Recent findings have brought out the fractal or self-similar nature of network traffic. Self-similar processes are defined in terms of their auto-correlation functions that are mathematically complex and are not very tractable. In this chapter, a simple distribution function that models self-similar arrivals by capturing their heavy tailed property (Long Range Dependence) is proposed and presented. Also jitter analysis is carried out for both Poisson and self-similar traffic models.

5.1 LONG-RANGE DEPENDENCE

One of the most significant properties of self-similar processes is referred to as LRD. This property is defined in terms of the behavior of the auto covariance $C(t)$ as $t$ increases. For many processes, the auto covariance rapidly decays with $t$. For example, for the Poisson increment process with increment $L$ and mean $\lambda$, the auto covariance for values of $t > L$ is, $C(t) = R(t) - \lambda^2 = \lambda^2 - \lambda^2 = 0$. In general, a short-range dependent process satisfies the condition that its auto covariance decays at least as fast as exponentially:

$$C(k) \sim a^{|k|} \text{ as } |k| \to \infty, \quad 0 < a < 1$$
where \( \sim \) denotes that the expressions on the two sides are asymptotically proportional to each other. The types of data traffic models typically considered in the literature employ only short-range dependent processes. Using the equality \( \sum_{k=0}^\infty x^k = 1/(1-x), \) for \( |x| < 1, \) it is observed that \( \sum_k C(k) \) is finite. In contrast, a long-range dependent process has a hyperbolically decaying auto covariance as \( C(k) \sim |k|^{-\beta} \) as \( |k| \to \infty, \) \( 0 < \beta < 1. \) In this case, \( \sum k C(k) = \infty. \)

Long-range dependence intuitively reflects the persistence phenomenon in self-similar processes—namely, the existence of clustering and burst characteristics in all time scales.

### 5.2 HEAVY-TAILED DISTRIBUTIONS

Self-similar data traffic can be characterized in terms of heavy-tailed distribution. One attraction of the heavy-tailed distribution approach is that it leads to manageable simulation models. Heavy-tailed distribution can be used to characterize probability density values that describe traffic processes such as packet interarrival time and burst length. The distribution of a random variable \( X \) is heavy-tailed if

\[
\Pr \{ X > x \} \sim x^{-\alpha}, \text{ as } x \to \infty, \ 0 < \alpha
\]

(5.1)

The simplest heavy-tailed distribution is the Pareto distribution having parameters \( k \) and \( \alpha (k, \alpha), \) with density and distribution functions as,

\[
f(x) = \alpha k^\alpha x^{-\alpha-1}
\]

(5.2)

\[
F(x) = 1 - (k/x)^\alpha \quad (x > k; \alpha > 0)
\]

(5.3)

and a mean value \( \mathbb{E}[X] = [\alpha/(\alpha - 1)]k \quad (\alpha > 1) \)
The parameter $k$ represents the smallest possible value of the random variable. Heavy-tailed distribution has a number of properties that are qualitatively different from those of exponential, normal, and Poisson distribution. If $\alpha \leq 2$, then the distribution has infinite variance; if $\alpha \leq 1$, then the distribution has infinite mean and variance.

5.3 A TWO PARAMETER DISCRETE PARETO PROCESS (2PDPP) TO MODEL SELF-SIMILAR ARRIVALS

One of the distinct properties of self-similar traffic is its heavy tailed property. This section details the modified Pareto Distribution function derived to model self-similar traffic by capturing its distinct behavior.

Generally, Pareto Distribution function is used to model the heavy tailed property of the self-similar process. For an ON-OFF source, defined by such a distribution, the active period is given by the density function

$$f(x) = \frac{\alpha}{k} \left( \frac{k}{x} \right)^{\alpha + 1}$$

(5.4)

where, $x > k$, $\alpha > 0$ and the Hurst parameter, $H = (3 - \alpha) / 2$. To model the discrete arrival process we have considered the discretised version of the Pareto process as suggested by Boris Tsybakov and Georganas (1997),

$$P_r \{ N = n \} = \frac{n^{-\alpha + 1}}{\sum_{i=1}^{\infty} n^{-\alpha + 1}}, \quad \alpha > 0$$

(5.5)
The mean of this process is,

\[ E[N] = \sum_{n=1}^{\infty} n \frac{n^{-(a+1)}}{\sum_{n=1}^{\infty} n^{-(a+1)}} \]  \hspace{1cm} (5.6)

