CHAPTER 2

ANALYSIS OF THE EFFECT OF SEPTUM THICKNESS ON IMPEDANCE OF TEM MODE OF AN ASYMMETRIC RECTANGULAR TEM CELL OPERATING AT EXTREMELY LOW AS WELL AS HIGH FREQUENCIES

2.1 INTRODUCTION

The chapter presents an analysis of the effect of septum thickness on characteristic impedance $Z_0$ of TEM mode of an asymmetric TEM cell. The cell has small gaps between the septum and the sidewalls perpendicular to the septum and operates at extremely low as well as high frequencies of the TEM mode. The vector magnetic potential integral is solved for the assumed current distribution on the septum to determine the electric and magnetic field components in the cross-section at low and high frequencies of the TEM mode. Green's function appearing in the integral expression of the vector magnetic potential is the same as that for the scalar electric potential (Spiegel et al 1987). Since current flowing in the septum is enclosed by the outer conductor, Green's function is derived taking the effect of boundaries and the thickness of the septum. Under this condition only current distribution on one of the conductors, viz. the septum, is considered for TEM mode analysis. The distributed electrical circuit parameters of the cell are determined in terms of energy stored in the cross-section and the energy losses, both in the medium (air) and in the conducting walls. The characteristic impedance is expressed in terms of these
circuit elements. The nature of variation of characteristic impedance of the cell as a function of thickness and width of the septum for its different offset positions are determined. Variations in characteristic and wave impedances with frequency of operation are also shown. The electric field distribution in the cross-section for different offset positions of the septum are also investigated and represented graphically. The present analysis is useful for the design and use of asymmetric TEM cell with septum having small but finite thickness.

2.2 FORMULATION OF THE IMPEDANCE AND FIELDS USING GREEN'S FUNCTION AND ENERGY CONSIDERATIONS

2.2.1 Expressions for line parameters and field components

The region of uniform cross-section of a TEM cell is a rectangular coaxial strip transmission line (RCTL) where almost uniform TEM mode fields exist, Fig. 2.1.

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**Fig. 2.1** Cross-section of uniform region of a typical TEM cell
In the present analysis the assumption is made that each of the tapered transition sections between the coaxial connector and the uniform section at the two ends of the cell produces perfect impedance match in the entire length of the line. The field solution inside the cell can be obtained from knowledge of either scalar electric potential or from vector magnetic potential and using Maxwell's equations. The expression for the characteristic impedance valid for all frequencies is given by

\[ Z_0 = \sqrt{\frac{R + j\omega L}{\gamma + j\omega C}} \]  

(2.1)

where \( R, L, \gamma \) and \( C \) are the distributed line parameters of the TEM transmission line per unit length. It will be seen that these parameters are conveniently evaluated from the vector magnetic potential \( \vec{A}(x,y) \) inside the cell which is related to the current density \( \vec{I}(x',y') \) on the septum, as

\[ \vec{A}(x,y) = \mu_0 \int_{S_1} \vec{I}(x',y') \cdot G(x,y|x',y') \, dl'; \]  

(2.2)

Where

\[ \mu_0 = \text{permeability of the medium inside the cell.} \]

\[ S_1 = \text{Septum surface per unit length.} \]

\( (x,y) \) and \( (x',y') \) = field point and source point, respectively

\[ G(x,y|x',y') = \text{Green's function.} \]

When the cell is terminated in its characteristic impedance a travelling wave with a propagation constant \( \beta \)
exists in the cell. For high and low frequency regions $\beta$ is expressed \cite{Spiegel et al 1987}, respectively, by

$$\beta = \frac{\omega \mu_0 L}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \left( \frac{\epsilon_0}{\mu_0} \right)^{1/2} \quad \text{(2.3)}$$

$$\beta = \sqrt{\frac{\mu_0}{\epsilon_0}} \left[ 0.5 \left( 1 + \left( \frac{R}{\omega L} \right)^2 \right)^{1/2} \right] \quad \text{(2.4)}$$

