CHAPTER 3

ESTIMATION OF CELL LOSS RATIO FOR SELF-SIMILAR TRAFFIC IN ATM NETWORKS
- A FUZZY APPROACH

In this chapter, a novel fuzzy logic algorithm based on the $\alpha$-cuts of equivalence relation to estimate CLR in real time for self-similar traffic in ATM networks is presented. The $\alpha$-cuts of fuzzy sets are used to construct the membership functions of the input and output variables, and to induce the necessary fuzzy rules. The new approach not only estimates accurately the real time CLR but also achieves it by using fewer appropriate theoretical data.

3.1 INTRODUCTION

The emergence of the internet has caused rapid growth in the size and complexity of networks in the world. It has also resulted in a massive increase in the amount of traffic being carried by the 'backbone' networks. This traffic is seen to be "self-similar" in nature when the number of sources is very large. The backbone in most cases is an ATM network. Thus, ATM network must be studied with "self-similar" background traffic.

A common assumption in modeling computer networks is that arrivals occur as a Poisson process. However, data communications traffic levels fluctuate over time, and time delays due to congestion can occur even on lightly utilized links. These wild fluctuations can occur over very short periods of time.
giving rise to the concept of bursty traffic. Poisson processes do not exhibit such burstiness and are, therefore, not consistent with observed data. Studies conducted at the Bellcore Research Laboratories have shown that network traffic is much more closely modeled by self-similar processes (Leland et al 1993). They suggest that the Poisson process is inadequate as a model of the arrival process and that a fractal renewal process is necessary to model the observed results.

Mandelbrot (1965) examines error clustering in digital telephone circuits. One of his observations is that there are hierarchies of error clusters and that they are invariant with respect to time scale. This forms a significant part of his pioneering work in self-similarity and fractal behavior, which has aided the investigation on the behavior of packet based networks. This includes very accurate measurements of packet arrival times, and studies of their self-similar behavior.

Self-similarity manifests itself in a variety of ways: a spectral density that diverges at the origin, a non-summable auto correlation function and an Index of Dispersion of Counts (IDC) that increases monotonically with the sample time T, etc. (Boris Tsybakov and Georganas 1997). A key parameter, which characterizes the self-similar processes, is the Hurst parameter, H, which gives the degree of self-similarity and assumes values between 0.5 and 1 (0.5 < H < 1.0).

Self-similarity is the property associated with one type of fractal - an object whose appearance is unchanged regardless of the scale at which it is viewed. Since the self-similar process has observable bursts at a wide range of time scales, it can exhibit long-range dependence (LRD); values at any instant
which are typically non-negligibly, positively correlated with values at all future instants. Surprisingly (given the counterintuitive aspects of long-range dependence), the self-similarity of Ethernet network traffic has been rigorously established. The importance of long-range dependence in network traffic is beginning to be observed in studies by Leland (1994), Paxson (1994) and Beran (1995), which show that packet loss and delay behaviour are radically different when simulations use either real traffic data or synthetic data that incorporate long-range dependence.

Adaptive fuzzy logic system as explained in Chapter 2 is applied here for the prediction of CLR for self-similar models, and it is observed that it leads to big differences in the predicted CLR values when compared to the theoretical CLR values. Hence, the modified fuzzy logic system given by the Equation (2.36) with and without the correction factor is implemented. Here also it is observed that there is a big difference between the theoretical and fuzzy estimated CLR values. Hence, a novel fuzzy logic system to predict real time CLR for self-similar traffic in ATM networks is implemented and analyzed. In this latest fuzzy approach, a learning algorithm based on the $\alpha$-cuts of equivalence relations and the $\alpha$-cuts of fuzzy sets are used to construct the membership functions of the input and output variables (Tzu-ping Wu and Chen 1999). All the training data required for this system are computed from the mathematical expressions derived for self-similar ATM traffic model (Boris Tsybakov and Georganas 1997). This fuzzy learning algorithm has a higher average classification ratio and it generates fewer rules than the existing algorithms.
3.2 DEFINITION OF SELF-SIMILARITY

