CHAPTER 7

DISCUSSION AND CONCLUSIONS

The application of Schwarz-Christoffel transformation permits evaluation of equipotential boundaries in the cross-section of a TEM cell based on which the effect of septum thickness on characteristic impedance can be taken into account. The method is quite rigorous and is applicable to symmetric location of the septum. Extension of the method for arbitrary location of the septum having finite thickness has not been found to be feasible. For arbitrary location of the septum the effect of its thickness can be taken into account from energy calculations or by applying the moment method.

From the information available in the literature it is found that Schwarz-Christoffel transformation which is a particular case of conformal transformation can be applied to lossless lines. Application of the moment method is based on the assumption that charge distribution evaluated in a particular cross-section of the line is independent of its position along the axis of the line. In this method, therefore, considerable complexity is expected when the effect of losses are considered. Hence the results obtained from the moment method are valid for the lossless case or at high frequencies. In the formula based on the energy calculations there is scope for taking the effect of losses into account and hence it is applicable to extremely low frequencies as well as very high frequencies of TEM mode.
From the results presented in chapter 2, it is found that the characteristic impedance is complex at frequencies below 1 kHz. Both the characteristic and the wave impedances reach constant values from their respective higher and lower values, only when the frequency exceeds 1 kHz. In spite of the considerable complexity in taking the effect of the septum thickness into account, the analysis presented in Chapter 2 is based on a relatively simple concept. Simplifying approximation like neglecting the effect of current on the side surface of the septum yields the formula which is valid for small thickness. About 2 ohms decrease in impedance has been observed for an increase of t/w from 0 to 0.02 for b/a=0.5 and w/a=0.6. The formulae obtained in this case are such that they can be extrapolated to the case of t/w=0. The computed results of impedance for this case show good agreement with the published results.

In the moment method two forms of Green's function are used: i) Green's function expressed as infinite summation of product of circular and hyperbolic functions which takes automatically into account the presence of two conductors through boundary conditions; ii) Green's function in the form of logarithmic function which is valid for line source of infinite length in free space. In the first case the moment method formulation requires the determination of electric charge distribution only on one of the conductors viz. septum. Consequently, the size of the matrix to be inverted is small. But each element of the matrix involves infinite summation. Although the computer time requirement is large, in view of the memory requirement which is not large, it is possible to calculate the impedance using a small personal computer. In the second case the formulation requires the determination of electric charge distribution on both the conductors and hence the matrix
to be inverted is of large order. But each element can be computed in a very short time. This requires the use of a high speed computer with higher memory size.

In order to consider the effect of the thickness of the inner conductor, which can vary from an extremely small \((t/w \to 0)\) to a large value, non-uniform segmentation on the periphery of the septum is adapted in the moment method analysis. Here the edge/bend elements taken are the smallest in size where charge or current is extremely high due to the edge effect. The sizes of the elements towards the centre of the width are made progressively larger in accordance with arithmetic progression. This avoids the unnecessarily large number of segments on the bottom and top surfaces of the septum which would have been, otherwise, taken for uniform segmentation with segment size equal to the septum thickness \(2t\) or its integral fraction when the solution converges. The solution converges for a total number of about 60 segments on the septum. This procedure allows wide latitude in the choice of subdomain or element size. Since the characteristic impedance depends on the total charge contained in one of the conductors viz. septum, complicated edge treatment is not used in this analysis. The effect of edges or bends of the conductors is taken into account by considering finer edge subdomain sizes compared to those of the centre of the bottom and top surfaces of the septum.

The moment method has the advantage that, unlike in the previous method described in chapter 2, there is no restriction on the thickness and width of the centre conductor. The impedance is computed very accurately as can be seen from the numerical results. This method is also advantageous in mapping the relative field.
distribution in the cross-section to find the region of field uniformity within a specified limit, viz., ±1 dB, as required in many applications. The MOM analysis shows that charge densities are much higher at the corners of the septum as compared with those in the middle region. The charge distribution becomes asymmetric when the septum is offset from the centre.

