods considered usually can be divided into three basic parts: a starting point, a solution generation scheme, and a termination rule. When the termination rule is such that the iteration stops if and only if a tour is optimal, the method is exact, and when the rule is such that the iteration stops, if but not only if a tour is optimal, the method is approximate.

Unfortunately, there is no such analytical method which can be used satisfactorily. However, a good number of algorithms have been proposed for the travelling salesman problem either optimally or sub-optimally by many research workers in different times. The methods and algorithms mainly include: Integer programming, Dynamic programming, Longest path problem approach, Job sequencing, partitioning and decomposition, Branch and Bound algorithm, Assignment technique etc.

One of the earliest formulations is suggested by Dantzig, Fulkerson and Johnson (1954). The difficulties in finding an optimal tour in solving the integer program are due to the appearance of enormous number of loop constraints. However they overcame the large number of loop constraints by beginning with only a few, and then adding new ones only as they were needed to block sub-tours. Combinatorial arguments were used to eliminate fractional solutions and to find an optimal tour. Finally it was demonstrated that for the problem at hand, an ordinary linear programming could be devised whose solution gave integer valued $x_i$'s representing the optimal tour. The constraints that rule out some fractional solutions but no integer solutions were forerunners to Gomory's "Cutting plane" constraint for solving any integer linear program (Gomory - 1963).

Other integer programming formulations have been obtained by Bock and Mudrov (1965). Dynamic programming algorithms have been developed by Bellman (1962), Gonzales (1962) and Held and Karp (1962).

Hardgrave and Nemhauser (1962) claimed that the travelling salesman problem is a special case of longest path problem. As they stated unfortunately no efficient computation scheme presently exists for the general Longest - path problem either. It is hoped that the longest path problem will prove to be easier than the travelling salesman problem has been and consequently that this approach will lead to an efficient algorithm for the travelling salesman problem.
However, as pointed out by Pandit (1964): The above two problems (i.e. the travelling salesman problem and the longest path problem) and the corresponding problem of finding the shortest non-looped paths with exactly k intermediate nodes in a n-node network are structurally so similar and their conversion into one another is so straightforward that they are practically equivalent to one another in computational difficulty. In fact, if would be more appropriate to say that the travelling salesman problem is another version - rather a special case of the 'longest -path - problem'.

Pandit also notes: 'As is obvious, the travelling salesman and assignment problems are structurally the same, except that the solution of the former exacting - the permutation matrix corresponding to the solution should be indecomposable'.

Gilmore and Gomery (1964) have related the travelling salesman problem to job sequencing problem. They formulated the problem of cyclical sequencing of n jobs on a machine with specified change over costs as a travelling salesman problem. It involves sequencing each of a set of jobs (e.g. visiting a city) for a facility (e.g. salesman) so as to minimize some characteristics of movement from one job to the next. They assumed that job i, i = 1, 2, ..., n could be started only when the machine in a particular state Ai and after the completion of the job the machine would be in a state Bi. Cij is the cost for changing the machine state Bi to Aj when job j follows job i on the machine.

In the Partitioning and Decomposition technique the size of a travelling salesman problem can be reduced by imposing restrictions on the order in which the nodes can be traversed. Suppose the nodes (i1, ..., i_{n+k}) es must be traversed consecutively in the given order. Then the original n node problem can be reduced to a k+1 node problem.

Rothkopf (1966) showed another possible simplifications. He was motivated by sequencing problems in which the cost of processing a job depends only on its class and the class of the previous job.

Held and Karp (1962) used partitioning to obtain approximate solutions. They gave some nodes for selecting 'good' partitions. The rules attempt to identify a partition such that an optimal tour on the partitioned problem is as short as possible. Karg and Thompson (1964) used partitioning in a slightly different way.
Branch and Bound algorithms are commonly used for the solution of TSP's. In the context of mathematical programming they can best be viewed as initially relaxing some of the problem's constraints and then regaining feasibility through an enumerative process. The quality of a branch and bound algorithm is directly related to the quality of the bound provided by the relaxation.