3.2.1 Continuous-time definition

A common definition of self-similar stochastic processes is based on a direct scaling of the continuous time variable, as follows. A stochastic process \( x(t) \) is statistically self-similar with parameter \( H \ (0.5 \leq H \leq 1) \) if for any real quantity \( a > 0 \), the process \( a^H x(at) \) has the same statistical properties as \( x(t) \) (William Stallings 1998). This relationship may be expressed by the following three conditions:

- \[ E[x(t)] = E[x(at)] / a^H = \text{Mean} \]
- \[ \text{Var} \ [x(t)] = \text{Var} \ [x(at)] / a^{2H} = \text{Variance} \]
- \[ R_x(t,s) = R_x(at, as)/a^{2H} = \text{Autocorrelation} \]

The parameter \( H \), known as the Hurst parameter, or the self-similarity parameter, is a key measure of self-similarity. More precisely, \( H \) is a measure of the persistence of a statistical phenomenon and is a measure of the length of the long-range dependence of a stochastic process. A value of \( H=0.5 \) indicates the absence of self-similarity. The closer \( H \) is to 1, the greater the degree of persistence on long-range dependence.

3.2.2 Discrete-time definition

For a stationary time series \( x \), the \( m \)-aggregated time series is defined as \( x^{(m)} = \{ x_{k}^{(m)}, k=0,1,2,\ldots \} \) by summing the original time series over nonoverlapping, adjacent blocks of size \( m \). This may be expressed as:

\[
x_k^{(m)} = \frac{1}{m} \sum_{i=km-(m-1)}^{km} x_i.
\]

For example, \( x^{(3)} \) is defined as \( x_k^{(3)} = 1/3 \ [ x_{3k-2} + x_{3k-1} + x_{3k}] \).
One way of viewing the aggregated time series is as a technique for compressing the time scale. $x^{(1)}$ can be considered to be the highest magnification or highest resolution possible for this time series. The process $x^{(3)}$ is the same process reduced in magnification by a factor of 3. By averaging over each set of three points, the fine detail available at the highest magnification level is lost. If the statistics of the process (mean, variance, correlation, etc.) are preserved with compression, then the process is self-similar.

Each point in the series $x^{(m)}$ can be viewed as a time average of the process $x$. For an ergodic process, a time average should equal an ensemble average, and the variance of the time average should go to zero relatively quickly as $m$ becomes large. This does not happen for a self-similar process the variance of which goes to zero but does so more slowly than for a stationary ergodic process.

A process $x$ is said to be exactly self-similar with parameter $\beta (0 < \beta < 1)$ if for all $m = 1, 2, 3, \ldots$, the variance is given by $\text{Var}(x^{(m)}) = \text{Var}(x) / m^\beta$ and the autocorrelation function is $R_x^{(m)}(k) = R_x(k)$. The parameter $\beta$ can be shown to be related to the Hurst parameter, as $H = 1 - (\beta/2)$. For a stationary, ergodic process, $\beta = 1$ and the variance of the time average decays to zero at the rate of $1/m$. For a self-similar process, the variance of the time average decays more slowly.

A process $x$ is said to be asymptotically self-similar if for all $k$, the variance is $\text{Var}(x^{(m)}) = \text{Var}(x) / m^\beta$, and the autocorrelation is $R_x^{(m)}(k) = R_x(k)$ as $m \to \infty$. Thus, according to this definition of self-similarity, the autocorrelation of the aggregated process has the same form as the original process.
This would suggest that the degree of variability, or burstiness would be the same at different time scales.

The autocorrelation of the aggregated self-similar process does not go to zero as \( m \to \infty \). This is in contrast to stochastic processes typically used for packet data models in which \( R^{(m)}(t) \to 0 \) as \( m \to \infty \). An autocorrelation function \( R(t) \) that is equal to zero is consistent with white noise. Another interesting feature is that the variance of \( x^{(m)} \) decreases more slowly than \( 1/m \) as \( m \to \infty \), that is, it decreases proportional to \( 1/m^\beta \). For stochastic processes typically used for packet data models, the variance decreases proportional to \( 1/m \). The variance of the sample mean is equal to the variance of the underlying random variable divided by \( m \). However, for the self-similar process, aggregating by a factor of \( m \) is not quite the same thing as taking a sample mean with sample size \( m \) because of the persistence of statistical properties across time scales.

Here, an asymptotically second order self-similar traffic in ATM network is considered. The theoretical CLR values are determined using the following equation, and these values are used for designing the fuzzy system.