In both the chapters 2 and 3 it is shown that the characteristic impedance decreases with the increase of thickness of the septum, of the width of the septum, and the amount of its offset in both horizontal and vertical directions. A decrease of impedance by 2-2.5 Ohms is observed for an increase of t/w ratio by an amount 0.02 from its zero value for the ratio of b/a = 0.5 and w/a = 0.6. From the distributions of electric field components in the cross-section of an asymmetric TEM cell it is seen that the electric field is essentially normal to the septum width and remains constant in the central region of the cell. The electric field component parallel to the width of the septum gradually becomes dominant towards the side walls. Reverse is the case of the magnetic field components as expected. From the MOM analysis it is found that the field is uniform within ±1 dB around the central point between the septum and the outer conductor wall parallel to the septum. The region of this field uniformity varies as the septum is offset from the centre of the cell. The area of this region decreases considerably when the distance between the septum and the parallel wall of the outer conductor increases. For a horizontal offset of the septum, the size of this region, however, is not altered significantly.
In chapter 4 the Schwarz-Christoffel transformation is applied to treat the thickness of the septum in symmetric TEM cell for impedance calculation. This analysis yields exact expressions for the characteristic impedance, the equipotential and the electric flux lines in a rectangular symmetric TEM cell. The technique is computationally very efficient. However, the method cannot be used for septum having finite thickness and simultaneously located asymmetrically in the cross-section. A very good agreement has been observed between the results of characteristic impedances of symmetric TEM cells as a function of septum thickness, computed by the three methods described in chapters 2, 3 and 4 when t/w is small. Discrepancies among the results increase as t/w increases. At t/w=0.02, the value of the characteristic impedance evaluated by using energy consideration is about 0.4 ohm higher than that obtained from the moment method. The same result, obtained by Schwarz-Christoffel transformation, agrees well with that of the moment method. The discrepancies of the results are due to the fact that the method employed in Chapter 2 neglected the current on the side surface of the septum. This approximation does not give the exact solution when t/w > 0.02.

The TEM cell supports the higher-order TE and TM modes when the excitation frequency exceeds the cut-off frequencies of the respective modes. The presence of the septum introduces considerable difficulty in evaluating the cut-off frequencies of the higher-order modes. Chapter 5 shows that the finite element method is very efficient for the evaluation of the cut-off frequencies of a large number of higher order modes in a TEM cell. The results obtained by this method are in good agreement with those reported by others in the
literature. The existing discrepancies in the results for TE_{21} and TM_{11} modes are explained with help of FEM results. The finite element solution is exact at the nodes and at any intermediate point the solution converges as the number of elements is increased.

The resonance frequencies of the first few higher order modes are theoretically predicted and results are experimentally verified. Good agreement has been observed between the theoretical and experimental results. This study helps in extending the frequency bandwidth of a TEM cell. The small discrepancies between the theoretical and measured values of resonance frequencies may be due to the use of average values of empirically determined factor X_{mn} taken from the results of Hill (1983).

It is shown in Chapter 6 that when the cell is excited with predominantly E-field the distance from the centre of the cell at which the electric field is lowered by 1 dB as compared to the value at the centre, decreases as the frequency of the operation increases. A similar situation arises for a uniform magnetic field excitation. The rate of decrease of the E/H ratio with the distance from the centre along the length, however, is large. This indicates that the equipment under test, whose size should be limited so as to fit in the predominantly uniform field region of the cell, will still experience a large variation of E/H ratio over its length. Thus the usable test region under this condition may be limited to the region where the variations of E/H ratio are not large and at the same time the predominantly electric field or magnetic field is fairly uniform. This method is useful for electromagnetic exposure studies when a small object is placed in such fields.
The measured and theoretical values of electric and magnetic fields of standing wave having a wide range of E/H ratios generated at the centre of the TEM cell, are found in good agreement at most of the frequencies. The deviation of results at some frequencies may be due to the limitations of the measurement accuracy and the prediction of length of the taper sections. The calibrated results of the small dipole probe when exposed to these fields are useful in estimating the level of radiated emission from a potential EMI source.
\[
\int \int = (n I o / \mu) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ K_1(g_{m1}g_{n1} + g_{m2}g_{n2} + 2g_{m1}g_{n2}) + K_4(g_{m1}g_{n1} + g_{m2}g_{n2} - 2g_{m1}g_{n2}) \right] \quad \text{(A 1.6)}
\]

where

\[
D-W
\]

\[
I_1 = \int_0^{m/n} \cos \left( \frac{m\pi x}{2a} \right) \cos \left( \frac{n\pi x}{2a} \right) \, dx = 1/2 \left( X_1 + X_2 \right) \quad \text{(A 1.7)}
\]

\[
I_2 = \int_{h-t}^{h+t} \sinh \left( \frac{m\pi}{2a} (2b-y) \right) \sinh \left( \frac{n\pi}{2a} (2b-y) \right) \, dy = 1/2 \left( Z_1 - Z_2 - Z_3 + Z_4 \right) \quad \text{(A 1.8)}
\]

