CHAPTER-IV
NON-LINEAR PERISTALTIC MOTION OF A CARREAU FLUID UNDER THE EFFECT OF A MAGNETIC FIELD IN AN INCLINED PLANAR CHANNEL
1. INTRODUCTION

The mechanics of peristaltic has been examined by a number of investigators. Latham (1966) discussed for the first time about peristalsis in his thesis. Later, Fung and Yih (1968) and Shapiro et al. (1969) worked on very similar lines. Lew et al. (1971) suggested chyme in the small intestine as a non-Newtonian fluid. Shukla et al. (1980) investigated the effects of peripheral -layer viscosity on peristaltic transport of a bio-fluid in a uniform tube and used the long wave length approximation as in Shapiro et al (1969). Bohme and Friedrich (1983) discussed the peristaltic flow of a viscoelastic liquid assuming that the relevant Reynolds number to be small enough to neglect inertia forces and ratio of the wave length and channel height to be large which implies that the pressure is constant over the cross-section. Pozrikids (1987) considered peristaltic flow under the assumption of creeping motion and used boundary integral method for Stokes flow. Srivastava and Srivastava (1985, 1988) showed the effects of power-law fluid in uniform and non-uniform tubes and in a channel under zero Reynolds number and long wavelength approximations. Siddiqui and Schwarz (1994) illustrated the peristaltic flow of a second order fluid in tubes and used a perturbation method to second order in dimensionless wave number. Provost and Schwartz (1994) have studied viscous effects in peristaltic pumping and assumed that the flow is free of inertial effects and that non-Newtonian normal stresses are negligible. El Misery et al. (1996) studied peristaltic transport of Carreau fluid through a uniform channel, under zero Reynolds number and long wave length approximations. Elshahawey et al. (1998) investigated peristaltic transport of Carreau fluid through non-uniform channel, under zero Reynolds number and long wave length approximations. Elshehawey et al (2000) analyzed peristaltic pumping of Carreau fluid through a porous medium in a channel. Abd El Hakeem et al. (2004) investigated separation in the flow through peristaltic motion of a Carreau fluid in an axisymmetric tube. Ali and Hayat (2007) investigated peristaltic motion of a Carreau fluid in an asymmetric channel.
Agrawal and Anwaruddin (1984) studied the effect of moving magnetic field on blood flow. They studied a simple mathematical model for blood through an equally branched channel with flexible outer walls executing peristaltic waves. The result revealed that the velocity of the fluid increases with an increase in the magnetic field. Hayat et al. (2007) discussed the peristaltic flow of a MHD third order fluid in a planar channel. The peristaltic flow of a MHD fourth grade fluid in a planer channel has studied by Hayat et al. (2007). Peristaltic transport of a Johnson-Segalman fluid under the effect of a magnetic field was developed by Elshahed and Haroun (2005). Mekheimer (2008) studied non-linear peristaltic flow of fluid in an inclined planar channel under the effect of a magnetic field.

However, the interaction of the magnetic field with peristaltic flow of a Carreau fluid in an inclined symmetric channel has received little attention. Hence, an attempt is made to model the peristaltic flow of a Carreau fluid in an inclined symmetric channel under the long wavelength and low Reynolds number assumptions, in the presence of a transverse magnetic field. The flow is examined in a wave frame of reference moving with velocity of the wave. A regular perturbation technique is employed to solve the present problem and solutions are expanded in a power of small Weissenberg number. Expressions for the velocity, axial pressure gradient, pressure rise and frictional force over a one wavelength are obtained. The effects of various emerging parameters on pumping characteristics and frictional forces are discussed in detail.

2. MATHEMATICAL FORMULATION

A two-dimensional flow of an incompressible electrically conducting Carreau fluid in an inclined symmetric channel with inclination $\alpha$ induced by sinusoidal wave trains propagating with constant speed along the channel walls is considered. A uniform magnetic field $B_0$ applied in the transverse direction to the flow. The fluid is assumed to be of small electrically conductivity so that the magnetic Reynolds number is very small and hence the induced magnetic
field is negligible in comparison with the applied magnetic field. The external electric field is zero and the electric field due to polarization of charges is also negligible. Heat due to Joule dissipation is neglected. Fig. 1 represents the physical model of the flow field.