For the self-similar model, the CLR is given by Boris Tsybakov and Georganas (1999),

\[
\text{CLR} = \frac{G}{\lambda \cdot E_t} \tag{3.1}
\]

where \( \lambda \) is the arrival rate and \( G \) is the buffer overflow probability given by,

\[
G = \frac{C[(h+e)/d + 2)]^{\gamma+1}}{\gamma (\gamma - 1) (E_i + E_k)^2} \tag{3.2}
\]
\[ E_i = C \sum_{i=1}^{\infty} n^{-\gamma} \quad (3.3) \]

\[ C = \frac{1}{\sum_{N=1}^{\infty} n^{-(\gamma-1)}} \quad (3.4) \]

\[ E_k = \frac{1}{(1-e^{-h})} - 1; \quad d = (E_i + E_k)^{1} \leq 1; \quad e = d = 1; \quad h \text{ is the buffer size and } \gamma = 1.5. \]

The theoretical CLR values are plotted for various buffer sizes by varying the arrival rate and is shown in Figure 3.1. The CLR values estimated theoretically and using the fuzzy system as a function of the buffer size are compared in Figure 3.2. Figures 3.3 and 3.4 show the comparison of the CLR values estimated theoretically and using the modified fuzzy system with and without correction factor as a function of the buffer size respectively. From these figures it is observed that there is big difference between the theoretical and the fuzzy estimated CLR values. Hence a novel fuzzy algorithm based on the \( \alpha \)-cuts of equivalence relation to estimate CLR in real time for self-similar traffic is chosen. It is observed that for low buffer sizes (cell region), the CLR variation is linear and for large buffer size (burst region), the CLR does not decrease significantly as shown in Figure 3.5. Hence, fuzzy membership functions are designed separately for the cell region and for the burst region. For self-similar traffic, the CLR values estimated by adaptive fuzzy, modified fuzzy with and without correction factor and novel fuzzy or compared in Figure 3.6 by varying the buffer size from 0-200 from this figure it is observed that the performance of novel fuzzy method is better compared to the other methods.
Figure 3.1 Theoretical CLR vs buffer size for self-similar traffic

Figure 3.2 Comparison of theoretical and Adaptive fuzzy estimated CLR values for self-similar traffic
Figure 3.3  Comparison of theoretical and modified fuzzy estimated CLR values for self-similar traffic without CF

Figure 3.4  Comparison of theoretical and modified fuzzy estimated CLR values for self-similar traffic with CF
Figure 3.5 Variation of CLR in the Cell and Burst regions for Self-similar model

Figure 3.6 Performance comparison of fuzzy techniques for CLR estimation
3.3 CONCEPTS OF FUZZY SETS

Zadeh (1965) proposed the theory of fuzzy sets. A fuzzy set can be characterized by a membership function. Let \( U \) be the universe of discourse, \( U = \{ x_1, x_2, \ldots, x_n \} \). A fuzzy set \( A \) in the universe of discourse \( U \) is a set of ordered pairs \( \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \ldots, (x_n, \mu_A(x_n))\} \), where \( \mu_A : U \rightarrow [0,1] \) is the membership function of the fuzzy set \( A \), and \( \mu_A(x_i) \) indicates the membership degree of \( x_i \) in the fuzzy set \( A \). If the universe of discourse \( U \) is an infinite set, then the fuzzy set \( A \) can be represented by,

\[
A = \int_{U} \mu_A(x) \, dx \quad x \in U
\]  

(3.5)

The triangular fuzzy set \( A \) shown in Figure 3.7 can be represented by a triplet \( (a, b, c) \) where \( a \) and \( c \) are respectively the left and right vertices of the triangular fuzzy set \( A \), and \( b \) is the mid point. The \( \alpha \)-cut \( A_\alpha \) of a fuzzy set \( A \) in the universe of discourse \( U \) is defined as follows:

\[
A_\alpha = \{x | \mu_A(x) \geq \alpha, x \in U\}, \alpha \in [0,1]
\]  

(3.6)

Based on the definition of \( \alpha \)-cuts, the fuzzy set \( A \) can be represented as follows:

\[
A = \bigcup_{\alpha \in [0,1]} A_\alpha
\]  

