1.1 TWO AND MULTIDIMENSIONAL SIGNAL PROCESSING

The field of digital signal processing is not only confined to one-dimensional (1-D) signals but it has grown from 1-D to the two-dimensional (2-D) and to multi-dimensions (n-D). The multidimensional signal processing deals with the processing of discrete signals that are functions of two or more integer variables. There are many situations in the real world, where signals are inherently two and multidimensional. For example, the picture data such as satellite images, x-rays, electron micrographs, aerial photographs for detection of forest fires or crop damages, infrared pictures constitute largest subset of two-dimensional signals. Seismic, magnetic, gravitational and hydrophobic data represent some of the signals that are multidimensional. Whether it is singular or multidimensional signal processing the purpose of such a processing may be to convert one sequence (called input) into another sequence (called output) by means of an algorithm, which is more desirable in some sense and to estimate the parameters that are characteristics of the system which give rise to the sequence of numbers. Thus the principal categories of two and multidimensional signal processing are

(i) Deterministic approaches: filter design, stability testing, stabilization.
(ii) Statistical approaches: estimation / detection, modeling.

(iii) Implementation: special purpose microprocessors and DSP processors.

(iv) Applications: geophysics, biomedicine, military, remote sensing etc.

Of all the above digital filtering area is the most important. So the problem of design and analysis of two and n-D filters have been extensively studied. The multidimensional data can be produced by 1-D techniques. But it is preferable to use 2-D and n-D methods because there are advantages by filtering 2-D data by 2-D filters. For instance on considering the particular application to images it is possible to correlate the data with neighboring pixels in all directions by filtering with 2-D filters, unlike the 1-D filter that allows correlation between information lying in the same line.

From filtering point of view different kinds of filtering can be performed by using either Finite Impulse Response (FIR) filters or Infinite Impulse Response (IIR) filters. Non-recursive filters are always stable as they use only weighted values of inputs. In addition they can have frequency response function, which is purely real. They can be implemented using 2-D Fast Fourier Transform or convolution methods.

Where linear phase is not a constraint the recursive method of filtering has often proved to be more efficient than non-recursive filtering. In IIR filtering technique the filter algorithm uses previously computed output values as well as input values and these filters are generally implemented by difference equations. For a given degree of approximation the order of IIR
filters will be less than the order of corresponding FIR filters, which means that the number of coefficients in IIR filtering operation is smaller than that in FIR filtering operation. Thus the recursive filters require less storage in their implementation than non-recursive filters and they may require less computation as well for the same performance. Hence recursive structures appear more compatible with small-scale hardware implementation such as those involving special purpose processors.

There are two steps in the development of 2-D or n-D recursive digital filters:

(i) The design stage involves the determination of transfer function \( H(z_1,z_2) \) of the filter as a ratio of two polynomials \( A(z_1,z_2) \) and \( B(z_1,z_2) \) with real coefficients to meet the specified space domain or frequency domain requirements.

(ii) After suitable transfer function has been determined the corresponding filter is usually implemented on a general-purpose computer and thereby one can evaluate its performance before committing it to the special purpose hardware.

But there are many problems associated with the design of stable recursive filters.
1.2 PROBLEMS IN DESIGNING STABLE 2-D RECURSIVE DIGITAL FILTERS

Stability testing for 2-D polynomials is not a straightforward task as in 1-D case where any 1-D polynomial may be factored in terms of its roots. Stability testing simply reduces to checking the location of these roots with respect to unit circle \( |z| = 1 \). The stability of two dimensional (2-D) systems depends on the impulse response of the system. In other words if the input is a bounded signal the result of convolving the input sequence with the filter response should yield a bounded output sequence.

The transfer function of a recursive digital filter can be represented as follows,

\[
H(z_1, z_2) = \frac{Y(z_1, z_2)}{U(z_1, z_2)} = \frac{N(z_1, z_2)}{D(z_1, z_2)} = \sum_{i=0}^{M} \sum_{j=0}^{N} a(i, j)z_1^{-i} z_2^{-i} / \sum_{i=0}^{M} \sum_{j=0}^{N} b(i, j)z_1^{-i} z_2^{-i}
\]