The equation of the channel walls is given by

\[ Y = \pm H(X,t) = \pm a \pm b \sin \left( \frac{2\pi}{\lambda} (X - ct) \right), \]

(2.1)

where \( b, \lambda, c \) and \( a \) are amplitude, wavelength, phase speed of the wave, mean-half width of the channel respectively, \( t \) is the time and \((X, Y)\) are the Cartesian co-ordinates in a fixed frame.

Fig. 1 The physical model
We introduce a wave frame of reference \((x, y)\) moving with the velocity \(c\) in which the motion becomes independent of time when the channel length is an integral multiple of the wave length and the pressure difference at the ends of the channel is a constant (Shapiro et al., 1969).

The transformation from the fixed frame of reference \((X, Y)\) to the wave frame of reference \((x, y)\) is given by

\[
x = X - ct, \quad y = Y, \quad u = U - c, \quad v = V, \quad p'(x) = P'(X, t).
\]

(2.2)

where \((u, v)\) and \((U, V)\) are the velocity components, \(p\) and \(P\) are the pressures in the wave and fixed frames of reference respectively.

The constitutive equation for a Carreau fluid [following Bird et al. (1977)] is

\[
\tau = -\left[\eta_\infty + (\eta_0 - \eta_\infty) \left(1 + (\Gamma \dot{\gamma})^2\right)^{\frac{n-1}{2}}\right]\dot{\gamma}
\]

(2.3)

where \(\tau\) is the extra stress tensor, \(\eta_\infty\) is the infinite shear rate viscosity, \(\eta_0\) is the zero shear rate viscosity, \(\Gamma\) is the time constant, \(n\) is the dimensionless power-law index and \(\dot{\gamma}\) is defined as

\[
\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij} \dot{\gamma}_{ij}} = \sqrt{\frac{1}{2}\pi}
\]

(2.4)

where \(\pi\) is the second invariant of strain-rate tensor. We consider in the constitutive Equation (2.3) the case for which \(\eta_\infty = 0\) and so we can write

\[
\tau = -\eta_0 \left(1 + (\Gamma \dot{\gamma})^2\right)^{\frac{n-1}{2}} \dot{\gamma}
\]

(2.5)

The above model reduces to a Newtonian model for \(n=1\) (or) \(\Gamma = 0\).

The equations governing the flow field, in the wave frame of reference are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(2.6)

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p'}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \sigma \mu \dot{\gamma}^2 + B_0 (u + c) + \rho g \sin \alpha
\]

(2.7)
\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p'}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \rho g \cos \alpha \tag{2.8}
\]

where \( \rho \) is the density, \( \sigma \) - the electrical conductivity, \( \mu_r \) - the magnetic permeability and \( B_0 \) - constant transverse magnetic field.

Due to symmetry, the problem is studied only for upper half of the channel.

The boundary conditions for the velocity are

\[
\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{2.9}
\]

\[
u = -c \quad \text{at} \quad y = H \tag{2.10}
\]

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

\[
\tilde{x} = \frac{x}{\lambda}, \quad \tilde{y} = \frac{y}{a}, \quad \tilde{u} = \frac{u}{c}, \quad \tilde{v} = \frac{v}{c \delta}, \quad \tilde{\tau} = \frac{\tau}{\lambda}, \quad \tilde{p'} = \frac{\rho' a^2}{\eta_c \lambda}, \quad \tilde{h} = \frac{H}{a}, \tag{2.11}
\]

\[
\tilde{c} = \frac{ct}{\lambda}, \quad \tilde{\tau}_{xx} = \frac{\tau_{xx}}{\eta_c}, \tilde{\tau}_{xy} = \frac{\tau_{xy}}{\eta_v}, \quad \tilde{\tau}_{yy} = \frac{\tau_{yy}}{\eta_v}, \quad \tilde{\eta} = \frac{\eta_c}{\eta_v}, \quad \tilde{R}e = \frac{\rho a c}{\eta_v},
\]

where \( R e \) is the Reynolds number, \( W e \) - Weissenberg number and \( \delta \) - the wave number.

