CHAPTER 1

INTRODUCTION

1.1 NONLINEAR FILTERS – AN OVERVIEW

Linear filters were the major tools that played an important role in the early development of signal and image processing. They are particularly effective when the spectrum of the desired signal is distinctly different from that of the noise components. Sound theoretical basis and mathematical simplicity make them attractive for signal and image filtering applications (Proakis and Manolakis 1997, Lyons 1997, Mitra 1998, Pratt 1991, Jain 1989 and Gonzalez and Woods 1993). Mathematically, a filter can be defined as an operator \( L(\cdot) \), which maps an input signal \( x \) into an output signal \( y \):

\[
y = L(x)
\]  

The Equation (1.1) describes the linear system, if the operator \( L(\cdot) \) satisfies i) the principle of superposition theorem and ii) the principle of proportionality. Despite the well developed theory of linear systems, not all signal processing problems are satisfactorily solved by the use of linear techniques. Quite often, it is expected of a filter to perform on signals having a broad spectrum, e.g., image signals. Unfortunately, the linear filters tend to lower the high frequency content of edges and blur them (Pratt 1991 and Pitas and Venetsanopoulos 1990). Moreover, they perform poorly in the presence of non-additive and/or non-Gaussian noise (Pratt 1991 and Pitas and Venetsanopoulos 1990). For these
reasons, researchers began to explore nonlinear techniques for signal and image filtering in the middle of the last century.

The filter defined by Equation (1.1) becomes nonlinear, when the operator \( L(\cdot) \) fails to obey the principle of superposition theorem or the principle of proportionality or both. It is well known that the output of a nonlinear filter cannot be obtained by convolving the input with its impulse response functions. Moreover, the lack of sound underlying theory rendered nonlinear methods ad hoc and application specific. For these reasons, different families of nonlinear filters have been studied in the literature for meeting the specific requirements of various types of applications.

Many digital signal processing problems entail the processing of multiplicative signals. For example,

\[
x(i) = x_1(i) x_2(i)
\]

is a multiplicative signal. Such signals are encountered in image processing applications. Linear filters cannot be used to process the signals described by Equation (1.2), as they do not obey the principle of superposition theorem. The class of filters that has been developed for processing the multiplicative and convolved signals is referred to as homomorphic filters (Oppenheim et al 1968 and Rabiner and Schafer 1978). The principle of operation involved in homomorphic filtering is that the filter transforms the multiplicative signals to additive signals and then processes them using linear filters. The output of the linear filter is transformed to multiplicative signal using exponential nonlinearity. The homomorphic filter exhibits good noise filtering characteristics in the presence of multiplicative and signal-dependent noise. Its
applications include seismic signal processing, speech processing, echo removal and image processing.

Mathematical morphology has been recognized as an effective tool for quantitatively describing image geometric structures. It segments the images into objects and analyzes their properties. The filters which stem from mathematical morphology are called morphological filters (Pitas and Venetsanopoulos 1990 and Maragos and Schafer (1987 and 1987 a)). They differ from the other families of nonlinear filters in the sense that they use the geometric features of the signal to perform their function. Morphological operations are simple and are easy to implement using parallel or pipelined architectures. Morphological filters find their use in image analysis (Haralick et al 1987) and image processing (Maragos and Schafer 1987, Sternberg 1983 and Sternberg 1986).

Filters based on Volterra and Wiener series constitute a family of nonlinear filters, namely, polynomial filters (Schetzen 1980). These filters are preferred for suppressing noise having non-Gaussian statistics and in such applications where linear filters tend to exhibit sub-optimal performance. Polynomial filters are characterized by a set of functions called Volterra kernels. Due to the algebraic complexity involved in computing the high-order Volterra kernels, the order of Volterra filters is limited to two (Koh and Powers 1985, Chiang et al 1986, Mertzios et al 1989 and Sicuranza 1992). Wiener suggested an alternative solution for modeling high-order nonlinear systems (Schetzen 1980). He used the Gram-Schmidt orthogonalization procedure to derive an orthogonal set of functionals from the Volterra functionals. This method made the computation of Wiener kernels easy and the filters based on these kernels are known as Wiener filters. Polynomial filters are used in image
processing (Ramponi and Sicuranza 1988) and in compensating for nonlinearity that affects adaptive echo cancellers (Agazzi et al 1982). Adaptive polynomial filters have been proposed by Mathews (1991).