Assuming only longitudinal current flow $I_z$ for TEM mode, the magnetic and electric fields are related by

$$\bar{H} = (\bar{E} \times \hat{z}) (\epsilon_0 \omega / \beta) \quad \text{(2.5)}$$

where $\hat{z}$ represents unit vector in $z$-direction. At high frequency regions Equation (2.5) reduces to

$$\bar{H} = (\bar{E} \times \hat{z}) / 120\pi \quad \text{(2.6)}$$

Therefore, the electric field can be determined from the magnetic field or vice versa from Equations (2.5) or (2.6) depending on the frequency region of interest. The expressions for the components of the electric and the magnetic fields in the cross-section are derived from the following relations

$$H_x = \frac{1}{\mu_0} \frac{\partial A_z}{\partial y}, \quad H_y = -\frac{1}{\mu_0} \frac{\partial A_z}{\partial x} \quad \text{(2.7)}$$

$$E_x = \frac{\beta}{\omega \epsilon_0} H_y, \quad E_y = \frac{\beta}{\omega \epsilon_0} H_x \quad \text{(2.8)}$$

where $\bar{A} = \hat{z} A_z$ in Equation (2.2).

At high frequencies, the current and magnetic field are confined to a small depth (skin depth) near the surface of the conductor. At these frequencies the inductance $L$ can be evaluated from the expressions for
the magnetic energies per unit length stored in the cross-section (Collin 1960)

\[
L = \frac{\mu_o \iint |\tilde{H}|^2 \, dx \, dy}{\left[ \oint_{S_1} I_a(x',y') \, dl' \right]^2}
\]

where the integration domain of numerator in Equation (2.9) is the cross-section of the cell between the inner conductor \( S_1 \) and the outer conductor \( S_2 \) and the denominator represents line integral over the periphery of the inner conductor in the transverse plane for unit length along the longitudinal direction. At lower frequencies, when the conductivity is finite, there exists an incremental inductance caused by the penetration of magnetic field into the conductor. This incremental inductance has a low value when the conductivity of the material is very high and is, therefore, neglected in the present analysis. The capacitance per unit length is then given by

\[
C = \frac{1}{v_o^2 L}
\]

where \( v_o \) is the velocity of light in medium inside the cell.

The series resistance \( R \) and shunt conductance \( \gamma \) can be found from the power losses in the inner and outer conductors and in the medium inside the cell (Collin 1960), respectively, as
\[ R = \frac{\int_{S_1+S_2} |H|^2 \, dl}{\left[ \int_{S_1} \Phi_2(x',y') \, dl' \right]^2} \] 

\[ \gamma = \sigma_o \frac{C}{\varepsilon_o} \] 

where \( \sigma \) = electrical conductivity of the inner and the outer conductors and \( \sigma_o = 1 \times 10^{-14} \) mhos/m, the electrical conductivity of free space (Spiegel 1987). The evaluation of the circuit parameters in terms of vector potential demands derivation of the Green's function \( G \).

### 2.2.2 Determination of Green's Function

Green's function depends on the configuration of the cross-section of the cell and is identical for both scalar electric potential \( V \) and vector magnetic potential \( A \) for TEM mode (Spiegel 1987). The expression for Green's function \( G(x,y|x',y') \) is derived following the method suggested by R.E. Collin (1960, pp 160-162) from the Poisson's Equation for an unit delta function source charge at \( (x',y') \):

\[ \Phi_t^2 G(x,y|x',y') = \delta(x-x') \delta(y-y') \] 

and using the following boundary conditions:

\[ V(0,y) = 0 = V(2a,y); 0 \leq y \leq 2b \]  
\[ V(x,0) = 0 = V(x,2b); 0 \leq x \leq 2a \]