(3.7)
A fuzzy relation among fuzzy sets $A_1, A_2, \ldots, A_n$ is a subset of the Cartesian product $A_1 \times A_2 \times \ldots \times A_n$ and is denoted by $R(A_1, A_2, \ldots, A_n)$, where $A_1 \times A_2 \times \ldots \times A_n = \{(x_1, x_2, \ldots, x_n) | x_i \in A_i \text{ and } 1 \leq i \leq n \}$. The membership function of the fuzzy relation $R(A_1, A_2, \ldots, A_n)$ is represented by $\mu_R(x_1, x_2, \ldots, x_n)$, where $x_i \in A_i$ and $1 \leq i \leq n$. Let $R_1(A, B)$ and $R_2(B, C)$ be two binary fuzzy relations with a common fuzzy set $B$. The composition of $R_1(A, B)$ and $R_2(B, C)$ is denoted by $R_1(A, B) \circ R_2(B, C)$ and is defined as:

$$
\mu_{R_1 \circ R_2}(x, z) = \max_{y \in B} \min \left[ \mu_{R_1}(x, y), \mu_{R_2}(y, z) \right] \quad (3.8)
$$

where $(x, y) \in A \times B$, $(y, z) \in B \times C$ and $(x, z) \in A \times C$.

A binary fuzzy relation $R(A, A)$ has the following properties.

$$
\begin{align*}
\mu_R(x, x) &= 1, \forall x \in A & \text{Reflexive} \quad (3.9) \\
\mu_R(x, y) &= \mu_R(y, x), \forall x, y \in A & \text{Symmetric} \quad (3.10) \\
\mu_R(x, z) &\geq \max \min [ \mu_R(x, y), \mu_R(y, z) ] & \forall (x, z) \in A \times A & \text{Transitive} \quad (3.11) \\
R_i^i(A, A) &= R^i & \text{Transitive closure} \quad (3.12)
\end{align*}
$$

where $R^i = R^i \circ R$, $R^{i+1} = R^i$ and $i \geq 2$.

If $R$ is reflexive and symmetric, then it is a compatibility relation. If $R$ is reflexive, symmetric and transitive, then it is called a fuzzy equivalence relation. The $\alpha$-cuts of a binary fuzzy relation $R(A, A)$ can be defined as follows:

$$
R_\alpha = \{(x, y) | \mu_R(x, y) \geq \alpha, (x, y) \in A \times A \} \quad (3.13)
$$
where $\alpha \in [0,1]$. Thus, every binary fuzzy relation $R$ can be represented in terms of its $\alpha$-cuts $R_\alpha$ as,

$$R = \bigcup_{\alpha \in [0,1]} aR_\alpha$$  \hspace{1cm} (3.14)

Furthermore, it can be easily shown that a fuzzy equivalence relation $R$ (A.A) effectively groups elements into equivalence classes whose members are similar to each other in some specified degree by taking an $\alpha$-cut $R_\alpha$, where $\alpha \in [0,1]$. Each of these equivalence classes forms a partition of $A$. Two elements $x,y$ of $A$ belong to the same block of partition if and only if $R(x, y) \geq \alpha$.

3.4 NOVEL FUZZY INFERENCE SYSTEM DESIGN

If a fuzzy system has $n$-inputs and a single output, then its fuzzy rules can be of the following general format:

$$R_j: \text{If } X_1 \text{ is } A_{1j} \text{ AND } X_2 \text{ is } A_{2j} \text{ AND } X_3 \text{ is } A_{3j} \ldots \text{, } X_m \text{ is } A_{mj} \text{ THEN } Y \text{ is } B_j$$

where the variables $X_i$ ($i=1,2,\ldots,n$) appearing in the antecedent parts of the fuzzy rules $R_j$ are called the input linguistic variables, the variable $Y$ in the consequent part of the fuzzy rule $R_j$ is called the output linguistic variable, the fuzzy sets $A_{ij}$ are called the input fuzzy sets of the input linguistic variable $X_i$ of the fuzzy rule $R_j$, and the fuzzy set $B_j$ is called the output fuzzy set of output linguistic variable $Y$ of the fuzzy rule $R_j$.