The most popular family of nonlinear filters is the Order-Statistic (OS) filters (Longbotham and Bovik 1989 and Pitas and Venetsanopoulos 1986). Median filters (Gallagher, Jr. 1988 and Fitch et al 1984), stack filters (Wendt et al 1986 a), median hybrid filters (Heinonen and Neuvo 1987), trimmed mean filters (Bednar and Watt 1984 and Lee and Kassam 1985) and L-filters (Bovik et al 1983) are the important members of the OS filter class. OS filters behave well in an additive noise environment. They have good edge preservation properties and can become adaptive. In addition, the OS filters are suitable for VLSI implementation (Ataman et al 1980, Huang et al 1979, Ahmad and Sundararajan 1987 and Oflazer 1983) and therefore, are proven to be quite useful for real-time applications. The objective of this thesis is to develop some nonlinear filtering strategies based on order-statistics. Therefore, a fairly detailed treatment of OS filters is undertaken in the next section.

1.2 MEDIAN FILTERS

The median filter is the most extensively used OS filter. John Tukey suggested it in 1971 as a time series tool for robust noise suppression (Tukey 1971). It is also referred to as rank-order (RO) filter.

1.2.1 Definition of Median Filter

Let \( \{x(i)\} \) and \( \{y(i)\} \) denote the input and output sequences respectively. A one-dimensional median filter slides a \( 2N+1 \) point wide window.
over \(x(i)\). At each point the samples inside the window are sorted out and the median or middle value is used as the filter output. The median output is associated with the time sample at the center of the window. The filtering procedure can be expressed as:

\[
y(i) = \text{median}(x(i-N), x(i-N+1), \ldots, x(i), \ldots, x(i+N)), \ i \in \mathbb{Z}
\]  

(1.3)

The Equation (1.3) is also called moving median or running median. For the window to reach the front and rear ends of the input signal sequence, \(N\) number of samples are appended both at the beginning and at the end. The front endpoints take the value of the first sample while the rear endpoints take the value of the last sample. An example of one-dimensional median filtering with window length 5 is illustrated in Figure 1.1.

The two-dimensional median filtering process has the following definition:

\[
y(i, j) = \text{median}(x(i+r, j+s)), \ (r, s) \in A \text{ and } (i, j) \in \mathbb{Z}^2
\]  

(1.4)

The set \(A \subseteq \mathbb{Z}^2\) defines the neighbourhood of the central pixel \((i, j)\). It is called the filter window. The commonly used window structures in two-dimensional median filtering are shown in Figure 1.2. The border samples of two-dimensional signals are processed by replicating them as done in one-dimensional median filtering. The median filters have been used with success in speech (Jayant 1976 and Rabiner et al 1975) and image (Pratt 1991 and Pitas and Venetsanopoulos 1990 and Perlman et al 1987) processing applications. In addition, fast algorithms and hardware implementation for median filtering
Figure 1.1 Illustration of one-dimensional median filtering with window size 5
Figure 1.2 Window structures used for two-dimensional median filtering
have been developed (Ataman et al 1980, Huang et al 1979, Ahmad and Sundararajan 1987 and Oflazer 1983).

1.2.2 Properties of Median Filters

Conventional tools such as frequency response and impulse response cannot be used for analyzing median filters as they do not come within the scope of linear system theory. As a result, new tools had to be developed to analyze and characterize the behavior of these nonlinear filters deterministically and statistically (Gallagher, Jr. and Wise 1981, Nodes and Gallagher, Jr. 1984, Ataman et al 1981, Kuhlmann and Wise 1981, Bovik 1987a, Fitch et al 1985, Wendt et al 1986, Arce and Gallagher 1982, Astola et al 1987, Zeng 1994, Eberly et al 1991 and Mao and Gan 1993). The deterministic properties describe the effect of filtering on the structure of the signal. On the other hand, the statistical analysis shows how effective the filter is in removing the different types of noise.