At all points except \( (x',y') \), Green's function satisfies Laplace's Equation

\[ \Phi_t^2 G = 0 \]
Since Laplace's Equation is separable in $x,y$ coordinates, and $G(x,y|x',y')$ is symmetrical in $(x,y)$ and $(x',y')$, one can write

$$G = \sum_{n=1}^{\infty} f_n(x)f_n(x')g_n(y)g_n(y'); \quad n=1,2,3,\ldots,\infty \quad \ldots(2.16)$$

The Fourier series expansions of $\delta(x-x')$ and $\sum f_n(x)f_n(x')$ which satisfy the boundary conditions (2.14a) at $x = 0$ and $2a$, become (R.E. Collin 1960):

$$\delta(x-x') = \frac{1}{a} \sum_{n=1}^{\infty} \sin \left( \frac{n \pi x}{2a} \right) \sin \left( \frac{n \pi x'}{2a} \right); \quad \ldots(2.17)$$

$$\sum f_n(x)f_n(x') = \sum_{n=1}^{\infty} \sin \left( \frac{n \pi x}{2a} \right) \sin \left( \frac{n \pi x'}{2a} \right); \quad \ldots(2.18)$$

Substituting Equations (2.17) and (2.18) into the differential Equation (2.13) and simplifying, it can be expressed that

$$g_n(y') \left[ \frac{d^2 g_n(y)}{dy^2} - \left( \frac{n \pi}{2a} \right)^2 g_n(y) \right] = (-) \frac{\delta(y-y')}{a}; \quad \ldots(2.19)$$

It can be shown that $\frac{dg_n(y)}{dy}$ is equal to the negative electric field component which terminates on the unit line charge and therefore, it is discontinuous at $y=y'$ by the amount:

$$\frac{dg_n(y)}{dy} \bigg|_{y'-}^{y'+} = - \frac{1}{a g_n(y')}; \quad \ldots(2.20)$$
The solution of Equation (2.19) can be taken as

\[ g_n(y) = A_n \ e^{n\pi y/2a} + B_n \ e^{-n\pi y/2a}; \ y \leq y' \quad \ldots (2.21) \]

\[ g_n(y) = A_n^2 \ e^{(n\pi/2a)(2b-y)} + B_n^2 \ e^{-(n\pi/2a)(2b-y)}; \ y > y' \quad \ldots (2.22) \]

The unknown coefficients \( A_n \)'s and \( B_n \)'s will depend on \( y' \). These constants can be determined by applying the boundary condition \( g_n(y) = 0 \) at \( y = 0 \) and \( 2b \), using continuity condition of \( g_n(y) \) at \( y = y' \) and the discontinuity condition (2.20) at \( y = y' \). This leads to Green's function

\[
G(x, y | x', y') = \sum_{n=1}^{\infty} \frac{2}{n\pi} \ \frac{\sin \left( \frac{n\pi x}{2a} \right)}{\sinh \left[ \frac{n\pi b}{a} \right]} \ \frac{\sin \left( \frac{n\pi x'}{2a} \right)}{\sinh \left[ \frac{n\pi b'}{a} \right]} \ \sinh \left[ \frac{n\pi y}{2a} \right] \ \sinh \left[ \frac{n\pi (2b-y')}{2a} \right]; \ \text{for} \ y \leq y' \quad \ldots (2.23)
\]

\[
G(x, y | x', y') = \sum_{n=1}^{\infty} \frac{2}{n\pi} \ \frac{\sin \left( \frac{n\pi x}{2a} \right)}{\sinh \left[ \frac{n\pi b}{a} \right]} \ \frac{\sin \left( \frac{n\pi x'}{2a} \right)}{\sinh \left[ \frac{n\pi b'}{a} \right]} \ \sinh \left[ \frac{n\pi y}{2a} \right] \ \sinh \left[ \frac{n\pi (2b-y')}{2a} \right]; \ \text{for} \ y > y' \quad \ldots (2.24)
\]

Here \( y' = h \) when the septum thickness \( t = 0 \).