The expression for mean is simplified as \( E[N] = \frac{\alpha}{(\alpha - 1)} \) by using the approximation

\[ \sum_{n=1}^{\infty} n^{-(a+1)} = \int_{1}^{\infty} x^{-(a+1)} \, dx \]  \hspace{1cm} (5.7)

where \( x \) is a continuous variable. This implies that the process (5.5) becomes a single parameter process, i.e. the process depends only on the mean. The function as it is, cannot be used to model self-similar arrival processes because there is a need to simulate various traffic traces with different mean values for a Hurst parameter. The existing models cannot be used to define the discretised self-similar arrival process. Hence, a modified version of the discrete Pareto process has been formulated called Two Parameter Discrete Pareto Process (2PDPP) and given by;

\[ \Pr\{ N(t)=n \} = \text{Probability of } n \text{ arrivals in time } t \]

\[ = \frac{(nk+1/t)^{-(a+1)}}{\sum_{0}^{\infty} (nk+1/t)^{-(a+1)}} \]  \hspace{1cm} (5.8)

where \( \alpha \) is a function of \( H \) and \( k \), a constant that helps to get the desired mean values for a fixed value of \( H \) and vice-versa. In this process, mean and \( H \) are the
two parameters. The mean, $E[N(t)]$, of this process, following the approximation in Equation (5.7), is obtained as,

$$E[n(t)] = \frac{tk\alpha + 1}{tk(\alpha - 1)}$$  \hspace{1cm} (5.9)

The motivation to add $1/t$ term in Equation (5.8) stemmed from the fact that in Equation (5.5) the quantity $n$ is not allowed to take the value 0. But, zero arrivals should be possible in an arrival process. Hence, there is a need to add a term to $n$ in the distribution function to make it possible for $n = 0$. It is seen from Equation (5.8) that as $t$ increases $E[N(t)]$ decreases. It is also seen that by fixing $\alpha$ the mean does not get fixed i.e., for the same value of $\alpha$, by varying $k$ different mean values can be obtained. This is the reason for including $k$ in Equation (5.8). A random number generator using the distribution function given in Equation (5.9) is created. It is found that $\alpha = 1.5$ yields a Hurst parameter of around 0.8. The exact relationship between $\alpha$ and $H$ is not found.

### 5.4 JITTER IN ATM NETWORK

In ATM based B-ISDN, the connections are established with widely different bit rates over an access link. The price of this flexibility is the risk of network performance degradation when a user, having contracted a communication at 10Mbits/sec, in fact, uses his access to emit data at a much greater rate. Also, the periodic cell stream experiences random delays at multiplexing stages notably in customer premises equipment. This is the phenomenon of jitter and requires a standard estimation of the source rate. To estimate jitter analytically it is assumed that the main stream of traffic is CBR and the background is self-similar traffic.
In the analysis given by (Robert and Gullemin 1992), the jitter is considered as a discrete time process where the time-unit is arbitrary. As shown in Figure 5.1, a CBR periodic stream, which has an inter-cell interval of \( d \) time slots, is considered. The \( i^{th} \) cell has a sojourn time of \( D + W_i \) in the system (multiplexer or network) where \( D \) is a constant (propagation time, etc.) and \( W_i \) is a non-negative delay component introduced by the multiplexer (waiting time in the multiplexer queue). Without loss of generality, \( D \) is assumed to be zero (\( D \) depends only on the route followed by the cells of the connection considered). \( W_i \) is assumed to constitute a stationary ergodic process with a probability distribution

\[
w_k = P_r\{ W_i = k \} \quad \text{for} \quad k \geq 0
\]  

(5.10)

It is further assumed that the dependence of successive delays on the earlier delay is first-order Markovian (i.e. delay of the \( i^{th} \) cell depends only on the delay of the \((i-1)^{th}\) cell), characterized by the transition probabilities

\[
q_{jk} = P_r\{ W_i = k / W_{i-1} = j \} \quad \text{for} \quad j, k \geq 0
\]  

(5.11)

and \( w_k \) satisfies the equation

\[
w_k = \sum_{j \geq 0} w_j q_{jk}
\]  

(5.12)

Let 0 be an arbitrary time instant and let \( \tau_0 \) be the cell instant immediately preceding 0. Let \( \tau_i, i \geq 1 \), be the exit instants of subsequent cells. The random variable \( U_n \) is defined as,
where $U_n = W_n - W_0$ is the variation of the $n^{th}$ order inter-exit time with respect to the inter-arrival time $nd$. Jitter is characterized by the distribution values of the random variable $U_n, n \geq 1$, and especially that of

$$U_i = W_i - W_0$$ (5.14)

The distribution of $U_i$ allows comparisons between the inter-arrival times in the jittered process with that in the initial flow that is constant and exactly equal to $d$.

