3.1 INTRODUCTION

Most of the analytical methods of solution for asymmetric TEM cell having finitely thick septum require specific choice of electric charge distribution on the conductors or specific choice of potential distribution in the cross-section for impedance and field computations. These methods encounter mathematical complexity due to the presence of edges or bends of the conductors. With the advent of high-speed digital computer that performs massive calculations in seconds, numerical methods become more advantageous in such cases.

The present chapter describes a conceptually simple numerical technique for the treatment of the effect of thickness of the asymmetrically located inner conductor on the characteristic impedance of TEM cell. The technique uses moment method (MOM) and is applied to uniform rectangular cross-section region of the TEM cell to find the distribution of electric charge on the periphery of the conductors. The method converts the integral equation for the electrostatic potential involving the Green’s function into matrix form for operation of the cell in TEM mode. The pulse function is used here as the subdomain basis function and the matrix equation is derived applying point matching technique. The procedures which are described in this chapter allow
wide latitude in the choice of subdomain or element size and do not require complicated treatment of edges and/or bends. The numerical results of the characteristic impedance obtained by this method for different conductor sizes of the cell are in good agreement with theoretical predictions reported by Costamagna and Fanni (1990). The region of uniform electric field distribution in the cross-section is also found for different offset positions of the septum. The analysis is valid for any thickness of the septum located in any offset position in the cross-section.

3.2 FORMULATION USING MOMENT METHOD

For the operation of TEM cells well below the cell's cutoff frequency, quasi-static approximations are used where it is assumed that the dimensions of the cell are much smaller than the wavelength of the TEM mode field. It is also assumed that the end taperings do not affect TEM mode of propagation. Under these conditions, only the cross-section of the uniform region (Fig. 3.1) is needed to be considered in this analysis. The characteristic impedance of the cell is expressed as

\[ Z_0 = \frac{1}{\frac{v_0}{C}} \]  \hspace{1cm} ....(3.1)

Here \( v_0 \) is the velocity of light and \( C \) is the capacitance per unit length of the line. \( C \) can be obtained from a knowledge of potential \( V_0 \) of the inner conductor (septum) \( S_1 \) and by finding the total charge on it:

\[ C = \frac{\int \rho(x,y) ds}{V_0} \]  \hspace{1cm} ....(3.2)
Fig. 3.1 Segmentation of TEM cell used in method of moments
where the potential of the outer ground conductor $S_2$ is zero. The charge distribution $\rho(x',y')$ on the conductors are determined by the moment method (Harrington 1969) using the following expression for the solution of the Poisson's equation in the cross-section of the TEM cell (Collins 1960):

$$V(x,y) = \frac{1}{\varepsilon_0} \int \limits_{S} G(x,y|x',y')\rho(x',y')ds' \quad \ldots (3.3)$$

where $V(x,y)$ is scalar electric potential, $G(x,y|x',y')$ is Green's function for the cell, $\varepsilon_0$ is the permittivity of the charge free medium (air) in the cross-section and $S$ is the conductor surfaces. Since $\rho = 0$ in the medium between the conductors $S_1$ and $S_2$, the integration is performed over the inner and outer conductor surfaces $S_1$ and $S_2$, respectively. The following boundary conditions are satisfied by the potential function $V(x,y)$:

$$V(x,y) = 0 \text{ over the cell walls } S_2 \quad \ldots (3.4a)$$

$$V(x,y) = V_0 \text{ over the septum } S_1 \quad \ldots (3.4b)$$

3.2.1 Moment method solution

An unit length of uniform region of the TEM cell is considered for MOM solution. The periphery of the inner conductor $S_1$ and the outer walls $S_2$ are divided into $N$ small segments. The segments are numbered as $n=1,2,3,\ldots,N$ and each has length $\Delta S_n$. The scheme of numbering of the segments on the two conductors are such that, $n=1,2,3,\ldots,M$ on the septum and $n=M+1,M+2,\ldots,N$ on the outer conductor ($M<N$). The length of all the segments are not necessarily the same for all cases as shown in Fig. 3.1. It is assumed that $\rho$ is constant over
Mathematically, \( \rho \) is expressed in terms of pulse function \( f_n \) as

\[
\rho = \sum_{n=1}^{N} \alpha_n f_n; \quad f_n = 1 \text{ on } \Delta S_n \quad \text{.....(3.5)}
\]

\[
= 0 \text{ elsewhere}
\]

where \( \alpha_n \) represents the constant coefficient corresponding to each \( \Delta S_n \) and is equal to the charge per unit length and per unit width of an element \( \Delta S_n \). We define \( M \) unit pulse functions along the periphery of the septum and \( N-M \) unit pulse functions along the inner periphery of the outer conductor. Wilton and Govind (1977) have shown that the pulse basis set enjoys a number of advantages in a numerical determination of the source function. The match points are logically chosen at the centre of the pulses so that the observation points are not taken directly at the edges of the septum where the charge can be singular due to the edge condition (Jones D.S. 1964). Equations (3.4) and (3.5) yield

\[
V(x,y) = \frac{1}{\varepsilon_0} \sum_{n=1}^{N} \iint_{S} G(x,y|x',y') \alpha f_n \, ds' \quad \text{.....(3.6)}
\]