(1.1)

where \( Y(z_1, z_2) \) and \( U(z_1, z_2) \) are the output and input functions respectively, while \( a \) and \( b \) are the coefficients of the system equation. Equation (1.1) can be expressed in spatial domain in terms of linear difference equation as

\[
y(m, n) = \sum_{i=0}^{M} \sum_{j=0}^{N} a(i, j)u(m-i, n-j) - \sum_{i=0}^{M} \sum_{j=0}^{N} b(i, j)y(m-i, n-j)
\]
by Shanks et al (1972). They stated that if $D(z_1, z_2)$ is polynomial in $z_1$ and $z_2$, the necessary and sufficient condition for the coefficients of the expansion of $D(z_1, z_2)$ in negative powers of $z_1$ and $z_2$ to converge absolutely, and hence for $h(m, n)$ to be absolutely summable as

$$D(z_1, z_2) \neq 0 \quad \text{for} \quad \bigcap_{i=1}^{2} |z_i| \geq 1 \quad (1.6)$$

The above condition suggests a test procedure for checking the stability of the filter by finding the continuum of $(z_1, z_2)$ value for which $D(z_1, z_2) = 0$. This is done by assigning values to the variable $z_1$ and finding the roots of $D(z_1, z_2) = 0$ as a function of $z_2$. For stability it follows that all roots of $z_2$ must be less than unity in magnitude when $|z_1|$ is greater than one. The testing for stability as stated above is very tedious to apply since it involves mapping an infinite number of points from the $z_1$ plane into the $z_2$ plane.

A considerably simplified version has been achieved by Huang (1972). Here the two dimensional bilinear transform was utilized to transform the system function so that results from multidimensional continuous theory by Ansell (1964) could be used. For the first quadrant filters which have the form

$$B(\omega, z) = \sum_{m=0}^{M} \sum_{n=0}^{N} b(m, n) \omega^{m} z^{n} \quad (1.7)$$

with $b(0, 0) \neq 0$, instead of testing $B(\omega, z) \neq 0$ on $0^2 = \{ \omega \leq 1, |z| \leq 1 \}$ the change of variables $s_1 = (1-\omega)/(1+\omega), \quad s_2 = (1- z)/(1+z)$ is performed on $H(\omega, z) = A(\omega, z)/B(\omega, z)$ to obtain $G(s_1, s_2) = E(s_1, s_2)/F(s_1, s_2)$ and $F(s_1, s_2)$ is
tested for zeros in $\Gamma = \{(s_1, s_2) : \text{Re}\{s_1\} \geq 0, \text{Re}\{s_2\} \geq 0\}$ using methods developed by Ansell (1964). This method was shown to be incorrect by Goodman (1978). Several extra one dimensional tests must be performed to check the if and only if condition. Once these one-dimensional procedures have been performed the Hermite algebraic procedure employed by Huang (1972) is valid.

The stability problem reduced to determining whether a $M \times M$ symmetric matrix polynomial $D(\omega)$, whose entries are real polynomials of order $2M$, is positive definite for all values of $\omega$. Huang (1972) suggested that positive definiteness of $D(\omega)$ could be verified by showing that successive principal minors of $D(\omega)$ are positive for all real $\omega$. It was noted that Sturm’s method could be used to test the postivity of each minor. Using Siljak’s (1974) proof it can be proved that it is not necessary to test all leading minors for postivity. Instead of $M$, the following two conditions have to be met

(i) $D(0)$ be positive definite.

(ii) Determinant ($D(\omega)$) > 0 for all real $\omega$.

A major difficulty arises in the determination of the polynomial determinant $D(\omega)$. It can be seen that while Ansell’s test require only a finite number of steps, the mathematics involved is tedious. This is mainly due to the calculation of the bilinear transformation which when efficiently implemented requires an order of $M^3$ calculations.
Huang (1972) simplification of the stability test rests in the fact that the denominator in the z-transfer function $D(z_1, z_2) \neq 0$ for $|z_1| \geq 1 \cap |z_2| \geq 1$ if the following two conditions hold good

$$D(z_1,0) \neq 0, \quad |z_1| \geq 1$$  \hspace{2cm} (1.8)

and

$$D(z_1,z_2) \neq 0, \quad |z_1| = 1 \cap |z_2| \geq 1$$  \hspace{2cm} (1.9)

The Anderson and Jury (1973) stability test is divided into two parts: first, condition (1.8) is checked for the values of $D$ with $z_1$ restricted and second the condition (1.9) is checked for the values of $D$ with both $z_1$ and $z_2$ restricted.

For checking condition (1.8)

(i) The Bilinear transformation of $D(z_1,0)=0$ is taken and Hurwitz method is applied to transformed polynomial.

(ii) An alternative test involves forming the Schur-Cohn matrix from $f(z_1,0)=0$ and testing this for positiveness.

