CHAPTER 3

CIRCUIT MODELING OF THE QUANTUM WELL LASER DIODE

3.1 INTRODUCTION

A two-port circuit model for quantum-well lasers has been developed from the rate equations. The model is implemented using a circuit simulator PSPICE and validated by simulating the response to dc and transient characteristics. This model, in conjunction with the optical fiber and photodetector models constituting an optical link, has also been simulated. The response of this link with variation in time constant \( RC \) is studied by generating eye patterns and observing horizontal and vertical eye closures. Eye diagrams are generated for different pulse formats that are commonly used in fiber optics systems. In addition, the frequency effects on the modulation properties of QW lasers have been studied and analyzed using a small signal model. The details of these studies are discussed in the following sections.

3.2 THE QUANTUM WELL LASER DIODE

Quantum-well lasers have been found to have superior characteristics associated with their two-dimensional-like nature [93,94]. The main difference between a QW laser and the conventional semiconductor laser is mostly in the thickness \( L_z \) of the active region. The active region is about 2000 Å thick in a conventional semiconductor laser, while it is between 50 Å and 200 Å in the QW laser. The electrons and holes in such thin layers display quantum size effect [95-97] that is, the particle is constrained in the
thin layer and shifts to a higher energy or a shorter wavelength ($\lambda=L_z$). At $L_z<\lambda$, the size quantization results in a series of discrete energy levels given by the bound state energies of a finite square well. A potential well exists in both the conduction band and the valence band giving rise to a series of bound states $E_n$ for electrons, $E_{hhn}$ for heavy holes, and $E_{lh}$ for light holes (Fig. 3.1) [98].

The confinement of carriers in the extremely thin active layer leads to major qualitative differences in the energy distribution of the electrons and holes and thus to major modifications of all the basic optical properties of these new media. In Fig 3.2 [98], the half-parabolas that originate from the conduction band edge $E_c$ and valence band edge $E_v$ correspond to the densities of states of a bulk semiconductor. The step-like densities of states are a characteristic of a quantum well structure. Interband recombination transitions occur from a bound state in the conduction band $E_n$ to a bound state in the valence band $E_{hhn}$ or $E_{lh}$.

The main advantages of the QW laser are the large gain at small currents. The low threshold current is due not only to the large differential gain, but also, to the major reduction in the active volume that occurs when one switches from conventional active regions ($L_z>1000\text{Å}$) to quantum wells ($L_z<200\text{Å}$). The threshold current is so low that in many practical situations the laser may be considered by the optoelectronic designer as a thresholdless device converting, in a proportional manner, current to coherent light. This makes it possible to eliminate most of the optical power monitoring and stabilizing circuitry used in conjunction with present day lasers. Hence there is a need for a suitable model for quantum-well lasers, which represents a more logical choice of semiconductor laser technology for integrated optoelectronic circuit design [98,99]. Circuit simulations could be a powerful tool for the design and analysis of optical interconnections, the driving circuit and the parasitics. The methodology adopted for the
Fig. 3.1 Well potential of a QW heterostructure
Fig. 3.2  Density of states for a QW heterostructure
construction of circuit models for QW lasers and the results obtained are discussed in the following sections.

3.3 ANALYSIS OF RATE EQUATIONS

The electrical and optical properties of the active region of a QW laser can be described by the rate equations [100,101]. The single mode rate equations are expressed as

\[
\frac{dn}{dt} = \frac{J}{qN_wL_z} - \frac{Bn^2 \cdot \Gamma_{gm} V_g p}{p} \tag{3.1}
\]

\[
\frac{dp}{dt} = \Gamma_{gm} V_g p \cdot \beta Bn^2 - \frac{p}{r_p} \tag{3.2}
\]

Equation (3.1) requires that the current flow into the active region minus the current lost due to spontaneous and stimulated emissions, is equal to the time rate of change of the charge density in this region. Similarly, equation (3.2) requires that the time rate of change for the average photon density in the Fabry - perot resonator is equal to the sum of stimulated emission and spontaneous emission minus the absorption in the active region. The portion of the optical field outside the active region is accounted for by the confinement factor \( \Gamma \).