Since \( f_n = 1 \) on \( \Delta S_n \) and zero elsewhere, applying point matching technique, Equation (3.6) reduces to

\[
V(x_m,y_m) = \frac{1}{\varepsilon_0} \sum_{n=1}^{N} \alpha_n (x_m,y_m) \int_{\Delta S_n} G(x_m,y_m|x_n,y_n) \, ds_n \quad \text{.....(3.7)}
\]

where \( (x_m,y_m) \) and \( (x_n,y_n) \) represent the co-ordinates of the centre of observation and source points on the conductors, respectively. \( m=1,2,\ldots,N \), and \( n=1,2,\ldots,N \). The Equation (3.7) can be written in matrix form (Harrington 1969):

\[
\{V\} = \frac{1}{\varepsilon_0} \left[ G_{mn} \right] \{\alpha_n\}
\]

or,
\[ \{\alpha_n\} = \varepsilon_0 \left[ G_{mn} \right]^{-1} \{V\} \quad \ldots(3.8) \]

where \( V = V_0 \) for segments \( n = 1, 2, \ldots, M \), on the septum and \( V = 0 \) for segments \( n = M + 1, \ldots, N \), on the outer conductor. Here the \((m, n)\) element of the matrix \( [G_{mn}] \) is given by

\[ G_{mn} = \int_{\Delta S_n} G(x_m, y_m | x_n, y_n) \, ds_n \quad \ldots(3.9) \]

The charge distribution \( \alpha_n \) is computed from knowledge of \( [G_{mn}]^{-1} \) and the values of \( V \) using Equations (3.8) and (3.9). Matrix inversion is carried out by partitioning \( [G_{mn}] \) into submatrices in view of taking the effect of two conductors into account in finding the potential. The problem is similar to that worked out by Harrington (Chapter 2, 1969) for parallel plate capacitance. The capacitance is expressed in terms of total charge on one of the conductors, say, septum and the potential difference between the two conductors:

\[ C = \frac{\sum_{n=1}^{M} \alpha_n \Delta S_n}{V_0} \quad \ldots(3.10) \]

The characteristic impedance \( Z_0 \) can be computed from Equations (3.1) and (3.10).

Using Equation (3.6) the components of the electric field and the magnetic field in the cross section can be expressed as

\[ E_x(x, y) = -\frac{\partial V}{\partial x} = -1/\varepsilon_0 \sum_{n=1}^{N} \alpha_n (x_n, y_n) \int_{\Delta S_n} \frac{\partial G(x, y | x_n, y_n)}{\partial x} \, ds_n \quad \ldots(3.11) \]
\[ E_y(x,y) = - \frac{\partial V}{\partial y} = - \frac{1}{\varepsilon_0} \sum_{n=1}^{N} \alpha_n \left( x_n, y_n \right) \int_{\Delta S_n} \frac{\partial G(x,y|x_n,y_n)}{\partial y} \, ds_n \] 
\[ \ldots (3.12) \]

\[ H_x(x,y) = - \sqrt{\frac{\varepsilon_0}{\mu_0}} E_y(x,y) \] 
\[ \ldots (3.13) \]

\[ H_y(x,y) = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_x(x,y) \] 
\[ \ldots (3.14) \]

where the TEM cell is considered as a loss-less transmission line with propagation constant \( \beta = \omega \sqrt{\frac{\mu_0 \varepsilon_0}{\mu_0}} \).

The field components can be evaluated by taking the source points \((x_n, y_n)\) on the conductor surfaces and the observation points \((x, y)\) in the cross-section region between the septum and the outer conductor. Since the entire periphery of the septum is considered in this analysis, the effects of the thickness and the width of the septum are taken into account in computations of impedance and fields.

### 3.2.2 Evaluation of charge, impedance and field distributions

In order to consider the effect of the septum thickness, which can vary from an extremely small \((t/w \to 0)\) to a large value, non-uniform segmentation on the periphery of the septum is adapted in this analysis. In selecting a segment we make engineering judgement that the charge distribution is almost uniform at the centre of the bottom and top surfaces of the septum and it takes higher gradient towards the edges of the septum. Hence the length of the segment at the edges of the septum should be small and that at the centre may be larger. This avoids unnecessarily large number of segments on the bottom and top surfaces of the septum which would
have been, otherwise, taken for uniform segmentation with segment size equal to the septum thickness 2t or its integral fraction when the solution converges. In this numerical computation a pulse basis solution is used in which it is assumed that the pulses at the edge or bend tend to represent numerically the correct average charge contained in the edge/bend subdomain. Since the characteristic impedance depends on the total charge contained in the septum and the uniform field regions correspond to the region around the centre of the cross-section, complicated edge treatment is not considered here. The effect of edges or bends of the conductor is taken into account by considering finer edge subdomain sizes compared to those at the centre of the bottom and top surfaces of the septum as described in the following paragraph.