The algorithm developed by Eastman and extended by Shapiro is a search technique in which one partitions the set of tours into subsets and calculates lower bounds on the cost of all tours in a subset. Shapiro reported considerable difficulty with symmetric problem. For this reason he took the integer programming formulation for the symmetric case.

Little, et al (1963) in their algorithm used different tactics for branching and bounding. The calculation of bounds is based upon matrix reduction. The reduced matrices are used for branching by partitioning the tours in a given subset into two subsets. This is done by committing an arc in one of the subsets and prohibiting that are in the other subset.

In the Carpaneto and Toth algorithm (1980), the problem solved at a generic node of the search tree is a modified assignment problem, in which some \( x_i \) variables are fixed at 0 or at 1. If the assignment problem solution consists of a unique tour over all vertices, it is then feasible for the TSP. Otherwise, it consists of a number of subtours. One of these subtours is selected and broken by creating subproblems in which all arcs of the sub tours are in turn prohibited. Here the main steps are initialization, node selection, subproblem partitioning, bounding, sub-problem selection and lastly feasibility check. Using their algorithm, Carpaneto and Toth have consistently solved randomly generated 240 vertex TSP's in less than one minute. The main limitation of this algorithm appears to be 'computed memory' rather than CPU time.
The Balas and Christophides algorithm uses a stronger relaxation than the assignment problem relaxation. Its description and the computational effort required for the lower bound computations, but the resulting search trees are smaller and the procedure is more powerful.

More recently Miller and Pekny have proposed a new powerful branch and bound algorithm based on Assignment problem relaxation, considering Dual Assignment Problem.

Pandit has developed an "Intelligent Search" approach viz., Lexiscographic search to the assignment problem and has modified it to solve the travelling salesman problem (Das - 1976; 78 Pandit - 1961, 1963, 1964, 1965)

For Assignment problem one is given an n-by-n matrix \( S \) (distance, times etc.,). The object is to find an n-by-n matrix \( x \), with

\[
X_{ik} = 0 \text{ or } 1 \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, n
\]

\[
\sum_{i=1}^{n} x_{ik} = 1, \quad k = 1, 2, \ldots, n
\]

\[
\sum_{k=1}^{n} x_{ik} = 1, \quad i = 1, 2, \ldots, n
\]

and such that \( \sum_{i=1}^{n} \sum_{k=1}^{n} S_{ik} x_{ik} \) is a minimum.

Upto this point the problem is the Assignment problem. But for the TSP there is the additional restraint that no tour can return to its starting point until all n cities have been visited. The Assignment problem (AP) based algorithms are valid whether the cost matrix (or distance or time matrix) is symmetrical or assymmetrical. However in symmetrical cases the AP solutions will in general contain several subtours containing only two vertices, resulting in excessive computing times for their elimination. Symmetrical problems are better handled by specialized algorithms that exploit their structure. Christofides (1970) and Held and Karp (1962) were among the first to propose a TSP algorithm based on this relaxation. Improvement and refinements were latter
One more algorithm available is 'Closest - Unvisited - City' algorithm, which of course is not of much practical importance. If there are frequent ties in the selection of the closest city and if one is to try to enumerate all possible closest city solutions, the computation becomes burdensome.

**Travelling Salesman Problem with variations:**

There are many algorithms for usual one man TSP developed by researchers from time to time. But the problem has not received much attention in its restricted context. However, literatures which are available in regard to the TSP with variations are discussed (Das - 1976, 78, Jaillet P - 1985, Kubo & Kasugai - 1991, Pandit - 1961, 63, 64, 65, Ramesh - 1981, Raviganesh G.S. Murthy & Das - 1998 and Srivastava Kumar, R.C. Garg and P. Sen - 1969).

The revisit of city (cities) in TSP has certain significance. It is observed that sometimes revisit of city (cities) may be cheaper than when it (revisit) is not allowed. Revisit of city may be either 'a must' (due to lack of communication) or more economical. It has been shown in (Das - 1971) that the example discussed in Little, et al - 1963 problem yields cheaper route when revisit is allowed.