2.2.3 Derivation of the field components and wave impedance:

The cross-section of the cell is divided into four regions I, II, III and IV as shown in Fig 2.1. Using Equations (2.23), (2.24) and (2.2) the following vector potential functions in these four regions can be
written using respective expressions for Green's function for $y \leq y'$ or $y > y'$, where $h-t \leq y' \leq h+t$:

$$A_{zI}(x,y) = \mu_o \sum_{n=1}^{\infty} A_n \sin \left( \frac{n\pi x}{2a} \right) \sinh \left( \frac{n\pi y}{2a} \right)$$

$$\left\{ \begin{array}{l}
\sinh \left( \frac{n\pi y}{2a} (2b - h-t) \right) + \sinh \left( \frac{n\pi y}{2a} (2b - h+t) \right)
\end{array} \right.$$  

$$+ \int_{D+w} \sin \left( \frac{n\pi x'}{2a} \right) I_\nu(x') \, dx'$$

$$+ \int_{D-w} \sin \left( \frac{n\pi x'}{2a} (D-w) \right) + \sin \left( \frac{n\pi x'}{2a} (D+w) \right) \right\}$$

$$h+t$$

$$\int_{h-t}^{y \leq y'} \sinh \left( \frac{n\pi y'}{2a} (2b - y') \right) I_\nu(y') \, dy'$$

$$\cdots(2.25)$$

$$A_{zII}(x,y) = \mu_o \sum_{n=1}^{\infty} A_n \sin \left( \frac{n\pi x}{2a} \right) \sinh \left( \frac{n\pi (2b-y)}{2a} \right)$$

$$\left\{ \begin{array}{l}
\sinh \left( \frac{n\pi y}{2a} (h-t) \right) + \sinh \left( \frac{n\pi y}{2a} (h+t) \right)
\end{array} \right.$$  

$$+ \int_{D+w} \sin \left( \frac{n\pi x'}{2a} \right) I_\nu(x') \, dx'$$

$$+ \int_{D-w} \sin \left( \frac{n\pi x'}{2a} (D-w) \right) + \sin \left( \frac{n\pi x'}{2a} (D+w) \right) \right\}$$
\[
\int_{h-t}^{h+t} \sinh \left[ \frac{n\pi y'}{2a} \right] I_z(y') dy' ; \ y' > y
\] ... (2.26)

\[
A_{III, IV}(x, y) = \mu_0 \sum_{n=1}^{\infty} A_n \sin \left[ \frac{n\pi x}{2a} \right]
\]

\[
= \left\{ \begin{align*}
\sinh \left[ \frac{n\pi}{2a} (2b-y) \right] \sinh \left[ \frac{n\pi}{2a} (h-t) \right] \\
+ \sinh \left[ \frac{n\pi}{2a} (D-w) \right] + \sin \left[ \frac{n\pi}{2a} (D+w) \right]
\end{align*} \right\}
\]

\[
\int_{h-t}^{h+t} \sinh \left[ \frac{n\pi y}{2a} (2b-y') \right] \sinh \left[ \frac{n\pi y'}{2a} (h-t) \right] dy'
\] ... (2.27)

where, \( A_n = \frac{2}{n\pi \sinh \left[ \frac{n\pi b}{a} \right]} \) ... (2.28)

\( I_z(x'), I_z(y') \) = current densities on the horizontal and vertical surfaces of the septum, respectively.
From Equations (2.25)-(2.28) and (2.7) following expressions for the magnetic field components are obtained:

Region I ($y<y'$):

$$H_y = \pm \frac{\mu_0}{a} \sum_{n=1}^{\infty} \sin \left( \frac{n \pi x}{2a} \right) \frac{\cosh \left( \frac{n \pi y}{2a} \right)}{\sinh \left( \frac{n \pi b}{a} \right)}$$

$$\left\{ \sinh \left( \frac{n \pi y}{2a}(2b-h-t) \right) + \sinh \left( \frac{n \pi y}{2a}(2b-h+t) \right) \right\}$$

$$\sinh \left( \frac{n \pi b}{a} \right)$$

$$\int_{D-w}^{D+w} \sin \left( \frac{n \pi x'}{2a} \right) I_2(x') dx'$$

$$+ \left[ \sin \left( \frac{n \pi}{2a}(D-w) \right) + \sin \left( \frac{n \pi}{2a}(D+w) \right) \right]$$

$$\int_{h-t}^{h+t} \sinh \left( \frac{n \pi}{2a}(2b-y') \right) I_2(y') dy'$$

...(2.29)