To construct the circuit model, it is important to find the analytical expressions for all nonlinear model elements. The spontaneous emission term in equations (3.1) and (3.2) has quadratic dependence on carrier density when dominated by the direct band-to-band electron - hole recombination. Considering the lowest order quantum transition, the electron density in the quantum well is obtained from [102]

\[
n = \frac{m^*_c KT}{\pi \hbar^* L_z^2} \ln \left[ 1 + \exp \left( \frac{E_{fc} - E_{c1}}{KT} \right) \right] \tag{3.3}
\]
In the above equation, the charge density is expressed in terms of quasi-Fermi levels, hence the spontaneous emission term is also described in terms of quasi-Fermi energies. The stimulated emission, which depends linearly on the photon density present in the active region is expressed as $\Gamma V_g \varepsilon_{mp}$, where the optical gain $\varepsilon_m$ is a function of the geometry, material composition of the device and driving current. Assuming that band-to-band recombination is dominate between the first electron and heavy hole states and neglecting the light hole effect, the optical gain is expressed as a function of quasi-Fermi levels given by [103-105]

$$\varepsilon_m = \frac{4\pi q^2 m_r}{c_0 m_e^2 c\nu h^* L_z E_{ph}} \frac{M_b}{|M_b|^2} \left[ f_c(E_{fc}, E_c) - f_v(E_{fv}, E_v) \right]$$  \hspace{1cm} (3.4)

where the quantity $|M_b|^2 = 1.3$, $m_e E_q$ is the momentum matrix element of transition between the band edges, $f_c (E_{fc}, E_c)$ and $f_v (E_{fv}, E_v)$ are the Fermi distribution functions in conduction and valence bands respectively, and $E_c$ and $E_v$ are given as

$$E_c = \frac{m_r}{m_e^*} (E_{ph} - E_g)$$  \hspace{1cm} (3.5)

$$E_v = \frac{m_r}{m_v^*} (E_{ph} - E_g)$$  \hspace{1cm} (3.6)

$$m_r = \frac{m_e^* m_v^*}{(m_e^* + m_v^*)}$$  \hspace{1cm} (3.7)

$$E_{ph} = \hbar \nu$$  \hspace{1cm} (3.8)

Assuming that the potential difference between the two quasi-Fermi levels is equal to the applied voltage across the active region $V_n$, the quasi-Fermi level in the conduction band can be calculated by using a data fitting method,
which is expressed as

\[
E_{fc} = \begin{cases} 
- 0.6755 + 0.4871 V_n & \text{if } V_n < 1.1 \\
- 0.2077 - 0.4253 V_n + 0.4895 V_n^2 - 0.03359 V_n^3 & \text{otherwise}
\end{cases}
\] (3.9)

\[
E_{fv} = E_{fc} - qV_n
\] (3.10)

which is valid over the range of -1 to 3.5 volts.

### 3.4 PHOTON DENSITY IN THE ACTIVE REGION

Considering the cavity structure of the active region as a Fabry-Perot resonator, the average photon density in the laser cavity is given by [105-107]

\[
P = \frac{(e^{glL_{cl}} - 1) [(1 - R_2) (1 + R_1 e^{glL}) + (1 - R_1) (1 + R_2 e^{glL})]}{gL (1 - R_1 R_2 e^{2glL})} \frac{r_s p}{\gamma V_g} \] (3.11)

where \( R_1 \) and \( R_2 \) are the respective back-facet and front-facet reflectivities, \( L \) is the cavity length, and \( r_s p = \beta B n^2 \) is the spontaneous emission rate. The gain index is given by

\[
g = \Gamma (g_{m} - \alpha_l)
\] (3.12)

where \( \Gamma \) is the confinement factor of the optical waveguide and is a function of the structure and material composition of the laser. It can be calculated from a simple formula [100,108] \( \Gamma = 0.3 L_\mu/1000 \).