In the present analysis vertical side surfaces of the septum are divided into segments of equal length of $\Delta y_n$ and the horizontal surfaces are divided into nonuniform segments of sizes $\Delta x_n = \Delta x_1 + (n-1)d$, which is progressively increasing from each of the bends to the centre of the septum in accordance with arithmetic progression (Fig. 3.1). Here $\Delta x_1$ represents the segment number 1 placed at the corner (D-w, h-t), and $d$ represents constant difference between two successive segment sizes [Bronshtein and Semendyayev 1985, pp.101-102]:

$$d = \Delta x_n - \Delta x_{n-1}$$

$$\sum_{n=1}^{M} \Delta x_n - n\Delta x_1 = 2 \cdot \frac{n(n-1)}{n(n-1)}$$

...(3.15)
Let \( n_x \) = total number (even) of segments on each of the bottom and the top horizontal surfaces of the septum, at \( y = h-t \) and \( y = h+t \), respectively.

\[ n_y = \text{total number of segments on each of the left side and the right side of the vertical surfaces of the septum, at } x_n = D-w \text{ and } x_n = D+w, \text{ respectively.} \]

\[ M = 2(n_x + n_y), \text{ the total number of segments on the periphery of the septum.} \]

Then it can be shown from Equation (3.15) that [Bronshtein and Semendyayev 1985, pp. 101-102]

\[ d = \frac{2(w-n_x/2) \Delta x_1}{n_x/2(n_x/2-1)} \quad \ldots(3.16) \]

where \( 2w \) = width of the septum. The segmentations of the outer wall may, however, be made uniform without much error. This is because the charge distribution on the outer conductor at the right angle corner is not expected to rise so sharply, as will be seen in numerical results.

Either of the following two different forms of Green's function can be used to find the solution of the Equation (3.8):

\[ G(x,y|x',y') = \sum_{p=1}^{\infty} \frac{2}{p\pi \sinh \left( \frac{p\pi b}{2a} \right)} \sin \left( \frac{p\pi x}{2a} \right) \sin \left( \frac{p\pi x'}{2a} \right) \sin \left( \frac{p\pi y}{2a} \right) \sinh \left( \frac{p\pi (2b-y')}{2a} \right); \text{ for } y \leq y' \quad \ldots(3.17) \]

\[ = \sum_{p=1}^{\infty} \frac{2}{p\pi \sinh \left( \frac{p\pi b}{2a} \right)} \sin \left( \frac{p\pi x}{2a} \right) \sin \left( \frac{p\pi x'}{2a} \right) \]
\[
\sinh \left( \frac{pny'}{2a} \right) \sinh \left( \frac{pn(2b-y)}{2a} \right); \text{ for } y>y' \quad \ldots \ldots \ldots (3.18)
\]

and

\[
G(x,y|x',y') = \frac{-1}{2\pi} \ln \left[ \frac{(x-x')^2 + (y-y')^2}{\left( \frac{\Delta S_n}{\Delta x} \right) ^2} \right] \quad \ldots \ldots \ldots (3.19)
\]

In the first case, for Equations (3.17) and (3.18), Green's function has been derived considering the boundaries of the TEM cell (Collin 1960, and chapter 2) and hence charge distribution on one of the conductors, say, septum only is needed to be found from Equation (3.8). Consequently, the number of unknowns \( p_n \), \( n=1,2,\ldots M \), need only correspond to division of the boundary of the septum alone and the size of the matrix reduces considerably (MxM). However, it will be seen that because of infinite summation in Green's function, which usually converges after large number of terms (about 2000), computer time requirement is very high, although, the computation can be possible using personal computer which need not have very large memory. On the other hand, Equation (3.19) represents Green's function corresponding to an infinite current filament in free space and can be used in the charge free space between the two coaxial line conductors, one containing positive charge and the other containing equal amount of negative charge so that net charge in the system is zero. In this case, the number of unknowns \( p_n \), \( n=1,2,3,\ldots M, M+1,\ldots N \), need correspond to division of the boundaries of the septum as well as the inner surface of the outer conductor and the overall matrix size in Equation (3.8) becomes larger, \( N \times N \). But each element of the matrix,

\[
G_{mn} = \frac{-1}{2\pi} \int \frac{\ln \left[ (x_m-x_n)^2 + (y_m-y_n)^2 \right]}{\Delta S_n} \, ds \quad \ldots \ldots \ldots (3.20)
\]

can be evaluated in seconds and total computer time
requirement is small although much larger memory is required for storing very large number of matrix elements. This requires very fast computer for finding the numerical solution. The following paragraphs give the details of the analysis using above two different forms of Green's functions.

Using above scheme of segmentations and Green's functions (3.17) and (3.18), Equation (3.7) can be written as

\[
\epsilon_{o \nu_0}(x_m, y_m) = \sum_{n=1}^{n_x} \alpha_n(x_n, h-t) \int G(x_m, y_m | x_n, h-t) dx_n
\]

\[
+ \sum_{n=n_x+1}^{n_x+n_y} \alpha_n(D+, y_n) \int G(x_m, y_m | D+, y_n) dy_n
\]

\[
+ \sum_{n=n_x+n_y+1}^{2(n_x+n_y)} \alpha_n(x_n, h+t) \int G(x_m, y_m | x_n, h+t) dx_n
\]

\[
+ \sum_{n=2n_x+n_y+1}^{2(n_x+n_y)} \alpha_n(D-, y_n) \int G(x_m, y_m | D-, y_n) dy_n
\]

\[
\ldots (3.21)
\]