The fuzzy rules can be learnt from the data obtained either numerically or experimentally. Let us assume that there are $m$ input-output pairs of numerical training data set $D$ of the $n$-input- single- output fuzzy system as,
where $x_{i,j}$ is the value of the $i$th input linguistic variable $X_i$ of the $j$th input-output pair and $y_j$ is the value of the output linguistic variable $Y$ of the $j$th input-output pair.

Generally, in all data networks the arrival rate $\lambda$, the occupancy of the buffer $h$, and the CLR exhibit stochastic behaviour. This uncertain nature of the network traffic process can be adequately captured by the fuzzy approach with $\lambda$ and $h$ as input variables and CLR as the single output variable, i.e. the process can be considered as Multiple Input and Single Output (MISO) system. Since the outputs of the MISO fuzzy model are independent, the general rule structure of the MISO fuzzy model can easily be represented as a collection of the rules of the multiple-inputs-multiple-outputs (MIMO) fuzzy models.

As explained in the previous section, a fuzzy equivalence relation is reflexive, symmetric and transitive and it can divide the crisp data into different groups by its $\alpha$-cuts. Instead of finding the fuzzy equivalence relation directly, a fuzzy compatibility relation (reflexive and symmetric) is determined in terms of an appropriate distance function applied to the data. Then, a fuzzy equivalence relation can be obtained by the max-min transitive closure of the fuzzy compatibility relation. (Tzu Ping and Chen 1999).

For the fuzzy system explained above, the input training data set $D$ is generated using the analytical expressions for CLR given in Equation 3.1. This training data set $D$ (Table 3.1) is then sorted and arranged in ascending order of CLR, the output variable (Table 3.2). The sorted training data set $D'$ is represented as:

$$D = \{(x_{i,j}, \ldots, x_{n,j}, y_j) | j = 1, 2, \ldots, m\} \quad 1 \leq i \leq n$$

(3.15)
where \( y_1 < y_2 < y_3 < \ldots y_m \).

Here \( x_{i1}, x_{i2}, \text{ and } y_i \) correspond to the arrival rate \( \lambda_i \), the buffer occupancy \( h_i \) and the output CLR respectively, where \( i = 1, 2, \ldots, m \). For convenience, we represent \( D' \) as,

\[
D' = \begin{pmatrix}
  x_{11} & x_{12} & y_1 \\
  x_{21} & x_{22} & y_2 \\
  x_{31} & x_{32} & y_3 \\
  \vdots & \vdots & \vdots \\
  x_{m1} & x_{m2} & y_m
\end{pmatrix}
\]  \quad (3.16)

The elements of the fuzzy compatibility relation matrix \( R \), between the output variables of the sorted training data set \( D' \), can be defined in terms of Euclidean distance as follows (Klir and Yuan 1995):

\[
T_{ij} = 1 - \frac{d_{ij}}{\delta} \quad i,j = 1,2,3,\ldots m
\]  \quad (3.18)

where \( \delta \) is a constant that ensures the compatibility relation \( R [d_{ij}, d_{ij}] \in [0, 1] \) and is given by,

\[
\delta = \frac{\sum_{i=1}^{m-1} (d_{ij} - d_{(max)})}{(m - 1)}
\]  \quad (3.19)
with \( d_{(\text{max})} = \max_{1 \leq i \leq m} (d_{i3}) \)

where \( d_{(\text{max})} \) is the maximum value of the output linguistic variable CLP in the sorted training data set \( D' \). In general, the relation defined in Equation (3.10) is a fuzzy compatibility relation, but it is not necessarily a fuzzy equivalence relation. The fuzzy equivalence relation \( R^T(d_{13}, d_{23}) \) between the output values \( d_{13} \) and \( d_{23} \) of the output linguistic variable CLP in the sorted training data set \( D' \) can be obtained from the max-min transitive closure of the compatibility relation \( R^T(d_{13}, d_{23}) \).

From the compatibility relation, the fuzzy equivalence relation matrix \( R^T \), can be obtained by the max-min transitive closure as \( R^T = R \circ R \). The elements of \( R^T \) are given by

\[
\gamma_{ij}^T = \max \left\{ \min( \gamma_{ji}, r_{ij} ) \right\} \tag{3.20}
\]

where \( 1 \leq i,j \leq m \).