Another type of TSP with variation may be precedence and / or positional constrained TSP (Das - 1971, Scroggs & A.L. Tharp). In precedence constrained problem each admissible tour has to satisfy some precedence relation(s). Similarly in fixed position constraints, admissible tours are required to satisfy the position constraints. Srivastava et al - 1969 have considered a problem where the salesman has to pass through n sets of nodes. The problem may be stated as : Consider a network which for convenience is denoted in any order of the sequence of nodes, 0, \( \alpha_1, \alpha_2, \ldots, \alpha_n \); \( \beta_1, \beta_2, \ldots, \beta_m \); \( \gamma_1, \gamma_2, \ldots, \gamma_p \); \( \delta_1, \delta_2, \ldots, \delta_r \). Here 0 denotes the starting place and \( \alpha, \beta, \gamma, \ldots, \delta \) denote the subset of n, m, p, \ldots, r nodes respectively. The problem considered is that of determining the optimal tour i.e. starting from 0 and returning to 0 under the condition that the route must contain at least one node from each
of the sets. A mathematical model is developed to solve the problem. Scroggs and Tharp have presented an algorithm for solving the TSP in restricted context, the restrictions being (a) the cities visited have partial ordering restrictions and / or (b) same cities are assigned particular positions on the tour.

Travelling purchaser problem (Ramesh T - 1981) is another kind of variation to the TSP. Here there is a of 'm' markets and a set of 'n' commodities. The cost of travel between each pair of markets and cost of each commodity at each market are known. A purchaser starts from one market and returns to it after purchasing all the commodities he needs. He may not visit all the markets, also may not purchase any commodity even if passes through a market. The purchaser is to find an optimal tour such that the total of the cost of travel and the cost of purchasing all the commodities is a minimum.

In 1985 Jaillet introduced the probabilistic travelling salesman problem (PTSP), a variant of the classical TSP in which only a subset of the nodes may be present in any given instance of the problem. The goal is to find an a priori tour of minimal expected length, with the strategy of visiting the present nodes in a particular instance in the same order as they appear in the a priori tour. In general PTSP's arise in practice whenever a company, on any given day, is faced with the problem of collection (deliveries) from (to) a random subset of its (known) global set of customers in area, etc.

Another interesting problem on variation to TSP has been discussed by Raviganesh, Kumar and Das (1998) where they have developed an algorithm to find the solution for a TSP when the set of nodes to be visited are changing. For example a particular manufacturer may have for his product a network of demand points which may change frequently due to changes experienced in the demand at those points. A system which changes for any reason is defined to be a protean system. Here a TSP in a protean network is considered and an efficient algorithm to solve the protean problem is discussed.
Time Dependent Travelling Salesman Problem (TDTSP):

The travelling salesman problem (TSP) can be generalized. The TDTSP can be stated as follows. "There are n cities and \( N = \{1, 2, \ldots, n\}\). The cost array \( C(i, j, k) \) is the cost of a salesman visiting from city \( i \) to city \( j \) at time/facility \( k \) is known \( (i, j, k = 1, 2, \ldots, n) \). The restriction for the travelling salesman is that at a point of time/facility he should not visit more than one pair of cities in his 'tour'. The problem is to find a feasible tour satisfying the above conditions such that the total tour cost is minimum".

The TDTSP is a generalised from the travelling salesman problem (TSP). The TDTSP may be stated as a scheduling problem in which \( n \) jobs have to be processed at a minimum cost on a single machine. The set-up cost associated with each job depends not only on the job that precedes it, but also on its position (time) in the sequence. In this problem, the cost of each transition (Set-up cost) depends not only on the two respective locations involved but also on their positions in the sequence that defines the "tour" (Picard J.C. and Queyranne, 1978). The problem is to find the "tour" of the salesman with a minimum total cost such that, he visits only one pair of cities at a specified time.

The TDTSP earlier was attempted by Bowman 1956, Miller 1960 etc. and Picard and Queyranne, 1978. The formulated the TDTSP in which it requires in the order of \( n^2 \) constraints and \( n^2 \) variables. TDTSP was also attempted by Fox, etc. 1979. They had suggested some improvements and it requires the order of \( 4n \) linear constraints. But in all the above attempts the simple combinatorial structure of the TDTSP is not at all taken into consideration. TDTSP was also attempted by Bhavani - 1997, Sobham Babu - 2000, Naganna - 2001, Balkrishna - 2006 and Venkatesulu - 2007, and the simple combinatorial structure of the TDTSP was taken into account.