The output light power from the laser can be expressed as

\[
P_{out} = \frac{r_s p E_{ph} N_{w} L_\mu}{2g} \frac{(1 - R_2) (e^{glL_{cl}} - 1) (R_1 e^{glL} + 1)}{(1 - R_1 R_2 e^{2glL})} \] (3.13)
For a quantum well laser without facet coatings, the reflectivities at both the ends are equal, \( R_1 = R_2 = R \). Then, above the threshold condition, the optical loss at the cavity ends is approximately equal to the gain, i.e.

\[
g_L = \ln \left( \frac{1}{R} \right)
\]

(3.14)

Hence

\[
P_{\text{out}} = \frac{V_g W_L L E_{\text{ph}}}{4}
\]

(3.15)

The expression for the average light power emitted from the front facet is given by

\[
P_{\text{avg}} = \frac{V_g W_L L E_{\text{ph}}}{4} \cdot P
\]

(3.16)

3.5 MODELING OF THE QW LASER DIODE

Due to the fact that the laser power \( P_{\text{out}} \) is the main physical quantity which carries the information, it was chosen to be a circuit variable and is equivalent to an "Optical Voltage" in the circuit simulation program. By including the volume of the active region in the rate equations and multiplying the rate equations by \( q N_w L_z W_L \) and rearranging, one gets:

\[
I = I_{\text{sp}}(V_n) + I_{\text{st}}(V_n) + C_N \frac{dV_n}{dt}
\]

(3.17)

\[
C_{\text{op}} \frac{dP_{\text{avg}}}{dt} = \alpha_{\text{sp}} I_{\text{sp}} + \alpha_{\text{st}} I_{\text{st}} - \frac{1}{R_L} P_{\text{avg}}
\]

(3.18)

where \( I_{\text{sp}} = B q N_w L_z W_L n^2 \) is the spontaneous emission component, \( I_{\text{st}} = 4(q/E_{\text{ph}}) L \Gamma_{\text{sm}} P_{\text{avg}} \) is the recombination current caused by stimulated emission, \( P_{\text{avg}} = (1/4) V_g W_L L_z E_{\text{ph}} P \) is the light output power emitted.
from the front face, \( C_N = qN_w L_z WL dN(V_n)/dV_n \) is the charge storage effect within the active region, \( \beta_p = \beta (E_{ph}/qL) \), \( \alpha_{st} = E_{ph}/(qL) \), \( C_{op} = 4/V_g \) and \( r_p = R_L C_{op} \). Combining equations (3.17) and (3.18), the circuit model is obtained as a two port network as shown in Fig.3.3, where the voltage drop outside the depletion region is represented by a series resistor \( R_B \) and the non-radiative recombination current is modeled as an ideal diode (D). \( C_D \) models the depletion and diffusion capacitance of the diode. Since both \( I_{st} \) and \( I_{sp} \) are nonlinear functions of voltage across the active region, both can be represented as nonlinear voltage controlled current sources \( \alpha_{sp} I_{sp} \), \( \alpha_{st} I_{st} \) are modeled as a pair of current controlled current sources in the optical portion. The photon loss and storage are modeled by the resistance \( R_L \) and capacitance \( C_{op} \) respectively, the voltage drop across \( R_L \) is therefore proportional to the photon density \( p \) inside the cavity and hence the light output from the laser.

As the amplitude of the lasing mode in the QW laser diode is much greater than the amplitude of the side modes, the spontaneous emission into modes other than the lasing mode is taken into account in the equivalent circuit, where \( \beta_{avg} (E_{ph}/qL) \). However, it is presumed that the average loss and photon storage effects in the non-lasing modes are the same as those of lasing mode (i.e \( R_L' = R_L \) and \( C_{op}' = C_{op} \)). Hence the total light output power consists of two voltage controlled voltage sources in series. The circuit model of the laser diode is validated by simulating the response to dc sweep and step input as shown in Fig.3.4 and 3.5 respectively. Table 3.1 gives the parameter values for the model. The response of the multimode laser to a step and pulse input of amplitude 100 mA is shown in Fig.3.6 and 3.7, respectively. It is observed that the intensity of light output from non-lasing modes is very small compared to the first mode.