In the above integral over \(\Delta x_n\), when \(y_n=h-t\) on the bottom surface of the septum, \(y_m<y_n\) and the expression (3.18) is used for Green's function. Similarly, for the top surface, \(y_n=h+t\) and Green's function (3.17) is used for \(y_m>y_n\). In the integrals over \(\Delta y_n\) on the side surfaces at \(x=D+w\), appropriate Green's function is used depending on the cases when \(y_m<y_n\) or \(y_m>y_n\). Under these conditions above four integrals in Equation (3.21) can be written as
\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sin(pnx_n/2a)dx_n
\]
\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dy_n; \quad \text{for } y_n \leq y_m
\]
\[
\int_{y_n - \Delta y_n / 2}^{y_n + \Delta y_n / 2} \sinh(pny_m/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
= \sum_{p=1}^{\infty} \frac{2}{p\pi} \sin (p\pi x_m/2a).f_{yt}.fxb
\]

\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dx_n
\]
\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
\int_{y_n - \Delta y_n / 2}^{y_n + \Delta y_n / 2} \sinh(pny_m/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
= \sum_{p=1}^{\infty} \frac{2}{p\pi} \sin (p\pi x_m/2a).f_{yt}.fxb
\]

\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dx_n
\]
\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
\int_{y_n - \Delta y_n / 2}^{y_n + \Delta y_n / 2} \sinh(pny_m/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
= \sum_{p=1}^{\infty} \frac{2}{p\pi} \sin (p\pi x_m/2a).f_{yt}.fxb
\]

\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dx_n
\]
\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
\int_{y_n - \Delta y_n / 2}^{y_n + \Delta y_n / 2} \sinh(pny_m/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
= \sum_{p=1}^{\infty} \frac{2}{p\pi} \sin (p\pi x_m/2a).f_{yt}.fxb
\]

\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dx_n
\]
\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
\int_{y_n - \Delta y_n / 2}^{y_n + \Delta y_n / 2} \sinh(pny_m/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
= \sum_{p=1}^{\infty} \frac{2}{p\pi} \sin (p\pi x_m/2a).f_{yt}.fxb
\]

\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dx_n
\]
\[
\int_{x_n - \Delta x_n / 2}^{x_n + \Delta x_n / 2} \sinh(pnx_n/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
\int_{y_n - \Delta y_n / 2}^{y_n + \Delta y_n / 2} \sinh(pny_m/2a)dy_n; \quad \text{for } y_m \leq y_n
\]
\[
= \sum_{p=1}^{\infty} \frac{2}{p\pi} \sin (p\pi x_m/2a).f_{yt}.fxb
\]
\[
\int \sum_{p=1}^{\infty} A_p \sin\left(\frac{p\pi n}{2a}\right) \sin\left(\frac{p\pi r}{2a}(D-w)\right) \sinh\left(\frac{p\pi}{2a}(2b-y)\right) \\
\Delta y_n \\
x_n = D - w
\]

\[
y_n + \Delta y_n /2
\int \sinh\left(\frac{p\pi y_n}{2a}\right) dy_n; \text{ for } y_m > y_n
\]

\[
y_n - \Delta y_n /2
\]

\[
\int \sum_{p=1}^{\infty} \sin\left(\frac{p\pi x_m}{2a}\right) \sin\left(\frac{p\pi}{2a}(D+w)\right) \sinh\left(\frac{p\pi}{2a}(2b-y)\right)
\]

\[
\Delta y_n
\]

\[
x_n = D - w
\]

\[
fy_l 2; \text{ for } y_m > y_n
\]

\[
\ldots (3.25)
\]

Similarly,

\[
\int \sum_{p=1}^{\infty} \sin\left(\frac{p\pi x_m}{2a}\right) \sin\left(\frac{p\pi}{2a}(D+w)\right) \sinh\left(\frac{p\pi y_m}{2a}\right)
\]

\[
\Delta y_n
\]

\[
x_n = D - w
\]

\[
fy_r 1; \text{ for } y_m < y_n
\]

\[
\ldots (3.26)
\]

\[
\sum_{p=1}^{\infty} \sin\left(\frac{p\pi x_m}{2a}\right) \sin\left(\frac{p\pi}{2a}(D+w)\right) \sinh\left(\frac{p\pi}{2a}(2b-y)\right)
\]

\[
\Delta y_n
\]

\[
x_n = D - w
\]

\[
fy_r 2; \text{ for } y_m > y_n
\]

\[
\ldots (3.27)
\]

