CHAPTER 6

DESIGN OF CONTROLLER USING VARIABLE TRANSFORMATIONS

In this chapter, a Globally Linearized Controller (GLC) for a first order nonlinear system with dead time is proposed and designed. This is similar to the GLC proposed by Ogunnalke (1986) for nonlinear systems without dead time. Two methods are proposed. One is based on the Smith prediction from the model in the transformed domain and the other is based on the Newton’s extrapolation method. The simulation study is made on the conical tank level process and the results are compared with those obtained using a conventional PI Controller and the Smith PI controller.

6.1 DESIGN OF GLC

6.1.1 Variable transformation

A single-input, single-output nonlinear control system with dead time can be represented in general by

\[
\frac{dx}{dt} = F(x, u(t - T_d)) \tag{6.1}
\]

where \( F(.) \) is an arbitrary nonlinear function of \( x \), the system state variable and \( u \), the control variable and \( T_d \) is the effective dead time in the process.
The function $F(x,u(t-T_d))$ is split up as

$$F(x,u(t-T_d)) = c_1f_1(x) + c_2f_2(x,u(t-T_d)) \quad (6.2)$$

where $f_1(x)$ is a function of $x$ alone and $f_2(x, u(t-T_d))$ is a function of both $x$ and $u(t-T_d)$. Both $f_1$ and $f_2$ are taken to be nonlinear. $c_1$ and $c_2$ are constants.

$$z = g(x) \quad (6.3)$$

is a transformation for mapping the non-linear system $F(.)$ to a linear system

$$\frac{dz}{dt} = a + bv \quad (6.4)$$

where $a$ and $b$ are constants, $v$ is a state variable and $g(.)$ is a function to be determined.

Differentiating equation (6.3) with respect to $t$

$$\frac{dz}{dt} = \frac{dg}{dx} \frac{dx}{dt} \quad (6.5)$$

From equations (6.1) and (6.2)

$$\frac{dx}{dt} = c_1f_1(x) + c_2f_2(x,u(t-T_d)) \quad (6.6)$$

Substituting equation(6.7) in equation(6.5)

$$\frac{dz}{dt} = c_1f_1 \frac{dg}{dx} + c_2f_2 \frac{dg}{dx} \quad (6.7)$$
g(x) may be chosen such that

\[ c_1 f_1(x) \frac{dg}{dx} = a \]  \hspace{1cm} (6.8)

and \[ c_2 f_2(x, u(t - T_d)) \frac{dg}{dx} = bv \]  \hspace{1cm} (6.9)

The nonlinear system is thus mapped to

\[ \frac{dz}{dt} = a + bv \]  \hspace{1cm} (6.10)

a linear system may be effectively controlled with a PI controller as in figure 6.1

![Figure 6.1 Conceptual configuration of the GLC controller.](image-url)
From equation (6.8), the transformation $g(x)$ which transforms the given nonlinear system with dead time into a linear system is

$$g(x) = \frac{a}{c_1} \int_{f_1(x)}^x \frac{dx}{f_2(x)}$$

(6.11)

Similarly from equation (6.9)

$$u(t) = \frac{c_1 b v(t + T_d)}{c_2 a f_2(x(t + T_d))}$$

(6.12)

where $v(t + T_d)$ is the predicted manipulated variable and $f_2(x(t + T_d))$ is the predicted function of state variable.

Thus a nonlinear controller $u(t)$ is designed based on a variable transformation for the first order nonlinear process with dead time. The controller performance is tested by simulation of the conical tank level process. The proposed controller is expected to be highly robust when the operating point of the process is shifted over the entire span of the tank.

A PI controller is designed and interfaced with the pseudo linear system shown in figure 6.2. This is made possible by the use of the transformations $g(x)$ and $u(t)$. Thus, the entire design procedure boils down to the determination of $g(x)$ to compute the corresponding $u(t)$.

6.1.2 Prediction of Process Variable and Controller Output

A conventional PI controller gives an output $v(t)$ which effectively controls $z$. The future value of $f_2$ is predicted either by using Newton’s method or by using the transformed model of the system. Since $f_2$ is function of $v$, once $f_2$ is
predicted, \( v \) can be easily got. The nonlinear control law can be obtained using equation (6.12).

### 6.1.3 Prediction using Newton's Extrapolation Method.

Newton's Extrapolation Method (NEM), derived from Taylor's series, is used in statistics to predict the future values of a random variable like population, quantity of future sales etc (Chidambaram et al 2003b). In this method, the available past data are assumed as input as in figure 6.2.

![Figure 6.2 Block diagram representation of the non-linear controller with Newton's Predictor.](image-url)
The Newton's Extrapolation formula, used to predict the future value is

\[ f(a + xh) = f(a) + xVf(a) + \frac{x(x + 1)}{2!} V^2 f(a) + \frac{x(x + 1)(x + 2)}{3!} V^3 f(a) + \ldots + \ldots \]  

(6.13)

where \( f(a + xh) \) is the future value to be predicted after \( x \) numbers of sampling periods, \( h \) is the sampling interval and \( a \) is the sampling time at which the latest value is available. \( V \) is the difference operator. \( V f(a) \) is the difference between the latest sampled value and the previous sampled value. \( V^2 f(a) \) is \( V(V f(a)) \) and so on etc. The number of terms that have to be considered in the above formula depends on the number of past data available and the designer's interest. In this work, only the first two terms are considered.