Region II ($y>y'$):

$$H_y = - \frac{\mu_0}{a} \sum_{n=1}^{\infty} \cos \left( \frac{n \pi x}{2a} \right)$$

$$\cosh \left( \frac{n \pi y}{2a}(2b-y) \right)$$

$$\sinh \left( \frac{n \pi b}{a} \right)$$

$$\left\{ \sinh \left( \frac{n \pi y}{2a}(h-t) \right) + \sinh \left( \frac{n \pi y}{2a}(h+t) \right) \right\}$$
\[ D+w \]
\[
\int_{D-w} \sin \left( \frac{n\pi x'}{2a} \right) I_z(x')dx'
\]
\[ + \left[ \sin \left\{ \frac{n\pi}{2a} (D-w) \right\} + \sin \left\{ \frac{n\pi}{2a} (D+w) \right\} \right] \]
\[ h+t \]
\[
\int_{h-t} \sinh \left( \frac{n\pi y'}{2a} \right) I_z(y')dy'
\]
\[ \ldots (2.30) \]

Region III and IV:

\[ H_y = \frac{\mu_o}{a} \sum_{n=1}^{\infty} \sin \left( \frac{n\pi x}{2a} \right) \]
\[
\cdot \left\{ -\cosh \left( \frac{n\pi}{2a} (2b-y) \right) \sinh \left( \frac{n\pi}{2a} (h-t) \right) \right\}
\]
\[ + \cosh \left( \frac{n\pi y}{2a} \right) \sinh \left( \frac{n\pi}{2a} (2b-h-t) \right) \int_{D-w} \sin \left( \frac{n\pi x'}{2a} \right) I_z(x')dx'
\]
\[ + \left[ \sin \left\{ \frac{n\pi}{2a} (D-w) \right\} + \sin \left\{ \frac{n\pi}{2a} (D+w) \right\} \right] \]
\[
\cdot \left\{ -\cosh \left( \frac{n\pi}{2a} (2b-y) \right) \right\}
\]
\[ y \]
\[
\int_{h-t} \sinh \left( \frac{n\pi y'}{2a} \right) I_z(y')dy' + \cosh \left( \frac{n\pi y}{2a} \right) \sinh \left( \frac{n\pi y}{2a} \right)
\]
Here upper case letters are used for the x-component and lower ones for the y-component. Components of electric fields are then obtained from Equations (2.3), (2.4), (2.8), (2.29) - (2.31).

The integral of the numerator of Equation (2.9) is evaluated from four regions I, II, III and IV as follows:

\[
\iint |H|^2 \, dx \, dy = \int \int I_I^2 \, dx \, dy + \int \int I_{II}^2 \, dx \, dy + \int \int I_{III}^2 \, dx \, dy + \int \int I_{IV}^2 \, dx \, dy
\]

\[\text{where the subscripts I, II, III and IV are used to represent the regions. Similarly, the integral of the numerator of the Equation (2.11) can be written as}
\]

\[
\phi \frac{|\overline{H}|^2}{|H|^2} \, dl = \int_{S_1} |H_{x'y=0}|^2 \, dx + \int_{S_2} |H_{x'y=2b}|^2 \, dx
\]

\[+ \int_{S_1} |H_{x=0}|^2 \, dy + \int_{S_2} |H_{x=2a}|^2 \, dy
\]
Characteristic impedance can be evaluated from Equations (2.1), (2.7) - (2.12), (2.32) and (2.33). Since the integrations are carried out in the cross-section of the cell outside the region of the conductors, the effect of the thickness of the septum is taken into account in the evaluation of fields and the characteristic impedance of the TEM cell.