A truncated time dependant TSP was also attempted by Balakrishna - 2006, but time is not continuous.

Multiple Travelling Salesmen Problem (M-TSP):

The formulation of M-TSP has been given by many researchers since early seventies. These are discussed briefly in this chapter.
A formulation of the TSP with more than one salesman is offered by Svestka and Huckfeldt-1973. The particular formulation is claimed to have computational advantages over other formulations. Experience is obtained with an exact branch - and - bound algorithm employing both upper and lower bounds (mean run time for 55 city problem is one minute). Due to the special formulation, certain sub-tours may satisfy the constraints, thus reducing the search a very good initial tour and upper bound are employed. The determination of these as well as pathology of the formulation and the algorithm are discussed. Their procedure requires to solve a sequence of assignment problems and test for the feasibility of the tours. They also proved with statistical data that their algorithm is better than that of Little et al (1963). The formulations forwarded are based on Miller - Tucker – Zemlin, Benzalel Gavish(1976) Svestka(1976) and Kulkarni&Bhave(1980) formulations.

The following note on M-TSP is offered by Hong and Padberg (1977). The M-TSP with fixed charges (MTSPF) is the following variation of the usual travelling salesman problem:

Given 'n' cities (number 1, 2, - - -, n ) and m salesmen located at city n, find an assignment of atmost m salesmen such that each city is visited exactly once by exactly one salesman and such that each salesman who is assigned to a tour starts and ends his tour in city n, the home city. By assigning the ith salesman to a tour, one incurs a fixed charge $f_i$ which is independent of the length of his tour. For travelling from city i to city j one incurs a cost $C_{ij}$ that does not depend upon which salesman makes that particular trip. The problem is to find the number of salesmen to be employed and their respective routes so as to minimize the total cost.

In this note they give a different construction and show that the symmetric MTSPF on n cities with salesmen is equivalent to the standard symmetric travelling salesman problem involving $n + m + 4$ cities. The fixed charge version of the M-TSP has immediate application. Though this MTSPF is claimed to be a variation to the M-TSP, it reduces to an ordinary M-TSP with fixed charges added to the different salesmen separately.
Later on M.R. Rao (1980) discussed on the formulation given by Bellmore and Hong (1974) and Hang and Padberg (1977). He shows how the transportation given in (1974) for the asymmetric case can be extended directly to the symmetric case. This transformation results in a standard symmetric travelling salesman problem with \( n + m - 1 \) cities only. A variation of the symmetric M-TSP's can also be considered. The variation involves specifying a base city in general a different one, for each salesman. If the base city is the same for all salesmen, one would have the asymmetric M-TSP without any fixed charges.

Another approach to M-TSP is 'Cutting Planes Algorithm' developed by Laporte and Nobert (1983) which is said to be a relaxed version of 1-TSP, when \( m \), the number of salesmen is not fixed a priori.

Also a model of printing press scheduling for Multi-Edition Periodicals similar to M-TSP has been offered by Gorenstein (1970). The problem of scheduling a printing press for a periodical with several editions, so as to minimize the costs has been discussed. A mixed integer programming has been formulated. Unfortunately, the method is not suitable for problem of sizes required for practical problems. A heuristic approach has been proposed based on TSP.

The Lexicographic search algorithm for \( m \) salesmen problem we have applied to the present investigation is found to be much more effective even if the number of constraints is more. It holds good if the number of 'nodes' and 'salesmen' are increased.

**M-TSP with variation:**

The study of M-TSP in restricted context is also found in Operations Research Literature. However, Das and Borah (1994) developed a solution procedure for a multiple salesmen problem. The works are done on precedence constrained cost minimization problem and time minimization M-salesmen problem.

Mrs. Vidyulatha's (1992) problem "M-TSP" and developed a lexi search algorithm.