Where

\[
A_p = \frac{2}{p\pi \sinh(p\pi b/a)}
\]

\[
\ldots (3.28)
\]

\[
f_y b = \frac{\sinh\left(\frac{p\pi}{2a}(2b-y)\right) \sinh\left(\frac{p\pi}{2a}(h-t)\right)}{\sinh(p\pi b/a)}
\]

\[
\ldots (3.29)
\]

\[
\cos\left(\frac{p\pi}{2a}(x_n - \Delta x_n/2)\right) - \cos\left(\frac{p\pi}{2a}(x_n + \Delta x_n/2)\right)
\]

\[
fx_b = \frac{p\pi /2a}{p\pi /2a}
\]

\[
\ldots (3.30)
\]
\[
\begin{align*}
\text{fyt} &= \frac{\sinh(p\eta_m/2a) \sinh \{p\pi/2a(2b-h-t)\}}{\sinh(p\eta/a)} \quad \ldots(3.31) \\
\text{fxt} &= \text{fxb} \quad \ldots(3.32) \\
\text{fyl}_1 &= \frac{\cosh\{p\pi/2a(2b-\eta_0+\Delta y_n/2)\} - \cosh\{p\pi/2a(2b-\eta_0-\Delta y_n/2)\}}{\eta/2a \cdot \sinh(p\eta/b/a)} \quad \ldots(3.33) \\
\text{fyl}_2 &= \frac{\cosh\{p\pi/2a(\eta_0+\Delta y_n/2)\} - \cosh\{p\pi/2a(\eta_0-\Delta y_n/2)\}}{\eta/2a \cdot \sinh(p\eta/b/a)} \quad \ldots(3.34) \\
\text{fyl}_1 &= \text{fyl}_2 \quad \ldots(3.35) \\
\text{fyl}_2 &= \text{fyl}_2 \quad \ldots(3.36)
\end{align*}
\]

Thus the elements of the matrix \( [G_{mn}] \) are evaluated from Equations (3.9), (3.17), (3.18), (3.21) - (3.36) for the solution of Equation (3.8) for charge distribution. The characteristic impedance \( Z_0 \) can be found from Equations (3.1) and (3.2) for offset septum having finite thickness.

Substituting appropriate Green's function in Equations (3.11) and (3.12) and simplifying, the field components at any point \((x,y)\) in the cross section of the cell can be written as

\[
\begin{align*}
E_x(x,y) &= \frac{1}{ao} N \sum_{n=1}^{\infty} a_n(x_n,y_n) \sum_{p=1}^{\infty} \cos(p\eta x/2a) \sin(p\eta x_n/2a) \\
&= \sum_{n=1}^{\infty} \int \frac{\sinh(p\eta/2a(2b-\eta_y)}{\sinh(p\eta/b/a)} \cdot ds_n \quad \text{for } y \leq \eta_n \quad \ldots(3.37)
\end{align*}
\]
\[ E_y(x,y) = \frac{1}{\varepsilon} \sum_{\alpha n=1}^{N} \alpha_n (x_n, y_n) \int_{\Delta S_n} \left[ \sum_{p=1}^{\infty} \cos(pnx/2a) \sin(pnx_n/2a) \right. \]
\[ \left. \frac{\sinh(pny/2a)}{\cosh(p/2a(y+n/2a))} \right] \frac{\sinh(pnb/a)}{\sinh(p/2a(y+n/2a))} ds_n; \text{ for } y > y_n \]

\[ \ldots (3.38) \]

where upper case letters are used for \( x \)-component and lower case letters are for \( y \)-component of the field.

Integrals in Equations (3.37) and (3.38) are evaluated in similar way as it is done in Equation (3.21). To find the field components inside the cell the entire cross-section of the cell is divided into four regions I, II, III and IV as shown in Fig. 3.1. The electric field components can then be expressed by

Region I \((0 \leq y \leq h-t, 0 \leq x \leq 2a)\):

\[ E_y(x,y) = \]
\[ \frac{1}{\varepsilon} \sum_{n=1}^{n_x} \left[ \sum_{p=1}^{\infty} X_p(x) \right. \frac{\sinh(p/2a(2b-h-t))}{\sinh(p/2a(y+n/2a))} \]
\[ \left. \times \frac{\sinh(p/2a(y+n/2a))}{\sinh(p/2a(y+n/2a))} \right] \]

\[ = \sum_{n=n_x+1}^{n_x+n_y} \alpha_n (D+w, y_n) \frac{\sinh(p/2a(D+w))}{\sinh(p/2a(y+n/2a))} \]

\[ \times X_p(x) Y_p(y) \sin \{pn/2a(D+w)\} \]

\[ \times fyr1 \]
\[ \sum_{n=n_x+n_y+1}^{2(n_x+n_y)} \alpha_n (x_n, h+t) \frac{\sinh(p/2a(2b-h+t))}{\sinh(p/2a(y+n/2a))} \]
\[ \times X_p(x) Y_p(y) \sin \{pn/2a(2b-h+t)\} \]

\[ \times fxb \]
Region II \( (h+t \leq y \leq 2b, \ 0 \leq x \leq 2a) \):

\[
E_{y}(x,y) = 
\]

\[
\frac{1}{a^{2}} \left[ \sum_{n=1}^{n_{x}} \alpha_{n(x_{y},y-t)} \sum_{p=1}^{\infty} X_{p}(x) Y_{p1}(y) \sin \left\{ \frac{\pi}{2a} (h-t) \right\} \right] 
\]

\[
* f_{xb} \]

\[
\sum_{n=n_{x}+1}^{n_{x}+n_{y}} \alpha_{n(D+w,y_{n})} \sum_{p=1}^{\infty} X_{p}(x) Y_{p2}(y) \sin \left\{ \frac{\pi}{2a} (D+w) \right\} 
\]

\[
* f_{yr2} \]