1.2.2.1 Deterministic Properties of Median Filters

The deterministic properties of median filters are described by their root signal set. The root signal set is defined as a set of signals, which remain invariant to further filtering. The concept of root signal with reference to the median filter is explained as follows. Under steady state conditions, when a sinusoidal signal is passed through a linear system, the frequency of the sinusoid is not changed; only its phase and amplitude are altered. This fact is not valid for median filters, because they are basically nonlinear systems. However, Gallagher and Wise (1981) proved that if any signal of finite-length is repeatedly median filtered using the same window, then the resultant signal
becomes invariant to further filtering at one point. Such a signal is called the root signal. That is, for a median filter of length \( k = 2N+1 \), this means that:

\[
x(i) = \text{median}(x(i-N), x(i-N+1), \ldots, x(i), \ldots, x(i+N))
\]  

(1.5)

If the above condition is satisfied for all \( i \), then \( \{x(i)\} \) is called the root signal of that particular median filter. The meaning of root signals for median filtering is analogous to the meaning of sinusoids in the passband of linear filters. If the original signal is of length \( L \) points (without counting the appended points at the beginning and the end), then

\[
3 \left( \frac{L-2}{2(N+2)} \right)
\]

filter passes are the maximum required to reach a root (Wendt et al 1986). The fact that the root signals are invariant to further filtering offers interesting possibilities. For example, in image filtering, a common approach is to design a median filter such that certain prescribed features, such as lines are root signals and thus not disturbed by the filtering operation. Root signals of median filters have been used for speech (Arce and Gallagher 1982) and image (Arce and Gallagher, Jr. 1983) coding.

### 1.2.2.2 Statistical Properties of Median Filters

Median is the best location estimator in the \( L_1 \) sense, because it minimizes:

\[
\sum_{i=1}^{n} |x(i) - T_n| = \min
\]  

(1.6)
The median filter is robust in the presence of long tailed noise. The effect of an outlier (impulse) on the performance of an estimator can be studied by a function called Influence Function (IF) (Pitas and Venetsanopoulos 1990). The IF of the estimator $T$, at the distribution $F$ for those $x \in X$, is denoted as $IF(x; T, F)$, where $X$ is the sample space. The IF of the mean and median estimators at the Gaussian distribution of zero mean and unit variance is shown in Figure 1.3. It can be seen from Figure 1.3 that the influence of an outlier on the mean estimator keeps increasing with the magnitude of $x$, while it gets bounded at $|1/(2\phi(0))|$ on the median estimator, irrespective of the magnitude of $x$.

The most important measure of robustness based on the IF is the gross error sensitivity $v^*$ of $T$ at the distribution $F$ (Pitas and Venetsanopoulos 1990):

$$v^* = \sup_x |IF(x; T, F)|$$ (1.7)

where $\sup$ denotes supremum. The gross error sensitivity measures the worst effect of contamination at any point $x \in X$. If $v^*$ is finite, $T$ is called the robust estimator. From the Equation (1.7) and Figure 1.3, it is evident that $v^*$ is unbounded for the mean estimator for unbounded values of $x$. Therefore, the mean is not a robust estimator; even one distant outlier can cause catastrophic effects on the arithmetic mean. The gross error sensitivity of the median estimator for zero mean Gaussian distribution of unit variance is:

$$v^* = \sup_x |IF(x; T, \Phi)|$$
Figure 1.3 Influence function of the mean and median estimators for zero mean Gaussian distribution of unit variance
The finite value of $v^*$ indicates that the median is a robust estimator.

To evaluate the median as an estimator of location, a measure called Asymptotic Relative Efficiency $ARE(T, S)$ of two estimators $T(F)$ and $S(F)$ is used and it is defined as:

$$
ARE(T, S) = \frac{V(S, F)}{V(T, F)}
$$

(1.9)

where $V(S, F)$ and $V(T, F)$, respectively, are the asymptotic variance of estimators $S$ and $T$ at the distribution $F$. To obtain an idea of the performance of the median, it is compared with the arithmetic mean $\bar{x}$ for different distributions. The asymptotic relative efficiency of the median estimator with respect to the mean at the distribution $F$ is:

$$
ARE(\text{median (} x(i), \bar{x} \text{)}) = \frac{V(\bar{x}, F)}{V(\text{median (} x(i)), F)}
$$

(1.10)

When $ARE(\text{median (} x(i), \bar{x} \text{)})$ is greater than one, the median performs better, that is, it exhibits lower output variance than the arithmetic mean. $ARE(\text{median (} x(i), \bar{x} \text{)})$ values evaluated at different distributions are summarized in Table 1.1.
Table 1.1 Asymptotic Relative Efficiency of Median Estimator with respect to Mean Estimator

<table>
<thead>
<tr>
<th>Noise Probability Density Function</th>
<th>ARE (median (x (i)), ( \bar{x} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1/3</td>
</tr>
<tr>
<td>Gaussian</td>
<td>( 2/\pi )</td>
</tr>
<tr>
<td>Laplacian</td>
<td>2</td>
</tr>
</tbody>
</table>

The median performs at its worst for the short tailed uniform distribution and performs at its best for the long tailed laplacian distribution.