### 6.1.4 Variable Transformation Predictor

The nonlinearity in the process and hence in the model limits the prediction. So, an attempt is made to predict the future variable using a linear model. The model which is used to map the nonlinear process by variable transformation is linear and this model is used here to predict the future variable as shown in figure 6.3. The configuration is a combination of Smith predictor and variable transformation and hence may be called variable transformation Predictor (VTP). The transfer function model of the transformed pseudo linear system (6.10) is

\[ \frac{z(s)}{v(s)} = \frac{b}{s} \]

The transfer function model \( \frac{b}{s} \) will give the predicted transformed process variable \( z' \).

The model prediction error \( e_m \) is the difference between the predicted output and the transformed process variable \( z \). \( e_m \) is added in the feedback path as an additional error.
6.2 APPLICATION OF GLC TO CONICAL TANK LEVEL PROCESS

The mathematical model of the conical tank liquid level system considered for the study is expressed as

\[
\frac{dh}{dt} = \frac{1}{\pi R^2 h^2} \left[ u - c \sqrt{h} \right]
\]  

(6.14)

where H, R, c are total height, top radius and out flow valve coefficient of the conical tank respectively. h and u are liquid level and inflow rate of the conical tank level process respectively. The transformation \( g(h) \) transforms the nonlinear system.

Figure 6.3 Block diagram representation of the variable transformation predictor.
represented by equation (6.14) into a linear system. The transformed system is a pseudolinear system, whose mathematical model is

\[ \frac{dz}{dt} = a + bv \]  

(6.15)

The transfer function of equation (6.15) is

\[ G(s) = \frac{b}{s} \]  

(6.16)

The forward path transfer function of the closed loop system with PI controller is

\[ G_c(s) = bK_t \frac{(1 + Ts)}{Ts^2} \]  

(6.17)

\( T_i \) and \( K_c \) can be obtained by assuming closed loop time constant and damping factor (Chidambaram 1998). Interestingly, the tuning parameters are independent of local time constant and local gain of the process.

### 6.3 RESULTS AND DISCUSSION

The responses are compared with a conventional PI controller tuned about nominal operating point of 39%. The time constant and gain of linearized model are 76 sec and 1.2 respectively. The process dead time is 32 sec. The values of a and b are as summed as - 0.7657 and 0.0177 respectively. Mathematically there is no restriction on the selection of values of a and b but practically it is found that selection of large values of a and b end up with overflow error during computation. It is important to select suitable values so that there is no overflow error in the transformed variable.
Regulatory responses for a 15% decrease in load at nominal operating point 39% as in figure 6.4 show that the proposed controller gives an improved response (28% lesser ISE) while the conventional Smith predictor (150% higher ISE) gives a poor performance than the ZN-PI controller.

Regulatory responses for a 15% increase in load at nominal operating point 39% as in figure 6.5 show that the proposed controller improves the response (35% lesser ISE) while the conventional Smith predictor (172% higher ISE) gives a poor performance than the ZN-PI controller.

Regulatory responses for a 15% decrease in load at 24% operating point but tuned at 39% as in figure 6.6 show that the proposed controller provides an improved response (36% lesser ISE) than the conventional Smith predictor (19% higher ISE). The ZN-PI controller gives an oscillatory response.

Regulatory responses for a 15% increase in load at 24% operating point but tuned at 39% as in figure 6.7 show that the proposed controller improves the response (26% lesser ISE) than the conventional Smith predictor. The ZN-PI controller gives an oscillatory response.

Regulatory responses for a 15% decrease in load at 54% operating point but tuned at 39% as in figure 6.8 show that the proposed controller gives an improved performance (58% lesser ISE) while the conventional Smith predictor (205% higher ISE) provides a poor performance than the ZN-PI controller.

Regulatory responses for a 15% increase in load at 54% operating point but tuned at 39% as in figure 6.9 show that the proposed controller improves the
performance (63% lesser ISE) while the conventional Smith predictor (209% higher ISE) gives a poor response than the ZN-PI controller.

Servo responses for a 20% decrease in the set point at nominal operating point 39% as in figure 6.10 show that the proposed controller gives an improved response (31% lesser ISE) than the conventional Smith predictor. The ZN-PI controller provides an oscillatory response.

Servo responses for a 20% increase in the set point at nominal operating point 39% as in figure 6.11 show that the proposed controller improves the response (32% lesser ISE) while the conventional Smith predictor (50% higher ISE) gives a poor performance than the ZN-PI controller.