The wave impedance inside the cell can be represented by using Equations (2.4) and (2.5) as

$$Z_w = \frac{120\pi}{\left[ \frac{\phi}{\delta} \left\{ 1 + \left( \frac{R}{\omega L} \right)^2 \right\} \right]^{1/2}} \quad \ldots (2.34)$$

which approaches the intrinsic impedance of free space at high frequencies.
2.2.4 Choice of current distribution on the inner conductor

At very low frequencies the current distribution can be taken as uniform on the septum, as suggested by R.E. Collin (1960) and Spiegel et al (1987):

\[
I_x = \frac{I_o}{4(\omega + t)} \quad \text{...(2.35)}
\]

As the frequency increases, at the edge of an infinitely thin perfectly conducting septum, the tangential current density for TEM mode of propagation, varies as \( r^{-1/2} \), where \( r \) is the radial distance from the edge (Collin 1960, page 362). Hence at the edge of an infinitely thin perfectly conducting septum this current component tends to infinity. The current distribution can be assumed to be of the same form as the charge distribution considered by earlier workers for the calculation of the capacitance. Cai and Li (1989) have shown that higher-order power form gives better accuracy. However, deviation of impedance values from the exact ones (Tippet and Chang 1976 a) was not much for Maxwell form (Das and Anandamohan 1985) of charge distribution. To investigate the variation of impedance with septum thickness \( (t \ll w) \) for a first order approximation to \( Z_0 \), the Maxwell's form of current distribution on the septum having small thickness, is assumed:

\[
I_z(x) = I_o \left[ 1 - \left( \frac{x-D}{w} \right)^2 \right]^{-1/2} \quad \text{...(2.36)}
\]

This satisfies the edge condition for infinitesimally thin septum. Here, only longitudinal component of current is considered for the TEM wave. It is further
assumed that for a thin septum \((t/w \ll 1)\) the contribution of the currents comes mainly from the distribution on the bottom \((y' = h-t)\) and the top \((y' = h+t)\) surfaces of the septum, which include the currents at the edges. The contributions of the current on the side surfaces \((h-t < y < h+t)\) of the septum at \(x = D+w\) are neglected for simplicity, considering the low values of thickness \((t \ll w)\). Therefore, the integrals in the denominator of Equations (2.9) and (2.11) are of the form

\[
\int_{D-w}^{D+w} I_s \, dx = 2 \int_{S_1} I_s(x) \, dx
\]

\[
\text{...(2.37)}
\]

A factor of 2 appears for the two surfaces of the septum parallel to x-axis.

Substitution of the expressions for current from Equations (2.35) and (2.36) - (2.37), in Equations (2.29)-(2.31) and in Equations (2.9)-(2.12) all the line parameters and field distribution in the cross-section of the TEM cell can be evaluated. Some details of the derivation of the integrals involved in these expressions are given in Appendices-1 and 2.

2.3 RESULTS AND DISCUSSION

The characteristic impedances of symmetric and asymmetric TEM cells are computed for different conductor sizes of the cell. Results obtained by this method for zero thickness of the septum are compared with those reported by others in Tables 2.1 and 2.2. It is seen
Table 2.1: COMPARISON OF CHARACTERISTIC IMPEDANCE
(Ohm) FOR SYMMETRIC TEM CELL (t=0)

<table>
<thead>
<tr>
<th>Aspect ratios: b/a = 0.333, h/b = 1.0, D/a = 1.0</th>
<th>w/a</th>
<th>0.2</th>
<th>0.267</th>
<th>0.333</th>
<th>0.667</th>
<th>0.8</th>
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<tbody>
<tr>
<td>Present</td>
<td></td>
<td>90.64</td>
<td>75.93</td>
<td>65.62</td>
<td>39.31</td>
<td>32.94</td>
</tr>
<tr>
<td>Exact</td>
<td></td>
<td>90.39</td>
<td>75.81</td>
<td>65.27</td>
<td>38.10</td>
<td>32.96</td>
</tr>
<tr>
<td>HOP</td>
<td></td>
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<td>75.94</td>
<td>65.99</td>
<td>38.10</td>
<td>31.98</td>
</tr>
<tr>
<td>NSC</td>
<td></td>
<td>90.59</td>
<td>75.81</td>
<td>65.35</td>
<td>38.12</td>
<td>31.97</td>
</tr>
</tbody>
</table>