In view of Equation (2.11), the Equations (2.6) - (2.8), after dropping bars, reduce to

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.12}
\]

\[
\text{Re} \delta \left( u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} \right) = - \frac{\partial p'}{\partial y} - \delta^2 \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - M^2 (u + 1) - \eta \sin \alpha \tag{2.13}
\]

\[
\text{Re} \delta^3 \left( u \frac{\partial v}{\partial x} + \nu \frac{\partial v}{\partial y} \right) = - \frac{\partial p'}{\partial y} - \delta^2 \frac{\partial \tau_{xy}}{\partial x} - \delta^2 \frac{\partial \tau_{yx}}{\partial y} - \eta \cos \alpha. \tag{2.14}
\]

where \( \tau_{xx} = -2 \left[ 1 + \left( \frac{n-1}{2} \right) W e^2 \right] \frac{\partial u}{\partial x}, \)

\[
\tau_{xy} = - \left[ 1 + \left( \frac{n-1}{2} \right) W e^2 \right] \left( \frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x} \right),
\]
\[ r_\nu = -2\delta \left[ 1 + \left( \frac{n-1}{2} \right) \text{We}^2 \right] \frac{\partial u}{\partial y}, \]

\[ \dot{\gamma} = \left[ 2\delta \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} - \delta \frac{\partial \dot{\gamma}}{\partial x} \right) + 2\delta \left( \frac{\partial v}{\partial y} \right)^2 \right]^{1/2}, \]

\[ \eta = -\frac{\rho g a^2}{\eta_c}, \quad \eta_h = \frac{\rho g a^2}{\eta_h c}, \]

and \( Da = \frac{k}{a^2} \) and \( M = a \mu B \sqrt{\frac{\sigma}{\eta_0}} \) are the Darcy number and Hartman number respectively.

Under lubrication approach, neglecting the terms of order \( \delta \) and \( \text{Re} \), from Equations (2.13) and (2.14), we get

\[ \left( \frac{\partial p'}{\partial x} - \eta \sin \alpha \right) = \frac{\partial}{\partial y} \left[ 1 + \left( \frac{n-1}{2} \right) \text{We}^2 \left( \frac{\partial u}{\partial y} \right)^2 \right] \frac{\partial u}{\partial y} - M^2 (u+1) \tag{2.15} \]

\[ \frac{\partial p'}{\partial y} = -\eta \cos \alpha. \tag{2.16}. \]

The corresponding dimensionless boundary conditions in wave frame of reference are given by

\[ \frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0. \tag{2.17} \]

\[ u = -1 \quad \text{at} \quad y = h = 1 + \phi \cos 2\pi x. \tag{2.18} \]

Let \( p' = p(x) - \eta_1 (\cos \alpha) y \) and \( \frac{\partial p'}{\partial x} = \frac{\partial p'}{\partial x} \). Then Equations (2.15) and (2.16) become

\[ \left( \frac{\partial p}{\partial x} - \eta \sin \alpha \right) = \frac{\partial}{\partial y} \left[ 1 + \left( \frac{n-1}{2} \right) \text{We}^2 \left( \frac{\partial u}{\partial y} \right)^2 \right] \frac{\partial u}{\partial y} - M^2 (u+1) \tag{2.19} \]

\[ \frac{\partial p}{\partial y} = 0. \tag{2.20} \]

Equation (2.20) implies that \( p \neq p(y) \). Therefore Equation (2.19) can be rewritten as
The volume flow rate $q$ in a wave frame of reference is given by

$$q = \frac{\partial}{\partial t} \int_0^h u dy.$$  \hspace{1cm} (2.22)

The instantaneous flux $Q(X,t)$ in a fixed frame is

$$Q(X,t) = \int_0^h (u + 1) dy = q + h.$$ \hspace{1cm} (2.23)

The time average flux $\bar{Q}$ over one period $T(=\lambda/c)$ of the peristaltic wave is

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = \int_0^1 (q + h) dx = q + 1.$$ \hspace{1cm} (2.24)

3. SOLUTION OF THE PROBLEM

The Equation (2.21) is non-linear and its closed form solution is not possible. Hence, we linearize this equation in terms of $We^2$, since $We$ is small for the type of flow under consideration. So we expand $u, p$ and $q$ as

$$u = u_0 + We^2 u_1 + O(We^4)$$

$$p = p_0 + We^2 p_1 + O(We^4)$$

$$q = q_0 + We^2 q_1 + O(We^4)$$ \hspace{1cm} (3.1)

Substituting (3.1) in the Equation (2.21) and in the boundary conditions (2.17) and (2.18) and equating the coefficients of like powers of $We^2$ to zero and neglecting the terms of $We^4$ and higher order, we get the following equations:

Equation of order $We^0$

$$\frac{dp_0}{dx} - \eta \sin \alpha = \frac{\partial^2 u_0}{\partial y^2} - M^2 (u_0 + 1)$$ \hspace{1cm} (3.2)

and the respective boundary conditions are

$$\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 0$$ \hspace{1cm} (3.3)
\( u_0 = -1 \) at \( y = h \) \hspace{1cm} (3.4)

Equation of order \( We^2 \)

\[
\frac{dp_0}{dx} = \frac{\partial^3 u_0}{\partial y^3} + \left( \frac{n-1}{2} \right) \frac{\partial}{\partial y} \left[ \left( \frac{\partial u_0}{\partial y} \right)^3 \right] - M^2 u_0 ,
\]
and the respective boundary conditions are

\[
\left. \frac{\partial u_0}{\partial y} \right|_{y=0} = 0 \hspace{1cm} \text{at} \hspace{1cm} y = 0 \hspace{1cm} (3.6)
\]
\[
u_i = 0 \hspace{1cm} \text{at} \hspace{1cm} y = h \hspace{1cm} (3.7)
\]

Solving the Equation (3.2) by using the boundary conditions (3.3) and (3.4), we get

\[
u_0 = \frac{1}{M^3} \left[ \frac{dp_0}{dx} - \eta \sin \alpha \right] \left[ \frac{\cosh My}{\cosh Mh} \right] - 1
\]
and the volume flow rate \( q_0 \) is given by

\[
q_0 = \int_0^h u_0 \, dy = \frac{1}{M^3} \left[ \frac{dp_0}{dx} - \sin \alpha \right] \left[ \frac{\sinh Mh - Mh \cosh Mh}{\cosh Mh} \right] - h
\]

From Equation (3.9), we get

\[
\frac{dp_0}{dx} = M^3 \left[ \frac{(q_0 + h) \cosh Mh}{\sinh Mh - Mh \cosh Mh} \right] + \eta \sin \alpha .
\]

Solving the Equation (3.5) by using the Equation (3.8) and the boundary conditions (3.6) and (3.7), we get

\[
u_i = \frac{1}{M^2} \left[ \frac{dp_0}{dx} \left[ \frac{\cosh My}{\cosh Mh} \right] - 1 \right] + \left( \frac{n-1}{2} \right) \left[ \frac{dp_0}{dx} - \eta \sin \alpha \right]^3 \frac{3}{M^2 \cosh^3 Mh} B
\]
where

\[
B = \left[ \frac{1}{32 M^4} \cosh 3Mh - \frac{h}{8M} \sinh Mh \right] \cosh My \cosh Mh - \frac{1}{32 M^4} \cosh 3Mh + \frac{y}{8M} \sinh My
\]

and the volume flow rate \( q_1 \) is given by

\[
q_i = \int_0^h u_i \, dy = \frac{1}{M^3} \left[ \frac{dp_0}{dx} \left[ \frac{\sinh Mh - Mh \cosh Mh}{\cosh Mh} \right] + \left( \frac{n-1}{2} \right) \left[ \frac{dp_0}{dx} - \eta \sin \alpha \right]^3 \right] \frac{3}{M^2 \cosh^3 Mh} B_i
\]
where

\[
B_i = \frac{3}{M^2 \cosh^3 Mh} \left[ \cosh 3Mh \sinh Mh - \frac{h}{32 M^3 \cosh Mh} \cosh Mh - \frac{h}{8 M^2} \sinh^2 Mh \right]
\]
\[-\frac{1}{96M^3} \sinh 3Mh + \frac{h}{8M^2} \cosh Mh - \frac{\sinh Mh}{8M^3}\]

From Equations (3.12) and (3.10), we have
\[\frac{dp}{dx} = \left( q_1 - \frac{3}{2} (n-1) \frac{M^3(q_o + h)^3 \cosh'Mh}{\cosh Mh (\sinh Mh - h \cosh Mh)} \right) \frac{M^3 \cosh Mh}{(\sinh Mh - Mh \cosh Mh)} \] (3.13)

where
\[k = \frac{\cosh 3Mh \sinh Mh}{32M^3 \cosh Mh} \frac{h}{8M^2} \cosh Mh - \frac{1}{96M^3} \sinh 3Mh + \frac{h}{8M^2} \cosh Mh - \frac{\sinh Mh}{8M^3}.\]