1.2.2.3 Edge Preservation Properties

Edges are fundamental features of images. They often contain valuable information and are important for human visual perception. In addition, edge information is used for image analysis and object classification. Median filters have good edge preservation properties (Ataman et al 1981). In fact, the median filter adapts to the signal characteristics in the sense that it behaves like a lowpass filter in the homogeneous regions for suppressing noise components, while it exhibits highpass behaviour close to the edges for preserving them.

Images may contain horizontal, vertical and/or diagonal edges. The effectiveness of the median filter in preserving these edges depends on the geometry of the filter window (Bovik et al 1987). Cross-shaped windows are good at preserving horizontal and vertical edges, while the X-shaped filters are
preferred for preserving the diagonal edges. Square windows do not have preference for the edge direction and are found to preserve the edges of all orientations fairly well. Square-shaped filters are commonly used, since most of the images contain edges along all directions.

1.2.3 Vector Median Filters

Multispectral satellite images and standard colour images on TV are referred to as vector-valued signals, as they consist of several components. The components of vector-valued signals in real applications are generally correlated and if each component is separately median filtered, this correlation is not utilized; this may cause edge jitter and/or distortion. To circumvent this problem, the vector median filters (Astola et al 1990) are used. The vector median operator processes the signals as vectors, utilizing the inherent correlation between the components and thus avoids jitter and distortion. The vector median filters are robust against impulse noise and have good edge preserving characteristics. Oistamo and Neuvo (1990) used the vector median filters with success for processing video signals.

1.2.4 Adaptive and Optimal Median Filters

Preservation of signal features and elimination of noise are two contradictory aspects in signal/image processing. For example, median filters produce blurring for large windows or insufficient noise suppression for small windows. Adaptive and optimal median filtering techniques offer a sound approach to achieve good noise filtering and feature preservation properties.
The signal adaptive median (SAM) filter, reported by Bernstein (1987), adjusts its window length automatically depending on the local signal-to-noise ratio and on the nature of the proposed part of the signal, that is, whether it is an edge or a flat region. The filter allows the simultaneous removal of a combination of signal-dependent and additive random noise in addition to mixed impulse noise in images.

Lin and Willson (1988) have proposed a class of easily hardware-realizable median filters (recursive and non-recursive) with adaptive window length. The window length gets adapted on the basis of impulse noise detection. The filters achieve a high degree of impulse noise suppression and still preserve the sharpness of the image.

An optimal median filtering method was introduced by Zeng (1995) to maximize noise attenuation under structural constraints imposed by the requirement of preserving certain signal or image features. The filter design is based on modifying minimally the Boolean function of a median filter such that only when a prescribed feature occurs in the window mask, the filter operates differently from the median filter; otherwise, it works as a median filter to provide the best noise attenuation. The noise attenuation ability of the filter is seen to improve as the noise distribution is more heavily tailed.

1.3 VARIATIONS OF MEDIAN FILTERS

The success of standard median filters led to a tremendous amount of activity in the area of filter design for median based filters. Several modifications of standard median filters have been reported in the literature.
1.3.1 Separable Median Filters

Separable median filter, suggested by Narendra (1981), is a useful tool for image filtering. The filter consists of a two-pass operation in which each row of a two-dimensional signal is first filtered by a horizontally oriented one-dimensional median filter; then each column of the resulting signal is filtered by a vertically oriented median filter. The output \( y(i, j) \) of a separable median filter is:

\[
y(i, j) = \text{median} \left( z(i-N, j), z(i-N+1, j), \ldots, z(i+N, j) \right)
\]

(1.11)

where

\[
z(i, j) = \text{median} \left( x(i, j-N), x(i, j-N+1), \ldots, x(i, j+N) \right)
\]

(1.12)

The separable median filter is faster and better in preserving signal structures than the standard median filter. However, it is not as effective as the standard median filter in noise removal. Nodes and Gallagher (1983) have investigated the deterministic properties of separable median filters.