For checking condition (1.9) two successive tests as follows are needed. First Schur-Cohn test is applied to $D(z_1,z_2)=0$ to get the self-inverse polynomial and positiveness of these polynomials checked.

Secondly, the positiveness of a number of polynomials of $D(z_1,z_2)=0$ on $|z_1|=1$ should be checked.
The Schur-Cohn matrix has polynomial elements of order $2M$ where $M$ is the order of the filter and it requires approximately an order of $6M^5$ operations to obtain the $M^2$ order polynomial. The positivity can be determined in an order of $M^4$ operations. Maria and Fahmy (1973) tested the condition that $B(\omega, z) \neq 0$ on $|\omega| = 1, |z| \leq 1$ by again considering

$$B(\omega, z) = \sum_{n=0}^{M} a_n(\omega)z^n \quad \text{where} \quad a_n(\omega) = \sum_{i=0}^{M} b(i,n)z^n \omega^i$$

and then using the Marden-Jury table. An $M^{th}$ order complex polynomial has all its zeros outside the unit circle if and only if the first column of the table is positive. The tables are polynomials in $\omega$ and $\omega^{-1}$ for a two-dimensional case. Positivity has to be checked for the first column entries. This can be done using Sturm's and Cohn's methods and they both require approximately an order of $N^2$ operations where $N$ is the order of the polynomial. It takes less than an order of $M^24^M$ real calculations to guarantee that a given $M^{th}$ order filter is stable. This means a $16 \times 16$ quarter plane filter stability test requires a trillion calculations to guarantee stability.

Subbarami Reddy et al (1984) proved the stability of Planar Least Square Inverse (PLSI) polynomials in 2-D by modifying Shank's conjecture and imposing restriction on the original polynomial. They considered a 2-D first quadrant polynomial not having zeros on the unit hyper circle. The restriction was necessary to stabilize an unstable polynomial eventually by taking double PLSI so as to maintain magnitude spectrum same.
Niranjan Damera, Subbarami Reddy (1999) have dealt with Discrete Hilbert Transform (DHT) method of stabilizing unstable two dimensional recursive digital filters originally proposed by Read and Treitel. They commented that DHT method cannot guarantee stability if the unstable 2-D polynomial has zeros on the unit bicircle. Stability testing of 2-D discrete linear Systems by Telepotation of an Immittance-Type Tabular Test was dealt by Bistritz.Y(2001,2002). Hence it is found that the problems of simplifying stability test for 2-D recursive filters still persist.

1.3 BRIEF LITERATURE REVIEW OF WORK DONE IN HIGH SPEED ARCHITECTURES

In linear systems a given transfer function can be realized by a variety of structures. A 2-D state-space structure can realize IIR digital filters with minimum round-off noise. Different realizations of a given filter will have the same linear response but may have different hardware complexity and different finite word length. In the past few years high speed image processing using digital filters has become a rapidly growing field as the need for fast processing is evident. Many applications involve data acquisition, transmission, processing and display in the real time. In these areas each pixel has to be processed in a very short time which is inversely proportional to the data rate that is very high. Therefore several architectures for digital image processing are developed.

Many researchers have implemented recursive filters using VLSI technology. Kung (1982) has done good work with the concept of systolic arrays and proposed the systolic arrays filtering. Rao and Kailath (1985) gave
a model identification approach to VLSI filter design. Mullis and Roberts (1976) have worked on synthesis and of computationally efficient, low round-off noise, state space digital filters. Zhang and Steenart (1990) proposed two high-speed VLSI architectures for high-speed applications. They presented advanced state update architecture that trades increased throughput rate for increased hardware. The technique is based on block processing. To achieve very high speed they proposed global speed up architecture exploiting the inherent spatial concurrency of two-dimensional filters. Sundarajan and Parhi (1992) used folding transformation to systematically determine the control circuits in DSP architectures where multiple algorithm operations are time multiplexed to a single functional unit resulting in an integrated circuit with low silicon area. Sundararajan and Parhi (1998) dealt with a novel multi-dimensional (n-D) folding transformation technique that can be used to synthesize control circuits for pipelined architectures for two-dimensional discrete wavelet transforms.

1.4 OBJECTIVE OF THE THESIS

The primary objective of the thesis is to present a simple method for testing the stability of first quadrant 2-D recursive digital filter for both necessary and sufficiency conditions. Following the Slice Projection Theorem suggested by Mersereau (1974) a very simple stability test for such filters is to be dealt with. The whole method of testing for stability reduces the checking of only one 1-D polynomial for zeros on and inside the unit circle.