\[
I_3 = \int_{h-t}^{h+t} \sinh \left( \frac{m\pi y}{2a} \right) \sinh \left( \frac{n\pi y}{2a} \right) \, dy = 1/2 \left( X_3 - X_4 - X_5 - X_6 \right) \quad \text{(A 1.9)}
\]

\[
D-W
\]

\[
I_4 = \int_0^{m/n} \sin \left( \frac{m\pi x}{2a} \right) \sin \left( \frac{n\pi x}{2a} \right) \, dx = 1/2 \left( X_1 - X_2 \right) \quad \text{(A 1.10)}
\]

\[
I_5 = \int_{h-t}^{h+t} \cosh \left( \frac{m\pi}{2a} (2b-y) \right) \cosh \left( \frac{n\pi}{2a} (2b-y) \right) \, dy = 1/2 \left( Z_1 - Z_2 + Z_3 - Z_4 \right) \quad \text{(A 1.11)}
\]

\[
I_6 = \int_{h-t}^{h+t} \cosh \left( \frac{m\pi y}{2a} \right) \cosh \left( \frac{n\pi y}{2a} \right) \, dy = 1/2 \left( X_3 - X_4 + X_5 - X_6 \right) \quad \text{(A 1.12)}
\]
\[ I_7 = \int_{h-t}^{h+t} \sinh \left( \frac{m\pi}{2a} (2b-y) \right) \sinh \left( \frac{n\pi y}{2a} \right) dy = 1/4 \{ X_7 - X_8 + X_9 - X_{10} \} \]  
\[ I_8 = \int_{h-t}^{h+t} \cosh \left( \frac{m\pi}{2a} (2b-y) \right) \cosh \left( \frac{n\pi y}{2a} \right) dy = 1/4 \{ X_{11} - X_{12} + X_{13} - X_{14} \} \]  
\[ K_1 = \int_{D+W} \cos \left( \frac{m\pi x}{2a} \right) \cos \left( \frac{n\pi x}{2a} \right) dx = 1/2 \{ Y_1 - Y_2 + Y_3 - Y_4 \} \]  
\[ K_4 = \int_{D+W} \sin \left( \frac{m\pi x}{2a} \right) \sin \left( \frac{n\pi x}{2a} \right) dx = 1/2 \{ Y_1 - Y_2 - Y_3 + Y_4 \} \]  
\[ B_{n1} = \sin \left( \frac{nnD}{2a} \right) J_0 \left( \frac{nnw}{2a} \right) \left[ \sinh \left( \frac{nn}{2a} (2b-h+t) \right) \right] 
+ \sinh \left( \frac{nn}{2a} (2b-h-t) \right) \]  
\[ B_{n2} = -\sin \left( \frac{nnD}{2a} \right) J_0 \left( \frac{nnw}{2a} \right) \left[ \sinh \left( \frac{nn}{2a} (h-t) \right) \right] 
+ \sinh \left( \frac{nn}{2a} (h+t) \right) \]
\[
\sin \left( \frac{\sinh D}{2a} \right) J_0 \left( \frac{\sinh w}{2a} \right) \sinh \left( \frac{n\pi}{2a} (h-t) \right)
\]

\[\text{(A 1.19)}\]

\[
\sin \left( \frac{\sinh D}{2} \right) J_0 \left( \frac{\sinh w}{2a} \right) \sinh \left( \frac{n\pi}{2a} (2b - h-t) \right)
\]

\[\text{(A 1.20)}\]

\[
\sin \left( \frac{m-n}{2a} \right) \frac{\pi (D - w)}{\pi}
\]

\[\text{(A 1.21)}\]

\[
\sin \left( \frac{m+n}{2a} \right) \frac{\pi (D - w)}{\pi}
\]

\[\text{(A 1.22)}\]

\[
\sinh \left( \frac{m+n}{2a} \right) \frac{\pi (h + t)}{\pi}
\]

\[\text{(A 1.23)}\]

\[
\sinh \left( \frac{m+n}{2a} \right) \frac{\pi (h - t)}{\pi}
\]