Region III \( (h-t < y < h+t, \ 0 \leq x \leq D-w) \) and

Region IV \( (h-t < y < h+t, \ D+w \leq x \leq 2a) \):

\[
\]
\[ E_y(x,y) = \]

\[
- \frac{1}{a e} \left[ \sum_{n=1}^{n_x} \sum_{p=1}^{\infty} \alpha_n(x_n, y-t) X_p(x) Y_{p2}(y) \frac{\sinh{\frac{pn}{2a}(h-t)}}{\sinh{pn(b/a)}} \right]
\]

\[
+ \sum_{n=n_x+1}^{n_x+n_y} \alpha_n(x_n, h+t) \sum_{p=1}^{\infty} X_p(x) Y_{p1}(y) \sin{\frac{pn}{2a}(D-w)}
\]

\[
E_x(x,y) = \]

\[
\frac{1}{a e} \left[ \sum_{n=1}^{n_x} \sum_{p=1}^{\infty} \alpha_n(x_n, y-t) X_p(x) Y_{p2}(y) \frac{\sinh{\frac{pn}{2a}(h-t)}}{\sinh{pn(b/a)}} \right]
\]

\[
+ \sum_{n=n_x+1}^{n_x+n_y} \alpha_n(x_n, h+t) \sum_{p=1}^{\infty} X_p(x) Y_{p2}(y) \sin{\frac{pn}{2a}(D-w)}
\]

\[ \text{for } y \leq y_n \] ....(3.41)
\[
2n_x^{n_y} + \sum_{n=n_x^{n_y}+1}^{2n_x^{n_y}} \alpha_n(x_n, y_n + t) \sum_{p=1}^{\infty} X_p(x) Y_{p1}(y) \frac{\sinh\left\{\frac{p\pi}{2a(2b-h+t)}\right\}}{\sinh(pnb/a)}
\]

\[
2(n_x^{n_y}) + \sum_{n=2n_x^{n_y}+1}^{n_x^{n_y}} \alpha_n(1\tilde{1}, y_n) \sum_{p=1}^{\infty} X_p(x) Y_{p2}(y) \sin \left\{\frac{p\pi}{2a(1\tilde{1}-w)}\right\}
\]

\[\text{for } y>y_n \] .... (3.42)

where

\[
X_p(x) = \frac{\cos(pnx/2a)}{\sin(pnx/2a)}; \quad \ldots (3.43)
\]

\[
Y_{p1}(y) = \frac{\sinh(pny/2a)}{\cosh(pny/2a)}; \quad \ldots (3.44)
\]

\[
Y_{p2}(y) = \frac{\sinh(pn/2a(2b-y))}{\cosh(pn/2a(2b-y))}; \quad \ldots (3.45)
\]

Equations (3.39)-(3.45) provide the electric field distribution in the entire cross-section of the TEM cell. Magnetic field distributions can also be obtained from Equations (3.39)-(3.45) and using the relations (3.13) and (3.14).

When Green's function of Equation (3.19) is used, the integral (3.20) becomes singular for a finite number of points when \( x_m = x_n \) and \( y_m = y_n \) simultaneously. This represents the potential at the centre of each AS due to a unit charge over its length and these form the diagonal elements of the matrix \( [G_{mn}] \). It can be shown that (Spiegel 1987) the diagonal elements yield after excluding the point of singularity \( (x_m = x_n, y_m = y_n) \):

\[
G_{nn} = \frac{\Delta S_n}{2\pi} \left( 1 - \frac{\Delta S_n}{2} \right) \quad \ldots (3.46)
\]
and off-diagonal elements

\[ G_{mn} = -\frac{\Delta S_n}{2\pi} \ln \left( \frac{(x_m-x_n)^2 + (y_m-y_n)^2}{(x_m-x_n)^2 + (y_m-y_n)^2} \right) \] ....(3.47)

The charge distribution on the conductors can be found from the Equations (3.8), (3.46) and (3.47). Then the characteristic impedance is obtained from Equation (3.1) and (3.10).

The resultant expressions for the x and y components of the electric field, \( E_x \) and \( E_y \), in the cross-section can be written in this case as (Spiegel 1987):

\[ E_x = \frac{1}{2\pi \varepsilon_0} \sum_{n=1}^{N} \rho_n \Delta S_n \cdot \frac{x-x_n}{(x-x_n)^2 + (y-y_n)^2} \] ....(3.48)

and

\[ E_y = \frac{1}{2\pi \varepsilon_0} \sum_{n=1}^{N} \rho_n \Delta S_n \cdot \frac{y-y_n}{(x-x_n)^2 + (y-y_n)^2} \] ....(3.49)

where the integration is achieved by merely adding the contributions from each of the elements of charge \( \rho_n \Delta S_n \) on both the conductors.

3.3 RESULTS AND DISCUSSION:

The charge distribution on the conductors and the characteristic impedance of symmetric and asymmetric TEM cells with septum having finite thickness are computed for different dimensions of the cell. Computations are carried out in a 486 computer system. For Green's function of Equation (3.17) and (3.18), the convergence of the solution for impedance or the total charge on the
conductor is obtained for a summation of about 2000 terms. For a given thickness of the septum, computation time for the determination of charge and a data for impedance is approximately twenty minutes for 60 segments, while the computer time requirement for the use of Green's function of Equation (3.19) is approximately less than 2 minutes for a total number of segments of 180. Therefore, the use of former Green's function is limited in numerical analysis of TEM cell due to requirement of large computer time.