1.3.2 Recursive Median Filters

Recursive Median Filter (Nodes and Gallagher 1982) is an intuitive modification of the standard median filter. Its output is the median of the last \( N \) output values and the last \( N+1 \) input values. The recursive median is defined as:

\[
y(i) = \text{median} \left( y(i-N), y(i-N+1), \ldots, y(i-1), x(i), \ldots, x(i+N) \right)
\]

(1.13)

An interesting characteristic of the recursive median is that any one-dimensional signal is reduced to the root in one pass (Nodes and Gallagher
This property, however, does not always hold good for two-dimensional recursive median filters (Boles et al. 1988). The statistical properties of recursive median filters have been analyzed (Arce 1986 and Arce and Gallagher, Jr. 1988). The recursive median filter usually provides better smoothing capability than the non-recursive filter but produces increased distortion. Recursive separable median filter is a variation of the recursive two-dimensional median filter. The filter, as expected, has stronger noise attenuation capability than its non-recursive counterpart; McLoughlin and Arce (1987) have described its deterministic properties.

1.3.3 Weighted Median Filters

Weighted Median (WM) filter is an extension of the median filter, which gives more weight to some samples within the window. It resembles linear finite impulse response (FIR) structures in certain respects and inherits the robustness and edge preserving capability of the median filter. Having a set of weights, however, the WM filter is much more flexible in preserving desired signal structures. This is explained as follows. A median filter of window length $2N+1$ can only preserve details lasting more than $N+1$ points. To preserve smaller details in signals, median filters with smaller windows must be used. Unfortunately, the smaller the filter window is, the poorer its noise reduction capability becomes (Gallagher, Jr. 1988). WM filters present a good solution to such a dilemma. That is, one can use a WM filter with a window long enough, say $2N+1$, to suppress noise effectively and at the same time preserve details lasting less than $N+1$ points. This property of WM filters has been the major reason for their successful applications in speech and image processing (Loupas et al. 1989, Jarske and Neuvo 1991, Salo et al. 1988, Brownrigg 1984, Ko and Lee 1991 and Yin et al. 1996).
Weighted median is the estimator that minimizes the weighted $L_1$ norm of the form:

$$\sum_{i=1}^{n} w_i |x(i)-T_n| = \min$$

(1.14)

where

$$T_n = \text{median} (w_1 \times x_1, ..., w_n \times x_n)$$

(1.15)

and $w \times x$ denotes $w$ times duplication of $x$:

$$w \times x = x, ..., x \text{ w times}$$

(1.16)

The WM filter was first introduced by Justusson (1981) and further studied by Brownrigg (1984 and 1986). The WM filtering process can be expressed as:

$$y(i) = \text{median} (w_N \times x(i-N), ..., w_N \times x(i+N))$$

(1.17)

Nieweglowski et al (1993) investigated WM filters with real positive weights. Like median filters, WM filters are usually characterized by their deterministic and statistical properties. The statistical properties of WM filters have been analyzed by Harja et al (1991) and Astola and Neuvo (1994). The root signal properties of several subclasses of WM filters have been studied and reported in the literature (Zeng 1994, Wendt 1990, Zeng 1991 and Sun et al 1994).

WM filters, admitting only positive weights, are, in essence, filters having lowpass characteristics. A large number of applications, such as equalization, deconvolution, prediction, beam forming and system identification require bandpass or highpass characteristics. To extend the use of WM filters to these applications, Arce (1998) proposed a general WM filter structure.
admitting negative weights. The WM filtering operation with real-value weights (both positive and negative) is defined as:

\[ y(i) = \text{median}(\{w_N \text{ sign}(w_N) x(i-N), \ldots, w_N \text{ sign}(w_N) x(i+N)\}) \] (1.18)

where \( \{w_i\}_{i=-N}^{N} \in \mathbb{R} \)

Design of frequency selective WM filters and an optimal highpass WM filter using real-value weights have been discussed by Arce (1998).

WM filters can also operate recursively (Han et al 1991). A generalized Recursive Weighted Median (RWM) filtering that can admit real-value weights (both positive and negative) has been recently introduced by Arce and Paredes (2000) and is defined as:

\[ y(i) = \text{median}(\{a_N \text{ sign}(a_N) y(i-N), \ldots, a_1 \text{ sign}(a_1) y(i-1), b_0 \text{ sign}(b_0) x(i), \ldots, b_N \text{ sign}(b_N) x(i+N)\}) \] (1.19)