3.6 SIMULATION OF AN OPTICAL LINK

The basic building blocks for any optical system are the optical source, the fiber and the receiver. The details of detector model used here are given
Fig. 3.3  Circuit model for QW laser

$V_{op}$ and $V_{opnl}$ are the equivalent output voltages for lasing and non-lasing modes respectively.
Fig. 3.4 The dc L-I characteristics of a QW laser

Voltage $V_{\text{out}}$ is proportional to output light intensity.
Fig. 3.5 Simulated response for step function drive current.
Fig. 3.6  Simulated response for step function drive current (multimode laser)
V(8) corresponds to dominant mode.
V(26) corresponds to nonlasing mode
V(8,26) corresponds to total output
Simulated optical equivalent output pulse from a multimode laser

- $V(8)$ corresponds to dominant mode.
- $V(26)$ corresponds to nonlasing mode.
- $V(8,26)$ corresponds to total output.
in Chapter 2 and the delay line model is used to simulate the optical fiber. The response of the photodetector to a pulse at the input of the laser source is shown in Fig. 3.8. It is observed that the time difference between the input signal and the output of the detector results because of the transmission through the fiber and at the output of the photodetector, it is seen that the signal smoothens considerably. The effect of RC time constant on the photodetector response is also studied. Eye diagrams are generated for the second bit of a 3-bit NRZ pulse sequence for different values of $C_i$ (1.5pF and 5pF) for a bit rate of 1Gb/s as shown in Fig 3.9. It is seen that for a given bit rate, for higher values of capacitance, the response of the detector becomes poor and the eye closure increases. It is inferred that degradation of the signal takes place not only in the fiber but also at the receiver. The response time of the photodiode depends on its time constant. Hence an optimum value of the capacitance should be chosen in order to reduce intersymbol interference.

The system performance has been investigated for input pulse formats corresponding to 3 level and 2 level alternate mark inversion pulses [109]. Fig 3.10 shows the intersymbol interference is more in the case of 3 level AMI. As the system utilizes more than two levels, the equivalent signal to noise ratio degradation is quite visible. Hence for the same average value of power, 2 level AMI gives greater SNR.

### 3.7 SMALL-SIGNAL MODEL OF THE QW LASER DIODE

A small signal model is derived from the steady state solution of the rate equations with a small perturbation $\Delta \delta e^{j\omega t}$ added to the external applied voltage. A linearized set of small signal equations is derived from equations (3.1) and (3.2). Assume that in steady state, a small ac signal is superimposed
Fig. 3.8 Waveforms at the output of an optical link
V(8) corresponds to laser output
V(21) corresponds to photodetector output
Fig. 3.9  Eye diagrams observed at 1 Gb/s for various values of $C_i$. 

$C_i = 1.5 \, \text{PF}$

$C_i = 5 \, \text{PF}$
Fig. 3.10  Eye-diagrams for different pulse formats
a) 3 level AMI  b) 2 level AMI
on the applied voltage, i.e., $V = V_0 + \Delta \phi e^{j\omega t}$ and the expression for $I$, $n$, $g_m$ and $P_{avg}$ can be given by

\[
I = I_0 + \Delta I e^{j\omega t} \quad \text{(3.19 a)}
\]
\[
n = n_0 + \Delta n e^{j\omega t} \quad \text{(3.19 b)}
\]
\[
g_m = g_{mo} + \Delta g e^{j\omega t} \quad \text{(3.19 c)}
\]
\[
P_{avg} = P_{avg_0} + \Delta P e^{j\omega t} \quad \text{(3.19 d)}
\]