Servo responses for a 14% decrease in the set point at 24% operating point but tuned at 39% as in figure 6.12 show that the proposed controller improves the response (33% lesser ISE) than the conventional Smith predictor. The ZN-PI controller gives an oscillatory response. At this operating point it is not possible to decrease the set point greater than 14% because of numerical round off error. Some values in the transformed domain are lesser than the tolerance that can be handled by the computer.

Servo responses for a 20% increase in set point at 24% operating point but tuned at 39% as in figure 6.13 show that the proposed controller improves the response (18% lesser ISE) while the conventional Smith predictor (43% higher ISE) gives a poor performance than the ZN-PI controller.

Servo responses for a 20% decrease in set point at 54% operating point but tuned at 39% as in figure 6.14 show that the proposed controller improves the response (36% lesser ISE) while the conventional Smith predictor (25% higher ISE)
gives a poor response than the ZN-PI controller. Servo responses for a 20% increase in set point at 54% operating point but tuned at 39% as in figure 6.15 show that the proposed controller gives a poor response (21% higher ISE) than the ZN-PI controller and the conventional Smith predictor (49% higher ISE) provides a very poor response.

The servo responses of the conical tank level process for a 20% decrease in set point at the nominal operating point of 39% using GLC controllers—one with NEM prediction and the other with VTP prediction but both with same tuning parameters are shown in figure 6.16. The response given by NEM has slight oscillations. This indicates that the prediction by NEM is very poor and hence all the simulations are carried out with VTP.

Consider the regulatory responses at 24% operating point for decrease in load as shown in figure 6.6. The ZN-PI controller gives highly oscillatory response. On the other hand, Smith PI and GLC give oscillation free response. Similarly consider the regulatory responses at 24% operating point for increase in load as shown in figure 6.7. Here also the ZN-PI controller gives highly oscillatory response but Smith PI gives oscillation free response. GLC gives oscillatory response but quickly settles than that of ZN-PI controller. In this case the draw back with the GLC is large under shoot. But it has lesser ISE than that of Smith PI. Consider the regulatory responses at 54% operating point (Figures 6.8 and 6.9). The Smith PI controller gives large under shoot and large over shoot than that of ZN-PI controller. But GLC gives very small over shoot and under shoot than that of ZN-PI controller. Consider the servo responses at 39% and 24% (Figures 6.10 and 6.12). The ZN-PI controller give oscillatory responses and Smith PI give sluggish responses. GLC gives oscillation free responses with quick rise time. These points show the robustness of the GLC.
Figure 6.4 Regulatory responses of conical tank level process for 15% decrease in load at nominal operating point 39% using ZN-PI, Smith PI controllers and GLC(VTP).
Figure 6.5 Regulatory responses of conical tank level process for 15% increase in load at nominal operating point 39% using ZN-PI, Smith PI controllers and GLC (VTP).
Figure 6.6 Regulatory responses of conical tank level process for 15% decrease in load at 24% operating point using ZN-PI, Smith PI controllers and GLC(VTP).
Figure 6.7 Regulatory responses of conical tank level process for 15% increase in load at 24% operating point using ZN-PI, Smith PI controllers and GLC(VTP).
Figure 6.8 Regulatory responses of conical tank level process for 15% decrease in load at 54% operating point using ZN-PI, Smith PI controllers and GLC(VTP).
Figure 6.9 Regulatory responses of conical tank level process for 15% increase in load change at 54% operating point using ZN-PI, Smith PI controllers and GLC(VTP).
Figure 6.10 Servo responses of conical tank level process for -20% step change at nominal operating point 39% using ZN-PI, Smith PI controllers and GLC (VTP)
Figure 6.11 Servo responses of conical tank level process for +20% step change at nominal operating point 39% using ZN-PI, Smith PI controllers and GLC (VTP).
Figure 6.12 Servo responses of conical tank level process for -14% step change at 24% operating point using ZN-PI, Smith PI controllers and GLC (VTP).
Figure 6.13 Servo responses of conical tank level process for +20% step change at 24% operating point using ZN-PI, Smith PI controllers and GLC(VTP)
Figure 6.14 Servo responses of conical tank level process for -20% step change at 54% operating point using ZN-PI, Smith PI controllers and GLC(VTP)
Figure 6.15 Servo responses of conical tank level process for +20% step change at 54% operating point using ZN-PI, Smith PI controllers and GLC(VTP)
Figure 6.16 Servo responses of conical tank level process for -20% step change at nominal operating point 39% GLC(VTP) and GLC(NEM) with same PI settings.
6.4 CONCLUSION

A nonlinear controller is designed based on the variable transformation for the first order nonlinear process with dead time. The performances with VTP are tested by simulation. The proposed controller with VTP is robust when the operating point of the process is shifted over the entire span of the tank. The proposed controller outperforms all dead time compensating controllers and the ZN-PI controller. The proposed GLC (NEM) gives poor dynamics and threatens the stability when compared with GLC (VTP).

The nonlinear controller proposed in this chapter is model based. The performance of the controller fully depends on the first principle model. It is very difficult in many processes to obtain the first principle model. Therefore, an attempt is made in the next chapter to design a nonlinear controller using fuzzy logic.