where \( \{a_k\}_{k=-N}^{-1} \in \mathbb{R} \) are the feedback coefficients and \( \{b_i\}_{i=0}^{N} \in \mathbb{R} \) are the feed forward coefficients. RWM filters, unlike linear Infinite Impulse Response (IIR) filters, are always stable under the bounded-input bounded-output criterion, regardless of the values taken by feedback coefficients \( \{a_k\}_{k=-N}^{-1} \). The bandpass and highpass RWM filters exhibit perfect stopband characteristics, not attainable with linear IIR filters. RWM filters are more robust than linear IIR filters and exhibit better noise filtering characteristics than their non-recursive counterparts. A disadvantage is that RWM filters produce some blurring artifacts on the output. These artifacts can be minimized by optimizing weights using the method suggested by Arce and Paredes (2000).
Center Weighted Median (CWM) filter is a special case of WM filter (Ko and Lee 1991). The filter gives more weight only to the central value of the window and thus it is easier to design and implement than the general WM filter. CWM filter is best suited for suppressing additive white noise and/or impulse noise without losing image details.

Yang et al (1995) have introduced an optimality theory for WM filters based on the new expressions for the output moments of weighted medians. The optimal WM filter reduces noise considerably besides preserving the desired signal structures. Its applications to 1-D signal processing and image processing have also been discussed by Yang et al (1995).

Matched median filter is a generalized WM filter useful for detection and estimation in communication systems. Astola and Neuvo (1992) have shown that when the matched median filter is used for detection, it outperforms the linear matched filter in the case of binary antipodal signaling under impulse noise. They have also described the statistical properties of matched median filters.

1.3.4 Max/Median, Min/Median and Multistage Median Filters

Median filters, when extended from the one-dimensional case to higher dimensions, have not always yielded satisfactory results. This is because, the ordering process destroys structural and spatial neighbourhood information. Several efforts have been made to modify the median filter to take into account structural information. Such a modification is the max/median filter (Arce and McLoughlin 1987).
\[ y(i, j) = \max(z_1, z_2, z_3, z_4) \]

where,

\[ z_1 = \text{median}(x(i, j-N), \ldots, x(i, j), \ldots, x(i, j+N)) \]
\[ z_2 = \text{median}(x(i-N, j), \ldots, x(i, j), \ldots, x(i+N, j)) \]
\[ z_3 = \text{median}(x(i+N, j-N), \ldots, x(i, j), \ldots, x(i-N, j+N)) \]
\[ z_4 = \text{median}(x(i-N, j-N), \ldots, x(i, j), \ldots, x(i+N, j+N)) \]

If the max operator in Equation (1.20) is replaced by min operator, min/median filter is obtained. The max/median and the min/median filters are biased estimators of location. The max/median (min/median) filter preserves positive (negative) image features well and attenuates noise adequately. In applications, where both positive and negative features exist, the max/median or min/median filter, alone, is not adequate.

To improve the performance of max/median (min/median), the median is used in the place of max (min) operator. Such a filter is referred to as multistage median filter (Arce and Foster 1989) and described by the relation:

\[ y(i, j) = \text{median}(\text{median}(z_1, z_2, x(i, j)), \text{median}(z_3, z_4, x(i, j)), x(i, j)) \]

(1.25)

Multistage median filters are good detail preserving smoothers in an impulse noise environment.
1.3.5 FIR-Median Hybrid (FMH) Filters

Median type filters are rank-order filters; therefore, they are good at preserving edge information and discarding impulses. On the other hand, linear filters do not consider rank-ordering of the observations in determining the output, leading to poorer performance at the signal edges and in the presence of non-Gaussian noise. To exploit the desirable properties of both the filter classes, median hybrid filters have been proposed in the literature. The combination of the median filter with the linear finite impulse response (FIR) filter produces the most widely used FIR-Median Hybrid (FMH) filters (Heinonen and Neuvo 1987). The one-dimensional FMH filtering operation with averaging substructures of window length $2N+1$ is defined as:

$$y(i) = \text{median} \left( \frac{1}{N} \sum_{k=1}^{N} x(i-k), x(i), \frac{1}{N} \sum_{k=1}^{N} x(i+k) \right)$$

(1.26)

In FMH filters, FIR filters are called substructures and the median of the output of these substructures is set as the output. FMH filter significantly reduces the computational complexity compared to median filter. Heinonen and Neuvo (1987) and Astola et al (1987) have analyzed the deterministic and statistical properties of FMH filters. Nieminen et al (1987) have introduced a variety of two-dimensional FMH filters and have demonstrated their usefulness in enhancing the images.
1.4 TRIMMED MEAN FILTERS