The impedance results obtained by MOM for zero thickness of the septum are compared with those of chapter 2 and other methods reported by Costamagna and Fanni (1990) in Tables 3.1 and 3.2. The same conductor sizes of the cell which were selected by these authors are used in the present computations for the purpose of comparison. It is seen that MOM results agree well with those of Tippet and Chang (1976 a), Cai and Li (1989) and average values of NSC results reported by Costamagna and Fanni (1990). Typical relative distributions of electric charge normalised with respect to the maximum value are shown in Figs. 3.2 - 3.4 for both symmetric and asymmetric TEM cells. It is seen that charge densities are much higher at the corners of the

<table>
<thead>
<tr>
<th>Table 3.1: COMPARISON OF CHARACTERISTIC IMPEDANCE (Ohm) FOR SYMMETRIC TEM CELL (t=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aspect ratios : b/a = 0.333, h/b = 1.0, D/a = 1.0</td>
</tr>
<tr>
<td>w/a               0.2      0.267     0.333     0.667     0.8</td>
</tr>
<tr>
<td>Energy            90.64    75.93     65.62     39.31     32.94</td>
</tr>
<tr>
<td>MOM               90.35    75.62     65.54     38.98     31.23</td>
</tr>
<tr>
<td>Exact             90.39    75.81     65.27     38.13     31.96</td>
</tr>
<tr>
<td>HOP               90.07    75.94     65.59     38.10     31.98</td>
</tr>
<tr>
<td>NSC               90.59    75.81     65.35     38.12     31.97</td>
</tr>
</tbody>
</table>
Table 3.2: COMPARISON OF CHARACTERISTIC IMPEDANCE UNDER ASYMMETRIC CONDITION (t=0)

<table>
<thead>
<tr>
<th>Aspect ratios</th>
<th>b/a</th>
<th>w/a</th>
<th>D/a</th>
<th>h/b</th>
<th>Impedance (Ohm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Energy MOM HOP NSC (Average)</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8</td>
<td>1.0</td>
<td>1.00</td>
<td></td>
<td>55.14 54.82 54.71 54.67</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8</td>
<td>1.0</td>
<td>1.50</td>
<td></td>
<td>48.61 48.97 48.86 48.44</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8</td>
<td>1.0</td>
<td>1.75</td>
<td></td>
<td>35.94 35.83 35.68 35.41</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6</td>
<td>1.2</td>
<td>1.00</td>
<td></td>
<td>71.96 71.35 71.62 70.87</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6</td>
<td>1.2</td>
<td>1.20</td>
<td></td>
<td>70.76 70.12 70.04 69.76</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6</td>
<td>1.2</td>
<td>1.20</td>
<td></td>
<td>66.65 66.56 66.84 65.93</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6</td>
<td>1.2</td>
<td>1.00</td>
<td></td>
<td>57.89 58.12 58.01 57.47</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6</td>
<td>1.2</td>
<td>1.80</td>
<td></td>
<td>40.44 39.52 39.89 39.69</td>
</tr>
</tbody>
</table>

Energy - Energy consideration in chapter 2
MOM - MOM results of present chapter
Exact - Conformal method (Tippet and Chang 1976)
HOP - Higher-order power form of charge distribution (Cai and Li 1989)
NSC - Numerical inversion of the SC conformal transformation (Costamagna and Fanni 1990)

The magnitudes of the x and y components of the electric field are normalised with respect to $V_o/b$. The normalisation factor $V_o/b$ corresponds to the uniform TEM mode field component $E_y$ at the centre of the cross-section between the septum and its parallel ground plane under symmetric condition. The relative
Fig. 3.2 Relative distribution of electric charge density on the septum and the inner periphery of the outer conductor of a symmetric TEM cell (b/a=1.0, w/a=0.6 and t/w=0.0)
Fig. 3.3 Relative distribution of electric charge density on the septum and the inner periphery of the outer conductor of an asymmetric TEM cell.

(b/a=1.0, w/a=0.6, h/b=1.6, D/a=1.3, t/w=0.0)
Fig. 3.4 Relative distribution of electric charge density on the outer periphery of the septum having non-zero but finite thickness (b/a=0.5, w/a=0.6 and t/w=0.04)

a) Symmetric TEM cell
b) Asymmetric TEM cell (b/b=1.6, D/a=1.3)
Fig. 3.5 Characteristic impedance vs $t/w$  
(b/a=0.5, h/b=1.0 and D/a=1.0)

Fig. 3.6 Characteristic impedance vs $w/a$  
(b/a=0.5, h/b=1.0 and D/a=1.0)
Fig. 3.7 Characteristic impedance vs $h/b$  
(b/a=0.5, D/a=1.0 and w/a=0.6)

Fig. 3.8 Characteristic impedance vs $D/a$  
(b/a=0.5, h/b=1.0 and w/a=0.6)
distributions of the electric field components in the cross-section are similar to those observed in chapter 2. This chapter shows 3-dimensional field distributions for both symmetric and asymmetric TEM cells, Fig. 3.9-3.12. For a symmetric cell the $E_y$ component is relatively constant at the centre of the cell and falls off rapidly near the edges of the septum, while $E_x$ is zero at the centre of the cell and becomes larger near the side walls. Therefore, the field strength is essentially in the vertical direction in the centre region of the symmetric cell, and the horizontal component gradually becomes dominant towards the side walls. The TEM field distribution is most uniform in the vicinity of the point $x/a=D/a, y/b=h/2b$. For offset positions of the septum the field distribution becomes asymmetric with respect to the centre of the cross-section as shown in Figs. 3.11 and 3.12.