Second objective is to propose high speed Folded architecture exploiting the inherent concurrency of 2-D IIR filters. Folded architecture is
used to reduce the functional units there by minimizing silicon area. Multiple outputs are computed in parallel to increase sampling speed and for reduction of power consumption. VLSI synthesis and simulation of signed sixteen bit fixed point Folded architecture of 2-D recursive filters using VERILOG is to be carried out. The concept is to be extended to synthesis and simulation thirty-two bit floating-point high-speed architecture.

Third objective is to develop microcontroller based system on chip architecture for 2-D IIR filtering applications by attempting to combine Harvard architecture and Folded architecture. Pipelining transformation leads to reduction in critical path to increase speed. Apart from ALU, program memory (PRAM), coefficient data memory (CROM), four register banks, two stacks are to be incorporated in the design. Control signals for Folded architecture is generated in the Harvard architecture of the RISC processor.

1.5 CHAPTER WISE ORGANISATION

Chapter 1 introduces the general class of two dimensions and multidimensional signal processing and their application. Problems in designing stable 2-D recursive digital filters are discussed. Confining to the following two topics. Stability test of two dimensional recursive filters and VLSI implementation of high speed architectures for two dimensional recursive filtering applications. A detailed literature survey describing the existing methods of solution and their drawbacks is presented. The desirable features and the scope of the present work are pointed out.
Chapter 2 addresses a very classical problem of testing for stability of a first quadrant 2-D recursive digital filter. Following the Slice Projection Theorem proposed by Mersereau [1974] a very simple stability test for such filters is arrived. Originally to verify Slice Projection Theorem infinite number of 1-D polynomials have to be tested for stability. The method proposed in the thesis reduces testing for stability to only one 1-D polynomial for zeros on and inside the unit circle. Also the method presented here for testing the stability of 2-D filters tests for both necessity and sufficiency. The method arrived at and presented here saves lot of time when compared to all the available methods and is more accurate.

In chapter 3 the Folded architecture provides a systematic technique for designing control circuits for hardware where operations of several algorithms are time multiplexed onto a single functional unit. The Global Speed-up Architecture (GSR) is discussed in this chapter, has an array of forty-nine processing elements with a throughput rate of one pixel per clock period. In the proposed Folded architecture the number of processing elements have been reduced to seven maintaining the same throughput rate of one pixel per clock period. The cores has been designed for signed sixteen bit fixed point and thirty-two bit floating point high speed architectures for two-dimensional recursive filters using VERILOG as the HDL language and synthesized with FPGA logic unit which targets XILINX FPGA XC2S300E series.

Chapter 4 presents VLSI design and simulation of fast eight bit and sixteen bit microcontroller based system on chip architecture for real time filtering applications. By exploiting inherent concurrency of two dimensional recursive filters the Folded architecture is designed and combined with Harvard
architecture. Due to pipelined features each instruction takes one clock period for execution. Apart from ALU, program memory (PRAM), data memory (CROM) a four register banks and two stacks are incorporated in design. Control signals for Folded architecture is generated in the Harvard architecture of RISC processor. The Folded architecture has seven column processors that are capable of handling forty-nine pixels in forty-nine clock periods. The core has been designed using VERILOG as HDL with XILINX FPGA XC2S300E series as target FPGA device. Waveform of functional simulation of core confirms the processor’s capability of having throughput rate of one pixel per clock period.

In chapter 5 a review of the work reported in the thesis and the major contributions made in the thesis are listed.

A simple stability test for 2-D recursive digital filters is arrived for both necessary and sufficiency conditions. The method arrived at and presented reduces complexity when compared to all the available methods and is more accurate.

Signed sixteen bit fixed point and thirty-two bit floating point high speed Folded architecture are designed for two dimensional recursive filter applications resulting in minimizing functional units, silicon area and maintaining the throughput rate of one pixel per clock period.

VLSI design and simulation of fast eight bit and sixteen bit microcontroller based system on chip architecture for real time filtering applications are implemented. Because of pipelining and parallel processing
features each instruction takes one clock period for execution. Control signals for Folded architecture is generated in the Harvard architecture of RISC processor. The Folded architecture has seven column processors that are capable of handling forty-nine pixels in forty-nine clock periods. Each column processor has fixed point adder, multiplier, latch and enable pin. The core has been designed using VERILOG as HDL with XC2S300E series as target FPGA device. Waveform of functional simulation of core confirms the microcontroller’s capability of having throughput rate of one pixel per clock period.