Substituting Equations (3.8) and (3.11) into the Equation (3.1) and using the relation \(\frac{dp}{dx} = \frac{dp}{dx} - We^2 \frac{dp}{dx}\) and neglecting terms greater than \(O(We^2)\), we get
\[u = \frac{1}{M^2} \left( \frac{dp}{dx} - \eta \sin \alpha \right) \left[ \frac{\cosh My}{\cosh Mh} - 1 \right] - 1 + We^2 \frac{3}{2} \left( \frac{n-1}{M^3 \cosh^3 Mh} \right) \left( \frac{dp}{dx} - \eta \sin \alpha \right)^3 \] (3.14)

Similarly,
\[\frac{dp}{dx} = \frac{M^3(q + h) \cosh Mh}{(\sinh Mh - Mh \cosh Mh)} - \frac{3(n-1)We^2 M^3 \cosh Mh}{2 \sinh Mh} (q + h)^3 B_2 + \eta \sin \alpha. \] (3.15)

The dimensionless pressure rise and frictional force per one wavelength in the wave frame are defined, respectively as
\[\Delta p = \int h \left( \frac{dp}{dx} \right) dx \] (3.16)
and
\[F = \int h \left( \frac{dp}{dx} \right) dx \] (3.17)

4. RESULTS AND DISCUSSION

In order to get a the physical insight of the problem, pressure rise and frictional force per one wavelength are computed numerically for different values of the emerging parameters, viz., Weissenberg number \(We\), power-law index \(n\), amplitude ratio \(\phi\), inclination angle \(\alpha\) and gravity parameter \(\eta\) and are presented in figures 2-13.
The variation of pressure rise \( \Delta p \) with time averaged flux \( Q \) for different values of Weissenberg number \( We \) with \( \phi = 0.6, \, M = 1, \, \eta = 1, \, \alpha = \pi / 6 \) and \( n = 0.398 \) is shown in Fig. 2. It is observed that, both the pumping (\( \Delta p > 0 \)) and free pumping decrease with an increase in \( We \), whereas the co-pumping (\( \Delta p < 0 \)) increases with an increase in \( We \).

Fig.3 shows the variation of \( \Delta p \) with \( \bar{Q} \) for different values of \( n \) with \( \phi = 0.6, \, M = 1, \, \eta = 1, \, \alpha = \pi / 6 \) and \( We = 0.2 \). It is observed that, the \( \bar{Q} \) increases with an increase in \( n \) in the pumping region, whereas in the co-pumping region the \( \bar{Q} \) decreases with an increase in \( n \). Further, the pumping is more for Newtonian fluid (\( n = 1 \)) than that of a Carreau fluid (\( 0 < n < 1 \)).

The variation of \( \Delta p \) as a function of \( \bar{Q} \) for different values of Hartmann number \( M \) with \( \phi = 0.6, \, \eta = 1, \, We = 0.2, \, \alpha = \pi / 6 \) and \( n = 0.398 \) is depicted in Fig. 4. It is found that, any of two pumping curves intersect at a point in the first quadrant and to the left of this point \( \bar{Q} \) increases and to the right of this point \( \bar{Q} \) decreases with an increase in \( M \).

Fig.5 shows the variation of \( \Delta p \) with \( \bar{Q} \) for different values of \( \phi \) with \( We = 0.2, \, \eta = 1, \, M = 1, \, \alpha = \pi / 6 \) and \( n = 0.398 \). It is found that, the \( \bar{Q} \) increases with an increase in \( \phi \) in both pumping and free pumping regions. But, in the co-pumping region, the \( \bar{Q} \) decreases with an increase in \( \phi \) for appropriately chosen \( \Delta p (<0) \).

Fig. 6 depicts the variation of pressure rise \( \Delta p \) with \( \bar{Q} \) for different values of \( \alpha \) with \( \phi = 0.6, \, We = 0.2, \, M = 1, \, \eta = 1 \) and \( n = 0.398 \). It is found that, as \( \alpha \) increases the \( \bar{Q} \) increases in pumping, free pumping and co-pumping regions.
The variation of pressure rise $\Delta p$ with $\bar{Q}$ for different values of $\eta$ with $\phi = 0.6$, $We = 0.2$, $M = 1$, $\alpha = \pi/6$ and $n = 0.398$ is shown in Fig.7. It is observed that, as $\eta$ increases, the $\bar{Q}$ increases in pumping, free pumping and co-pumping regions.