where $V_0$ is the dc bias voltage and $\omega$ is the small - signal angular frequency; $n_0$, $g_{mo}$, $I_0$ and $P_{avg_0}$ are the corresponding steady - state values. $\Delta n$, $\Delta g$, $\Delta I$ and $\Delta P$ are magnitudes of the corresponding small - signal perturbations. Applying this set of perturbation to equations (3.1) and (3.2) and neglecting higher order terms, the small - signal version of the rate equations can be written as

\[
j \omega C_N \Delta \theta = \Delta i - G_{st} (g_{mo} \Delta p + \Delta g P_{avg}) - 2G_{sp} Bn \Delta n \quad \text{(3.20)}
\]
\[
j \omega G_{op} \Delta p = 2z_{sp} G_{sp} Bn \Delta n + \alpha_{st} G_{st} (g_{mo} \Delta p + \Delta g P_{avg}) - \frac{\Delta p}{R_L} \quad \text{(3.21)}
\]

where

\[
G_{st} = \frac{4q\Gamma L}{E_{ph}} \quad \text{(3.22)}
\]

and

\[
G_{sp} = q (N_w L_z W_L) \quad \text{(3.23)}
\]

The steady state values for $n_0$, $g_{mo}$, $I_0$ and $P_{avg_0}$ can be calculated from equations (3.3), (3.4), (3.17) and (3.18) with $V = V_0$ and $\Delta n$ and $\Delta g$ can be derived by differentiating equations (3.3) and (3.4), and then multiplying them with $\Delta V$ as follows :

\[
\Delta n = -\frac{dn}{dV} \bigg|_{V = V_0} \quad \text{(3.24)}
\]
\[
\Delta g = -\frac{dg_m}{dV} \bigg|_{V = V_0} \quad \text{(3.25)}
\]
The small signal equivalent circuit model can be derived from equations (3.20) and (3.21) as shown in Fig. 3.11, where

\[ G_{s1} = \frac{d g_m}{d V} \bigg|_{V = V_0} G_{st} P_{\text{avgo}} \]  

(3.26)

\[ g' = G_{st} \, g_{mo} \]  

(3.27)

\[ G_{s2} = 2 \, G_{sp} \, Bn(\frac{dn}{dV}) \bigg|_{V = V_0} \]  

(3.28)

The stimulated component of the current \( \Delta_{ist} = G_{st} (g_{mo} \, \Delta p + \Delta g \, P_{\text{avgo}}) \) is the sum of two currents represented by \( R_{s1} = 1/G_{s1} \) and a voltage controlled current source \( g' \). \( \Delta_{isp} = 2G_{sp} \, Bn\Delta n \) is the spontaneous component represented by \( R_{s2} = 1/G_{s2} \). The optical side of the equivalent circuit is exactly similar to the large signal model (Fig. 3.3). \( C_D \) represents the depletion capacitance and \( R_D \) represents the junction resistance. \( C_N \) represents the charge storage effect. The spontaneous emission effects of the modes other than the lasing mode are neglected in the small signal model. However the effects of package parasitics have been considered. Fig. 3.12 shows the simulated frequency characteristics of the QW laser. It is seen that the 3 db modulation bandwidth of the QW laser is about 0.1 GHz.
Fig. 3.11 The small-signal model of a QW laser
Fig. 3.12 Frequency response of a QW laser
### TABLE 3.1

Model parameters for the QW laser diode

<table>
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<tr>
<th>Model Parameter</th>
<th>Value</th>
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<td>Laser length (L)</td>
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<tr>
<td>Laser width (W)</td>
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<td>Reflectivity (R)</td>
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<td>Wavelength ($\lambda$)</td>
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<td>Intrinsic loss ($\alpha_i$)</td>
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<td>Coupling coefficient ($\beta$)</td>
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<td>Refractive index ($n$)</td>
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<tr>
<td>Radiative recombination coefficient ($B$)</td>
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<tr>
<td>Nonradiative recombination current</td>
<td>$7.36 \times 10^{-11}$ (mA)</td>
</tr>
<tr>
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</tr>
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