Let the distribution of $U_n$ be $f_n(k) = \Pr \{ U_n = k \}$. It can be proved that

$$f_n(k) = \sum_{i=0}^{\infty} w_i q_{i,i+k}^{(n)}$$ (5.15)

where $q_{i,i+k}^{(n)}$ is the $n$-step transition probability, $q_{jk}$.

To calculate the transition probabilities $q_{jk}$, the conditional probabilities defined below are used.

$$Q(j,k) = \Pr \{ W_i > k / W_{i-1} = j \}$$

$$P_n(j,k) = \Pr \{ W_i > k / W_{i-1} = j \text{ and } n \text{ Poisson arrivals in } ((i-1)d,id) \}$$

Then, from the definitions of $q_{jk}$ and $Q(j,k-1)$ it can be written as

$$q_{jk} = Q(j,k-1) - Q(j,k),$$ (5.16)
Q(j,k) = \sum \limits_{s} P_{n}(j,k) \frac{\lambda^{d}}{n!} e^{-\lambda d}  \tag{5.17}

It can also be proved that

\[ P_{s}(j,k) = \begin{cases} 
0 & \text{for } (j+1 \geq d \text{ and } n \leq d + k - j - 1) \\
\sum \limits_{s=1}^{n-k} \frac{n}{s+k} \left( \frac{s}{d} \right)^{s-k} \left( 1 - \frac{s}{d} \right)^{d-n+k} \frac{d-n+k}{d-s} & \text{for } j+1 < d \text{ and } n \leq k \\
1 & \text{for } n > d + k - j - 1
\end{cases} \tag{5.18}

For \( n = 10 \) to 40, \( j = k = 1 \) to 25, \( d = 30 \) and \( \lambda = 0.75 \), the probabilities \( P_{n}(j,k) \) and \( Q(j,k) \) are computed. Using these values in Equations (5.11) and (5.12) the distribution function \( f_{1}(k) \), which gives the probability that the first cell is delayed with respect to the 0th cell by \( k \) slots, is computed. Since, there is no absolute time reference it can be inferred that \( f_{1}(k) \) gives the probability that the \( n^{th} \) cell is delayed with respect to the \((n-1)^{th}\) cell by \( k \). Next, the value of \( k \) which covers 99% of the area under the \( f_{1}(k) \) curve is determined. The significance of this value is that it gives us the idea of the buffer size required so that the \( n^{th} \) cell is not lost 99% of the time when compared to the \((n-1)^{th}\) cell. Since this is true for all \( n \), the value of \( k \) gives the buffer size required so that the CLR does not exceed 1%. For Poisson background traffic with \( \lambda = 0.75 \) and the CBR stream period \( d \) = 30, the buffer size required is found to be 11 slots.
The analytical method described above is applicable only for Poisson traffic. To apply this method for self-similar traffic, Equation (5.18) is replaced by 2PDPP developed in this thesis and \( Q(j,k) \) is obtained as

\[
Q(j,k) = \sum_n p_n(j,k) \frac{(nk + 1/d)^{-(\alpha+1)}}{\sum_1 (nk + 1/d)^{-(\alpha+1)}}
\]  

(5.19)

One important point to be noted here is that in deriving the Equation (5.19), it is assumed that Poisson arrivals are uniformly distributed in any finite interval. The same assumption is made here for self-similar traffic because as given by Erramilli et al (1996), self-similarity depends only on the number of arrivals in a given time interval and not on the inter-arrival distribution. It is reported by Erramilli et al (1996) that the same H parameter is obtained for traces in which all the arrivals are shuffled in a time interval maintaining the same number of arrivals.

In order to do jitter analysis, self-similar traces must be generated. This is done using the algorithm proposed in Paxson (1997). This algorithm is referred to as the FFT method. An ATM multiplexer, carrying self-similar background traffic and the periodic stream of CBR traffic is simulated and the cell delay variation is calculated.
Figure 5.1 The Multiplexer Model
5.5 GENERATION OF SELF-SIMILAR TRAFFIC USING FFT METHOD

The principle behind FFT method can be summarized as under. Suppose the power spectrum \( f(\lambda;H) \), of the Fractional Gaussian Noise (FGN) signal is known, a sequence of complex numbers \( z_i \) can be constructed corresponding to this power spectrum. \( z_i \) is in a sense a frequency domain sample path. An inverse Discrete Time Fourier Transform (DTFT) is then used to obtain the time-domain counterpart \( x_i \). Because \( x_i \) by construction, denotes the power of FGN, and because autocorrelation and power spectrum form a Fourier pair, \( x_i \) is guaranteed to have the autocorrelation properties of an FGN process.