The magnitudes of the normalised $x$ and $y$ components of the electric field for different conductor sizes of the TEM cell are shown in Tables 3.3 - 3.6. The values of the normalised $E_y$ components are shown within brackets ( ) and those of the normalised $E_x$ components evaluated at the corresponding points are shown without brackets. The regions within the dashed boundaries show approximately the optimal areas in the cross-section in which the vertical component ($E_y$) of TEM mode E-field is uniform within ±1.0 dB with respect to its reference value at the mid points between the septum and its parallel ground planes for different asymmetric locations of the septum. The horizontal component is found to be at least 20 dB below the corresponding reference vertical component in these regions. These areas indicate almost uniform plane wave regions. It is seen that for increase of vertical offset of the septum, area of field uniformity decreases. However, for
horizontal offset this effect could not be seen very large because of the limited scope of the amount of offset available due to large width of the septum and small gap regions at its two sides.

The method which has been proposed and described in this chapter has the advantage over others that no assumption is needed for specific choice of electric charge distribution on the conductors or potential distribution in the cross-section. The method is applicable to any asymmetric TEM cells having arbitrary thickness of the septum. The analysis by this method become simple and universal.
Fig. 3.9 Relative distribution of electric field component $E_y$ for symmetric TEM cell ($b/a = 0.5$, $w/a = 0.6$ and $t/w = 0.005$)

a) Region $0 \leq y \leq h-t$

b) Region $h+t \leq y \leq 2b$

Normalised to $V_0/b$
Fig. 3.10 Relative distribution of electric field component $E_x$ for symmetric TEM cell (b/a=0.5, w/a=0.6 and t/w=0.005)

a) Region $0 \leq y \leq h-t$

b) Region $h+t \leq y \leq 2b$
Fig. 3.11 Relative distribution of electric field component $E_y$ for asymmetric TEM cell ($b/a=0.5$, $h/b=1.4$, $D/a=1.2$, $w/a=0.6$ and $t/w=0.005$)

a) Region $0 \leq y \leq h-t$  
b) Region $h+t \leq y \leq 2b$
Fig. 3.12 Relative distribution of electric field component $E_x$ for asymmetric TEM cell (b/a=0.5, h/b=1.4, D/a=1.2, w/a=0.6 and t/w=0.005)

a) Region $0 \leq y < h-t$  
b) Region $h+t \leq y < 2b$
Table 3.3: Ex AND Ey COMPONENTS IN A TEM CELL OF ASPECT RATIOS: b/a=0.5, w/a=0.6, h/b=1.0, D/a=1.0 AND t/w=0.005 NORMALISED TO Vo/b.

<table>
<thead>
<tr>
<th>y/b</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Areas between dashed lines represent uniform TEM mode field regions.
Table 3.4: EX AND EY COMPONENTS IN A TEM CELL OF ASPECT RATIOS: $b/a=0.5$, $w/a=0.6$, $h/b=1.4$, $D/a=1.0$ AND $t/w=0.005$ NORMALISED TO $V_0/b$.

Areas between dashed lines represent uniform

20 log $E_y/(V_0/b)$ shown within brackets
20 log $E_x/(V_0/b)$ shown without brackets
Table 3.5: Ex AND Ey COMPONENTS IN A TEM CELL OF ASPECT RATIOS: $b/a=0.5$, $w/a=0.6$, $h/b=1.0$, $D/a=1.2$ AND $t/w=0.005$ NORMALISED TO $Y_0/b$.

Areas between dashed lines represent uniform TEM mode field regions.

![Diagram showing Ex and Ey components with normalized values.]

20 log $E_y/(Y_0/b)$ shown within brackets
20 log $E_x/(Y_0/b)$ shown without brackets

<table>
<thead>
<tr>
<th>$y/b$</th>
<th>$x/a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Note: The values for $E_x$ and $E_y$ are normalized to $Y_0/b$. The diagram shows the distribution of these components across the TEM cell with normalized values indicated within and without brackets.
Table 3.6: Ex AND Ey COMPONENTS IN A TEM CELL OF ASPECT RATIOS: b/a=0.5, w/a=0.6, h/b=1.4, D/a=1.2 AND t/w=0.005 NORMALISED TO Vo/b.

<table>
<thead>
<tr>
<th>h/b</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 log Ez/(Vo/b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 log Ex/(Vo/b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Areas between dashed lines represent uniform TEM mode field regions.

20 log Ez/(Vo/b) shown within brackets
20 log Ex/(Vo/b) shown without brackets