\[\text{(A 1.24)}\]
\[
X_5 = \frac{\sinh \left\{ \frac{m-n}{2a} \pi (h + t) \right\}}{\left\{ \frac{m-n}{2a} \pi \right\}} \quad \text{..(A 1.25)}
\]
\[
X_6 = \frac{\sinh \left\{ \frac{m-n}{2a} \pi (h - t) \right\}}{\left\{ \frac{m-n}{2a} \pi \right\}} \quad \text{..(A 1.26)}
\]
\[
X_7 = \frac{e^{-mnb/a} \left\{ \frac{m-n}{2a} \pi (h+t) - e^{\frac{m-n}{2a} \pi (h-t)} \right\}}{\left\{ \frac{m-n}{2a} \pi \right\}} \quad \text{..(A 1.27)}
\]
\[
X_8 = \frac{e^{mnb/a} \left\{ -\frac{m-n}{2a} \pi (h+t) - e^{-\frac{m-n}{2a} \pi (h-t)} \right\}}{\left\{ \frac{m-n}{2a} \pi \right\}} \quad \text{..(A 1.28)}
\]
\[
X_9 = \frac{e^{mnb/a} \left\{ -\frac{m+n}{2a} \pi (h+t) - e^{-\frac{m+n}{2a} \pi (h-t)} \right\}}{\left\{ \frac{m+n}{2a} \pi \right\}} \quad \text{..(A 1.29)}
\]
\[
X_{10} = e^{-mnb/a} \left\{ \frac{m+n}{2a} \pi (h+t) - e^{\frac{m+n}{2a} \pi (h-t)} \right\} \quad \text{..(A 1.30)}
\]
\[ X_{11} = \frac{e^{-m\pi b/a} \left\{ e^{\left( \frac{m-n}{2a} \right) \pi (h+t)} - e^{\left( \frac{m-n}{2a} \right) \pi (h-t)} \right\}}{e^{\left( \frac{m-n}{2a} \right) \pi}} \]  
..(A 1.31)

\[ X_{12} = -\frac{e^{m\pi b/a} \left\{ e^{-\left( \frac{m-n}{2a} \right) \pi (h+t)} - e^{-\left( \frac{m-n}{2a} \right) \pi (h-t)} \right\}}{e^{-\left( \frac{m-n}{2a} \right) \pi}} \]  
..(A 1.32)

\[ X_{13} = \frac{e^{-m\pi b/a} \left\{ e^{\left( \frac{m+n}{2a} \right) \pi (h+t)} - e^{\left( \frac{m+n}{2a} \right) \pi (h-t)} \right\}}{e^{\left( \frac{m+n}{2a} \right) \pi}} \]  
..(A 1.33)

\[ X_{14} = -\frac{e^{m\pi b/a} \left\{ e^{-\left( \frac{m+n}{2a} \right) \pi (h+t)} - e^{-\left( \frac{m+n}{2a} \right) \pi (h-t)} \right\}}{e^{-\left( \frac{m+n}{2a} \right) \pi}} \]  
..(A 1.34)

\[ Y_1 = \frac{\sin \left\{ \left( \frac{m-n}{2a} \right) \pi .2a \right\}}{e^{\left( \frac{m-n}{2a} \right) \pi}} \]  
..(A 1.35)
\[
Y_2 = \frac{\sin \left( \left( \frac{m-n}{2a} \right) \pi \right)}{\left( \frac{m-n}{2a} \right) \pi} \quad \text{(A 1.36)}
\]

\[
Y_3 = \frac{\sin \left( \left( \frac{m+n}{2a} \right) \pi \cdot 2a \right)}{\left( \frac{m+n}{2a} \right) \pi} \quad \text{(A 1.37)}
\]

\[
Y_4 = \frac{\sin \left( \left( \frac{m+n}{2a} \right) \pi \right)}{\left( \frac{m+n}{2a} \right) \pi} \quad \text{(A 1.38)}
\]

\[
Z_1 = \frac{\sinh \left( \left( \frac{m+n}{2a} \right) \pi \right)}{\left( \frac{m+n}{2a} \right) \pi} \quad \text{(A 1.39)}
\]

\[
Z_2 = \frac{\sinh \left( \left( \frac{m+n}{2a} \right) \pi \right)}{\left( \frac{m+n}{2a} \right) \pi} \quad \text{(A 1.40)}
\]

\[
Z_3 = \frac{\sinh \left( \left( \frac{m-n}{2a} \right) \pi \right)}{\left( \frac{m-n}{2a} \right) \pi} \quad \text{(A 1.41)}
\]

\[
Z_4 = \frac{\sinh \left( \left( \frac{m-n}{2a} \right) \pi \right)}{\left( \frac{m-n}{2a} \right) \pi} \quad \text{(A 1.42)}
\]