For an FGN process the power spectrum is given by (Paxson 1997)

\[
f(\phi;H) = A(\phi;H) \left[ \psi^{-2H-1} + B(\phi;H) \right] \quad \text{for } 0 < H < 1 \text{ and } -\pi < \phi < \pi
\]  

where,

\[
A(\phi;H) = 2 \sin(\pi H) \Gamma(2H+1) (1-\cos \phi)
\]  

\[
B(\phi;H) = \sum_{j=1}^{\infty} \left[ (2\pi j + \lambda)^{-2H-1} + (2\pi j - \lambda)^{-2H-1} \right]
\]

The infinite summation in Equation (5.22) can be approximated as

\[
B(\phi;H) = a_1^d + b_1^d + a_2^d + b_2^d + a_3^d + b_3^d + [(a_3^d + b_3^d + a_4^d + b_4^d) / 8H\pi]
\]

where \( d = -2H-1, \ a_k = 2k\pi + \phi, \ b_k = 2k\pi - \phi \).
The approximated power spectrum \( f'(\phi; H) \) is obtained by using Equation (5.23) in Equation (5.20). The algorithm for generating self-similar traces using \( f'(\phi; H) \) is given as under:

**Step 1**

Construct a sequence of values \( \{f_1, \ldots, f_{n/2}\} \) where \( f_j = f'(2\pi j/n; H) \), \( j = 1, 2, \ldots, n/2 \) and \( n \) is even.

**Step 2**

"Fuzz" each \( \{ f_j \} \) by multiplying it by an independent exponential random variable with mean 1. Call the fuzzed sequence \( \{ f'_j \} \).

**Step 3**

Construct \( \{ z_1, \ldots, z_{n/2} \} \), a sequence of complex values such that \( |z_i| = \sqrt{f'_i} \) and the phase of \( z_i \) is uniformly distributed between 0 and \( 2\pi \).

**Step 4**

Construct \( \{ z'_1, \ldots, z'_{n/2} \} \), an expanded version of \( \{ z_1, \ldots, z_{n/2} \} \) as

\[
z'_j = \begin{cases} 
0, & \text{if } j = 0 \\
z_i, & \text{if } 0 < j \leq n/2 \\
z_{n-j}, & \text{if } n/2 < j < n
\end{cases}
\]
where $\overline{z_{n,j}}$ denotes the complex conjugate of $z_{n,j}$. The set $\{ z'_j \}$ retains the power spectrum used in constructing $\{ z_j \}$, but it is symmetric about $z'_n/2$ and corresponds to the Fourier transform of a real-valued signal.

Step 5

Inverse Fourier transforms of $\{ z'_j \}$ are taken to obtain the approximate FGN sample path $\{ x_i \}$.

It is known that FGN has zero mean, implying the presence of negative samples in $\{ x_i \}$. But, in order to make use of the samples to simulate a certain number of cell arrivals per timeslot, the mean is shifted by an amount equal to the absolute value of $\min \{ x_i \}$. In order to get a desired mean for the trace, the entire trace is scaled by a constant. In the simulations, the period of the CBR traffic $d$ is assumed as 30 slots and hence to make the utilization of 75%, the FGN trace is scaled to a mean of 22.5. Since the number of cells arriving per slot is an integer, each sample is rounded off to the nearest integer.

5.6 ESTIMATION OF JITTER IN AN ATM MULTIPLEXER

To estimate jitter in an ATM network, the multiplexer model shown in Figure 5.1 is used. In between two CBR cells, one sample from the self-similar trace is introduced. The internal distribution of the arrival of cells per time interval is not important for the LRD, since it depends only on the number of arrivals per time interval. Hence, the self-similar cells are assumed to arrive uniformly in ‘d’ slots.
The delay is calculated for each of the CBR packets using the following algorithm:

1. The buffer occupancy is checked just before the arrival of a CBR packet using the following formula.
   Buffer occupancy \( h = \) the number of self-similar arrivals occurring in the previous ‘d’ slots + the delay of the previous CBR packet.