Bednar and Watt (1984) interpreted the $\alpha$-trimmed mean as a good compromise between the mean and median filters. The $\alpha$-trimmed mean is computed by sorting the data points inside the window and averaging the central part of the ordered array after trimming (removing) a few data points from both the ends. The filtering procedure can be expressed as:

$$\overline{x}_\alpha = \frac{1}{n - \lceil\alpha n\rceil} \sum_{j=\lceil\alpha n\rceil+1}^{n-\lceil\alpha n\rceil} x(j)$$

(1.27)

where $x(j)$, $j = 1, 2, \ldots, n$ are the ordered statistics of $x(i-N), \ldots, x(i), \ldots, x(i+N)$, $\alpha$ is the trimming parameter, $\lceil\cdot\rceil$ is the greatest integer function and $\overline{x}_\alpha$ is the output $y(i)$ of the $\alpha$-trimmed mean filter. In this data-smoothing scheme, the trimming ensures rejection of impulses and the averaging of the remaining samples reduces Gaussian noise.

Although the $\alpha$-trimmed mean filter is effective in suppressing medium and long tailed noise, it does not behave satisfactorily in the presence of thin tailed (e.g., uniform) noise. For the removal of thin tailed noise, the $\alpha$-trimmed complementary mean (Bednar and Watt 1984) is used:

$$R_\alpha = \frac{1}{2 \ast \lceil\alpha n\rceil} \left( \sum_{j=1}^{\lceil\alpha n\rceil} x(j) + \sum_{j=n - \lceil\alpha n\rceil+1}^{n} x(j) \right)$$

(1.28)

Restrepo and Bovik (1988) proposed an adaptive $\alpha$-trimmed mean filter by combining $\alpha$-trimmed mean and its complementary filters. The filter
adapts to the tail behaviour of the distribution and performs well in a variety of noise distribution environments.

Lee and Kassam (1985) introduced a different approach to trimmed mean filtering. Modified Trimmed Mean (MTM) filter, suggested by them, excludes samples $x_{i+r}$ in the filter window, which differ considerably from the local median $m_k$:

$$y(i) = \frac{\sum_{r=-N}^{N} a(r) x_{i+r}}{\sum_{r=-N}^{N} a(r)}$$

where

$$a(r) = \begin{cases} 
1 & \text{if } |x_{i+r} - m_k| \leq q \\
0 & \text{otherwise}
\end{cases}$$

and $q$ is a preselected constant.

A variation of MTM filter, called Double Window Modified Trimmed Mean (DW MTM) filter (Lee and Kassam 1985) uses two windows of sizes $2N+1$ and $2L+1$ with $L > N$. First, the sample median $m_k$ is computed from the smaller window of size $2N+1$. For some positive number $q$, an interval $(m_k-q, m_k+q)$ is chosen. Then, the mean of the points lying within this interval among the samples in the larger window of size $2L+1$ is computed as the output. DW MTM filter is robust against impulses, suppresses Gaussian noise and preserves signal details satisfactorily.
1.5 MIDPOINT FILTERS

Midpoint is the best estimator of location for uniform distribution (Pitas and Venetsanopoulos 1990). Midpoint filtering can be defined as:

\[
y(i) = \frac{1}{2} (x_{(1)} + x_{(n)})
\]

where \( x_{(j)}, j = 1, 2, \ldots, n \) are the ordered statistics of \( x(i-N), \ldots, x(i), \ldots, x(i+N) \). Bovik et al (1983) have shown that midpoint is the optimal-L filter for estimating the data confounded by short tailed noise. Arce and Fontana (1988 a) have analyzed the statistical properties of the midpoint filter.

1.6 SCOPE OF THE PRESENT WORK

The objective of this thesis is to introduce some nonlinear filtering strategies based on order-statistics. The proposed filters are useful in enhancing the images confounded by signal-independent additive white i.i.d. (independently and identically distributed) noise and/or impulse noise; in addition, they are shown to exhibit good edge and fine detail preserving characteristics. The work, presented in the thesis, is organized as follows.