Table 2.2: COMPARISON OF CHARACTERISTIC IMPEDANCE
UNDER ASYMMETRIC CONDITION (t=0)

<table>
<thead>
<tr>
<th>Aspect ratios</th>
<th>Impedance (Ohm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b/a w/a D/a h/b</td>
<td>Present</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------</td>
</tr>
<tr>
<td>1.0 0.8 1.0 1.00</td>
<td>55.14</td>
</tr>
<tr>
<td>1.0 0.8 1.0 1.50</td>
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</tr>
<tr>
<td>1.0 0.8 1.0 1.75</td>
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</tr>
<tr>
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<td>71.96</td>
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</tr>
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</tr>
<tr>
<td>1.0 0.6 1.2 1.60</td>
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</tr>
<tr>
<td>1.0 0.6 1.2 1.80</td>
<td>40.44</td>
</tr>
</tbody>
</table>

Exact - Conformal method (Tippet and Chang 1976)
HOP - Higher-order power form of charge distribution
(Cai and Li 1989)
NSC - Numerical inversion of the SC conformal
transformation (Costamagna and Fanni 1990)

that present results agree well with those of exact (Tippet and Chang 1976 a), HOP (Cai and Li 1989) and average value of three NSC methods reported by Costamagna and Fanni (1990). For a given cell, the resistive and reactive components of the characteristic impedance are plotted in Fig.2.2 as function of frequency. The resistive component approaches the high frequency value \((L/C)^{1/2}\) rapidly at about 1 kHz, while
Fig. 2.2 Variation of $Z_0$ with frequency ($b/a=0.5$, $h/b=1$, $D/a=1$, $w/a=0.6$ & $t/w=0.005$)
the reactive component tends towards zero more slowly. However, the magnitude of the characteristic impedance decreases rapidly to the stable high frequency value of \((L/C)^{1/2}\) when the frequency exceeds 1 kHz. Similar behaviour is also observed for the wave impedance \(E/H\) (Fig. 2.3) which increases and attains free-space impedance value \((120\pi\ \text{Ohms})\) when the frequency exceeds approximately 1 kHz. The effects of the thickness of the septum on the impedance of the cell are shown in Figs. 2.4 and 2.5. Fig. 2.4 gives a comparison between the results of variation of \(Z_0\) with \(t/w\), obtained by the present method, with that of the method of conformal transform (Kumar and Das 1976) of symmetric narrow strip transmission line neglecting the effect of the side walls and assuming the cross-section of the septum an ellipse with minor axis length \(2t\). Both the results agree well for small values of \(t/w\). Discrepancies at higher values of \(t/w\) may be due to several assumptions in both the methods. Fig. 2.5 shows a variation of \(Z_0\) with \(t/w\) for different values of \(w/a\). The impedance decreases with the increase of thickness of the septum. About 2 to 3 ohms decrease in impedance values are observed for a change of thickness from \(t/w=0\) to \(0.02\). In TEM cell design this variation in impedance should be taken into account for obtaining very good VSWR. Fig. 2.6 shows that the impedance varies significantly with change of \(w/a\) ratio. Figs. 2.7 and 2.8 show that when the septum is offset more and more, \(Z_0\) decreases slowly for values of \(h/b\) or \(D/a\) in the vicinity of 1. But the fall becomes more rapid when \(h/b>1.2\) or \(D/a>1.2\).