2. If \( h < 0 \), the delay for the CBR packet is 0 as the multiplexer buffer becomes empty before the arrival of the CBR packet.

3. If \( h > 0 \), the delay for CBR packet is \( h \).

The above algorithm is carried out for Poisson arrivals also. The Poisson arrivals are assumed to be uniformly distributed over ‘d’ time slots.

The calculations for both the self-similar and the Poisson cases are done for 32768 CBR samples. The histogram of the delays of the CBR packet is calculated and normalized to get the PDF of the delays. The buffer requirement is determined from the PDF in order to have a CLR not more than 1%. The buffer length required is the value of the delay at which the area under the PDF curve becomes 99%. Table 5.1 gives the buffer length required for CLR less than 1%, when the CBR traffic stream is multiplexed with self-similar background traffic for various values of \( H \). The ratio of buffer requirement for self-similar and Poisson traffic is also shown in the Table 5.1.
5.7 RESULTS AND DISCUSSIONS

Analytically it is found that when CLR < 1%, for the self-similar traffic, when H = 0.75 the required buffer size is 37 giving a ratio of 6.1 and when H = 0.85 the required buffer size is 51 giving a ratio of 8.5 respectively when compared with the Poisson traffic. Figure 5.2 shows the delay of CBR packets when multiplexed with self-similar traffic and Figure 5.3 shows the delay of CBR packets when multiplexed with Poisson traffic. From these results it is observed that a drastic increase in buffer size is required to maintain the CLR at a predetermined value. But, any increase in buffer size increases the cell delay. Figures 5.4 and 5.5 show the probability of delay for self-similar traces with H = 0.75 and 0.6 respectively.

It is found analytically that in self-similar traffic, for CLR < 1% and H = 0.8, the buffer requirement is 70 which is an increase by a factor of 6.3 compared to Poisson traffic. Table 5.1 gives the increase in buffer requirement obtained by simulations. For a Hurst parameter of 0.85, the increase in buffer size is by a factor of 8.5 and for H of 0.75 the increase in buffer size is by a factor of 6.

From the results obtained, it is seen that a drastic increase in buffer size is required to maintain the CLR at a predetermined value. But, any increase in buffer size increases the cell delay. Also, for the transport of CBR traffic in which the cell delay variation is very high, some additional measures have to be taken to maintain the guaranteed QoS. This could take the form of preferential treatment at the switches. Source level or switch level modeling of the traffic may be done in order to decrease the burstiness of the traffic.
If the relationship between $\alpha$ and $H$ is found, then the 2 Parameter Pareto Process will provide us with a general distribution function which can be used to model any self-similar process. This distribution can be used to make a random generator that can produce self-similar traces that can be used in simulation. Though, this self-similar trace generator maybe easier to understand than the existing methods, like the FFT method, it requires more time for self-similar trace generation.
Table 5.1 Buffer length requirements and the Ratio of Buffer length requirements as a function of the Hurst parameter

<table>
<thead>
<tr>
<th>Hurst Parameter (H)</th>
<th>Buffer length required for CLR &lt; 1% (self-similar case)</th>
<th>Ratio of buffer requirement of self-similar and Poisson traffic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.53</td>
<td>7</td>
<td>1.167</td>
</tr>
<tr>
<td>0.59</td>
<td>10</td>
<td>1.667</td>
</tr>
<tr>
<td>0.6</td>
<td>11</td>
<td>1.833</td>
</tr>
<tr>
<td>0.67</td>
<td>14</td>
<td>2.33</td>
</tr>
<tr>
<td>0.71</td>
<td>21</td>
<td>3.50</td>
</tr>
<tr>
<td>0.75</td>
<td>37</td>
<td>6.16</td>
</tr>
<tr>
<td>0.8</td>
<td>58</td>
<td>9.66</td>
</tr>
<tr>
<td>0.85</td>
<td>51</td>
<td>8.5</td>
</tr>
<tr>
<td>0.86</td>
<td>97</td>
<td>16.16</td>
</tr>
<tr>
<td>0.9</td>
<td>226</td>
<td>37.67</td>
</tr>
</tbody>
</table>
Figure 5.2 Delays of CBR packets multiplexed with Self-similar traffic 
\( (H = 0.6) \)
Figure 5.3 Delays of CBR packets multiplexed with Poisson Traffic
Figure 5.4 Delay Distribution of self-similar ($H=0.75$) and Poisson traffic