The resultant fuzzy equivalence relation matrix \( R^T \) for \( m=8 \) (for example) can be written as

\[
R^T = \begin{bmatrix}
1 & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} \\
S_{21} & 1 & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} \\
S_{31} & S_{32} & 1 & S_{34} & S_{35} & S_{36} & S_{37} & S_{38} \\
S_{41} & S_{42} & S_{43} & 1 & S_{45} & S_{46} & S_{47} & S_{48} \\
S_{51} & S_{52} & S_{53} & S_{54} & 1 & S_{56} & S_{57} & S_{58} \\
S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & 1 & S_{67} & S_{68} \\
S_{71} & S_{72} & S_{73} & S_{74} & S_{75} & S_{76} & 1 & S_{78} \\
S_{81} & S_{82} & S_{83} & S_{84} & S_{85} & S_{86} & S_{87} & 1 \\
\end{bmatrix} \tag{3.21}
\]
The symmetric sub matrices shown by dotted lines within $R^T$ of Equation (3.21) are obtained such that the elements about the diagonal are greater than or equal to the $\alpha$-cut value. In the above example of $R^T$, we have shown 3 sub matrices with the number of rows $r_1 = 3$, $r_2 = 3$ and $r_3 = 2$. A numerical example is given in Appendix A.

In general, there may be $N$ such symmetric sub matrices having $r_1$, $r_2$, ..., $r_N$ as their number of rows. The input-output pairs of the sorted training data matrix $D'$ of Equation (3.17) is divided into $N$ subsets viz., $S_1$, $S_2$, ..., $S_N$ having $r_1$, $r_2$, ..., $r_N$ elements respectively. The partitioned subsets are given by

$$
S_1 = \{ (x_{11}, x_{12}, y_1), (x_{21}, x_{22}, y_2), \ldots, (x_{r_1}, x_{r_2}, y_{r_1}) \}
$$

$$
S_2 = \{ (x_{(r_1+1)1}, x_{(r_1+1)2}, y_{(r_1+1)}), (x_{(r_1+2)1}, x_{(r_1+2)2}, y_{(r_1+2)}), \ldots, (x_{(r_1+r_2)1}, x_{(r_1+r_2)2}, y_{(r_1+r_2)}) \}
$$

$$
S_3 = \{ (x_{(r_1+r_2+1)1}, x_{(r_1+r_2+1)2}, y_{(r_1+r_2+1)}), (x_{(r_1+r_2+2)1}, x_{(r_1+r_2+2)2}, y_{(r_1+r_2+2)}), \ldots, \}
$$

$$
S_N = \{ (x_{(r_1+r_2+\ldots+r_{N-1}+1)1}, x_{(r_1+r_2+\ldots+r_{N-1}+1)2}, y_{(r_1+r_2+\ldots+r_{N-1}+1)}), \ldots, (x_{(r_1+r_2+\ldots+r_{N})1}, x_{(r_1+r_2+\ldots+r_{N})2}, y_{(r_1+r_2+\ldots+r_{N})}) \}
$$

(3.22)

The output value sets corresponding to the output linguistic variable $y$ are obtained from the subset $S_i$ as,

$$
A_1 = \{ y_1, y_2, \ldots, y_{r_1} \}; \hspace{1cm} A_2 = \{ y_{r_1+1}, y_{r_1+2}, \ldots, y_{r_1+r_2} \}; \ldots \text{ and } \hspace{1cm} A_N = \{ y_{r_1+r_2+\ldots+r_{N-1}+1}, y_{r_1+r_2+\ldots+r_{N-1}+2}, \ldots, y_{r_1+r_2+\ldots+r_{N}} \}.
$$

(3.23)
The corresponding input value sets for the input linguistic variable $x_i$ are

$$X_{i1} = (x_{i1}, x_{i2}, x_{r1.1}); \quad X_{i2} = (x_{i1+1}1, x_{i1+2}1, x_{i1+2}2) \quad \text{and}$$

$$X_{iN} = (x_{i1+1}1+i, x_{i1+2}1+i, x_{i1+2}2+i) \quad (3.24)$$

The input value sets corresponding to the input linguistic variable $x_2$ are given by

$$X_{21} = (x_{12}, x_{22}, x_{r1.2}); \quad X_{22} = (x_{12+1}2, x_{12+2}2, x_{12+2}3) \quad \text{and}$$