Chapter 2 introduces simple nonlinear highpass filter algorithms for detecting edge structures in one- and two-dimensional signals. One-dimensional highpass filter detects and preserves signal edges by sliding a 3-point wide time-ordered window over the input signal sequence. A high level structure for its VLSI implementation is proposed. VLSI simulation results obtained using the one-dimensional highpass filter are presented. The two-dimensional highpass filter slides a time-ordered window of size 3x3 over the image signals.
for detecting and preserving their edges. A novel feature of the two-dimensional highpass filter is that it is sensitive to the orientation of image edges. That is, the filter can detect and preserve the edges as horizontal, vertical, left diagonal and right diagonal edges. The two-dimensional highpass filter is compared to the commonly used edge detectors in terms of computational requirements. The filter requires fewer computations for detecting the image boundaries (edges). The only comparable edge detector to the proposed one on the basis of computational complexity is the range edge detector. Therefore, a qualitative comparison, on the basis of subjective visual criterion, is performed between the range edge detector and the proposed filter in both noisy and noise-free environments. The results obtained are presented and discussed.

Chapter 3 describes four nonlinear image-filtering schemes, namely, New Filter I, New Filter II, New Filter III and New Filter IV. The two-dimensional highpass filter algorithm, discussed in chapter 2, forms the basis for these new filters. New Filter I consists of a mean filter, a highpass filter and a combiner. The mean filter suppresses Gaussian noise, while the highpass filter preserves the image edges. The combiner, using the outputs of mean and highpass filters, produces the output of New Filter I. An alpha trimmed mean filter, a median filter of window size 3x3, a highpass filter and a combiner configure New Filter II. The alpha trimmed mean filter smoothes out both Gaussian and impulse noise. The highpass filter together with the median filter detects and preserves the image boundaries. The combiner synthesizes the output of New Filter II using the outputs of the alpha trimmed mean and highpass filters. New Filter III consists of a midpoint filter, a highpass filter and a combiner. The midpoint filter removes uniform noise, while the edges are detected and preserved by the highpass filter. The combiner, using the outputs of the midpoint and highpass filters, produces the output of New Filter III. New
Filter IV comprises an alpha trimmed midpoint filter, a 3x3 median filter, a highpass filter and a combiner. Uniform and impulse noise are filtered out by the alpha trimmed midpoint filter. The image edges are preserved by the highpass filter along with the median filter. The combiner produces the output of New Filter IV using the outputs of the alpha trimmed midpoint and highpass filters. All of these filters are evaluated for their performance in terms of two objective quality measures, namely, image enhancement factor $F_e$ obtained at different levels of noise and MSE/pixel produced at the output with windows of various sizes. Furthermore, the results of filtering of noisy images are also included for subjective evaluation.

Chapter 4 presents a new class of midpoint-type nonlinear filters, which is useful for recovering the images contaminated by uniform and impulse noise. To begin with, the Alpha Trimmed Midpoint (ATMP) filter is introduced. The ATMP filter sorts out the samples, trims (removes) a few and equal number of them from both the ends and then performs midpoint filtering. The filter discards impulses and attenuates uniform noise quite well. Next, the Adaptive Alpha Trimmed Midpoint (AATMP) filter is described. This filter performs in an adaptive manner using edge information. It operates as an ATMP filter in the homogeneous regions for eliminating noise components and works as a median filter (with the same window size as that of the ATMP filter) in the neighbourhood of edges for preserving them. Modified Adaptive Alpha Trimmed Midpoint (MAATMP) filter, presented in the end, is similar to the AATMP filter except that it operates as a median filter of window size 3x3 in the neighbourhood of edges, irrespective of the window size of the ATMP filter. The filter allows simultaneous removal of uniform and impulse noise, preserves edge structures and retains fine details. The proposed filters are evaluated for their performance by applying them to test images corrupted by
different levels of noise and using windows of various sizes. The extensive results obtained are presented and discussed.

Chapter 5 introduces a technique for improving the performance of median smoothers at the signal corners characterized by low order polynomials. It is a fact that the median filters with larger windows provide greater smoothing for non-impulsive noise components and are more robust against impulse noise than the median filters of smaller windows. However, larger median filters (median filters with larger windows) fail to track low order polynomial trends in the signals. Due to this, constant regions are produced at the signal corners leading to a loss of fine details. The proposed scheme, called combination smoother, consists of a 3-point median smoother and a larger median smoother of desired window length in parallel; besides, it has a combiner for appropriately combining the outputs of these median filters. The combiner algorithm, presented in this chapter, combines the detail-preserving characteristics of the 3-point smoother and the better noise filtering characteristics of the larger smoother. The efficacy of the proposed combination smoother is illustrated both objectively and subjectively, by applying it to a test image corrupted by different levels of impulse and/or non-impulse noise.

Chapter 6 concludes the thesis by highlighting the salient features of the proposed filtering strategies. The major contribution of this work is summarized. The possible directions for future work are indicated.