The electric field distributions for symmetric and asymmetric cases are shown in Figs. 2.9 - 2.14. The \(y\)-axis represents the magnitudes of the E-field components normalised with respect to the magnitudes of the electric field component, \(E_y\) calculated at the
Fig. 2.3 Variation of wave impedance with frequency
(b/a=0.5, h/b=1, D/a=1, w/a=0.6 & t/w=0.005)
Fig. 2.4 Comparison of $Z_0$ vs. $t/w$ obtained from energy consideration and from conformal transformation of symmetric narrow strip transmission line ($b/a=0.1$, $h/b=1.0$, $D/a=1.0$ & $w/a=0.1$)
Fig. 2.5 Variation of Zo with t/w for different w/a (b/a=0.5, h/b=1.0 and D/a=1.0)

Fig. 2.6 Variation of Zo with w/a (b/a=0.5, h/b=1.0, D/a=1.0 and t/w=0.005)
Fig. 2.7 Variation of Zo with h/b
(b/a=0.5, D/a=1.0, w/a=0.6 and t/w=0.005)

Fig. 2.8 Variation of Zo with D/a
(b/a=0.5, h/b=1.0, w/a=0.6 and t/w=0.005)
central position \((x/a=1.0, \ y/b=0.5)\) between the septum and its parallel ground plane under symmetric condition. It is seen that the strength of the \(E_y\) component increases with \(y\) from ground conductor to inner conductor. It is most uniform along the \(x\)-direction in the centre region between the two conductors. The strength of the \(E_x\) component is, however, lowest near the ground conductor. Asymmetry of the field distribution in the cross-section is shown in Figs. 2.10 - 2.14 for different offset positions \((h/b\) and \(D/a\)) of the septum. The relative distributions of the magnetic field components \(H_x\) and \(H_y\) will be similar to those of \(E_y\) and \(E_x\), respectively, as can be seen from the Equation (2.8).

The present method investigates the effect of septum thickness on impedance of a rectangular TEM cell when the maximum septum thickness is limited to \(t/w=0.02\). For greater thickness, this method does not give accurate results since the currents on the side walls of the septum need to be considered, which are neglected in this analysis at higher frequencies. The frequency dependence of the impedance is also studied. The method evaluates the magnetic and electric field components inside the cell at extremely low frequencies when \(E\) and \(H\) cannot be related by the intrinsic impedance of free space. This investigation is useful in the design of a TEM cell with a septum of small but finite thickness.
Fig. 2.9 Relative distribution of E-field components for a symmetric TEM cell (b/a=0.5, w/a=0.6 and t/w=0.005)
Fig. 2.10 Relation distribution of $E_y$ for an asymmetric TEM cell ($b/a=0.5$, $w/a=0.6$, $h/b=1.4$, $D/a=1.0$ and $t/w=0.005$)

a. Region $0\leq y \leq h-t$

b. Region $h+t \leq y \leq 2b
Fig. 2.11 Relative distribution of Ex for an asymmetric TEM cell (b/a=0.5, w/a=0.6, h/b=1.4, D/a=1.0 and t/w=0.005)

a. Region 0 \leq y \leq h - t  
b. Region h + t \leq y \leq 2b
Fig. 2.12 Relative distribution of E-field components for an asymmetric TEM cell (b/a=0.5, w/a=0.6, h/b=1.0, D/a=1.2 and t/w=0.005)
Fig. 2.13 Relative distribution of $E_y$ for an asymmetric TEM cell ($b/a=0.5$, $w/a=0.6$, $h/b=1.4$, $D/a=1.2$ and $t/w=0.005$)

a. Region $0 \leq y < h-t$

b. Region $h+t \leq y \leq 2b$
Fig. 2.14 Relative distribution of Ex for an asymmetric TEM cell (b/a=0.5, w/a=0.6, h/b=1.4, D/a=1.2 and t/w=0.005)

- Region $0<y\leq h-t$
- Region $h+t\leq y<2b$

a. Region $0\leq y\leq h-t$

b. Region $h+t \leq y \leq 2b$