Figures 8-13 depict the effects of $We$, $n$, $M$, $\alpha$, $\eta$ and $\phi$ on the frictional force. From Fig.8, it is found that the friction force first increases and then decreases with an increase in $We$. From Fig.9, it is seen that the friction force initially decreases and then increases with an increase in $n$. From Fig.10, it is observed that the friction force increases with an increase in $M$. Fig.11 indicates that friction force first decreases and then increases with an increase in $\phi$. From Fig.12, it is observed that the friction force decreases with increasing $\alpha$. From Fig.13, it is found that the friction force decreases with an increase in $\eta$. In general, figures 2-13 show that the friction force has opposite character in comparison to the pressure rise.

5. CONCLUSIONS

The peristaltic flow of a MHD Carreau fluid in an inclined channel under lubrication approach is investigated. It is found that the pumping is more for Newtonian fluid ($n=1$) than that of Carreau fluid ($0 < n < 1$). The magnitudes of pressure rise and friction force increase with an increase in $M$ or $\phi$ or $\alpha$ or $\eta$, whereas, the magnitudes of pressure rise and friction force decrease with an increase in $We$. 
Fig. 2 The variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for different values of Weissenberg number $We$ with $\phi = 0.6, M = 1, \eta = 1, \alpha = \pi / 6$ and $n = 0.398$. 
Fig. 3 The variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for different values of $n$ with $\phi = 0.6, M = 1, \eta = 1, \alpha = \pi / 6$ and $We = 0.2$. 

$n = 1, 0.728, 0.496$
Fig. 4 The variation of time averaged flux $\overline{Q}$ with pressure rise $\Delta p$ for different values of Hartmann number $M$ with $\phi = 0.6, Wc = 0.2, \eta = 1, \alpha = \pi / 6$ and $n = 0.398$. 
Fig. 5 The variation of time averaged flux $\bar{Q}$ with pressure rise $\Delta p$ for different values of amplitude ratio $\phi$
with $We = 0.2, \eta = 1, M = 1, \alpha = \pi / 6$ and $n = 0.398$. 
Fig. 6 The variation of time averaged flux $\bar{Q}$ with pressure rise $\Delta p$ for different values of inclination angle $\alpha$ with $We = 0.2, \eta = 1, M = 1, \phi = 0.6$ and $n = 0.398$. 
Fig. 7 The variation of time averaged flux $\bar{Q}$ with pressure rise $\Delta \rho$ for different values of gravity parameter $\eta$ with $We = 0.2, \alpha = \pi / 6, M = 1, \phi = 0.6$ and $n = 0.398$. 

$\Delta \rho$

$\bar{Q}$
Fig. 8 The variation of friction force $F$ with time averaged flux $\overline{Q}$ for different values of Weissenberg number $We$ with $\phi = 0.6, M = 1, \eta = 1, \alpha = \pi/6$ and $n = 0.398$. 
Fig. 9 The variation of friction force $F$ with time averaged flux $\overline{Q}$ for different values of $n$ with $\phi = 0.6, M = 1, \eta = 1, \alpha = \pi / 6$ and $We = 0.2$. 

$n = 1, 0.728, 0.496$
Fig. 10 The variation of friction force $F$ with time averaged flux $\overline{Q}$ for different values of Hartmann number $M$ with $\phi = 0.6, We = 0.2, \eta = 1, \alpha = \pi / 6$ and $n = 0.398$. 

$M = 3, 2, 1$
Fig. 10 The variation of friction force $F$ with time averaged flux $\overline{Q}$ for different values of amplitude ratio $\phi$ with $We = 0.2, \eta = 1, M = 1, \alpha = \pi/6$ and $n = 0.398$. 
Fig. 12 The variation of friction force $F$ with time averaged flux $\overline{Q}$ for different values of inclination angle $\alpha$ with $We = 0.2, \eta = 1, M = 1, \phi = 0.6$ and $n = 0.398$. 

\[ \alpha = \frac{\pi}{2}, \frac{\pi}{4}, 0 \]
Fig. 13 The variation of friction force $F$ with time averaged flux $\overline{Q}$ for different values of gravity parameter $\eta$ with $We = 0.2, \alpha = \pi / 6, M = 1, \phi = 0.6$ and $n = 0.398$. 