$$X_{2N} = (x_{12+1}2+i, x_{12+2}2+i, x_{12+2}3+i) \quad (3.25)$$

From these sets, the triangular membership functions are derived for the output value set as well as for the input value sets. The mid point of the triangular membership function $b_1$ and the left vertex $a_1$ and right vertex $c_1$ of the output set $A_1$ are given by

$$b_1 = \left( \text{first element of } A_1 + \text{last element of } A_1 \right) / 2$$

$$a_1 = b_1 - \frac{b_i - y_i}{1 - \alpha} \quad (3.26)$$

$$c_1 = b_1 + \frac{y_i - b_i}{1 - \alpha} \quad (3.27)$$

Then, the membership function for $A_1$ is given by,

$$\mu_{A_1}(y) = \begin{cases} 
\frac{y - a_1}{b_i - a_i} & \text{if } a_1 \leq y \leq b_1 \\
\frac{c_i - y}{c_i - b_i} & \text{if } b_1 \leq y \leq c_1 \\
0 & \text{otherwise}
\end{cases} \quad (3.29)$$
The above said procedure is followed to obtain the membership function for the sets \( A_2, A_3, \ldots, A_n \); \( X_{12}, X_{13}, \ldots, X_{1N} \) and \( X_{22}, X_{23}, \ldots, X_{2N} \).

After obtaining the membership functions, the necessary rules are generated based on the hierarchical relationship between the fuzzy set \( A_i \) of the output linguistic variable \( y \), and the corresponding fuzzy sets of the input linguistic variables \( x_1 \) and \( x_2 \).

From the hierarchical relationship, the required fuzzy rules corresponding to the input linguistic variables \( x_1, x_2 \) and the output variable \( y \) can be derived as:

- If \( x_1 \) is \( X_{11} \) AND \( x_2 \) is \( X_{21} \) THEN \( y \) is \( A_1 \)
- If \( x_1 \) is \( X_{12} \) AND \( x_2 \) is \( X_{22} \) THEN \( y \) is \( A_2 \)
- If \( x_1 \) is \( X_{13} \) AND \( x_2 \) is \( X_{23} \) THEN \( y \) is \( A_3 \)
- \( \ldots \)
- If \( x_1 \) is \( X_{1N} \) AND \( x_2 \) is \( X_{2N} \) THEN \( y \) is \( A_N \).
The membership functions for the input variables $x_1$, $x_2$ and the output variable $y$ are derived and the required fuzzy rules are generated, using the analytical CLR value. Then the output CLR is given by

$$y = \left\{ \frac{b_{k1}z_{1,k1} + b_{k2}z_{2,k2}}{z_{1,k1} + z_{2,k2}} \right\}$$

(3.32)

where $z_{1,k1} = \text{non zero min} \left\{ \mu_{x1j} (x_1) \right\}$

(3.33)

and $k1$ is the value for which $\mu_{x1j}(x_1)$ is minimum.

where $z_{2,k2} = \text{non zero min} \left\{ \mu_{x2j} (x_2) \right\}$

(3.34)

and $k2$ is the value for which $\mu_{x2j}(x_2)$ is minimum.

While deriving the membership functions, the output region is divided into cell region and burst region. In the cell region the buffer occupancy is small and the cell loss occurs because of large arrival rate, whereas in the burst region the occupancy of the buffer is large but the arrival rate is very high compared to the service rate. Therefore, the nature of CLR becomes non-linear for self-similar traffic. The sample example given below illustrates the prediction of CLR value using this novel fuzzy approach.

### 3.5 SAMPLE EXAMPLE

The training data set, $D^t$, given in Table 3.2, is used to predict the values of CLR using the proposed fuzzy logic approach. To illustrate this fuzzy approach, the values of $\lambda$ and $h$ are taken as 0.5 and 1300 respectively. The
membership values of input variables $x_1$ and $x_2$ which corresponds to $\lambda$ and $h$ respectively are determined.

The computed membership values of $x_1$ and $x_2$ namely, $\mu_{x_{12}}(x_1)$ and $\mu_{x_{22}}(x_2)$ respectively are given below.

$$\mu_{x_{12}}(x_1) = \frac{0.513 - 0.5}{0.06} = 0.2167; \quad \mu_{x_{22}}(x_2) = \frac{1437.5 - 1300}{187.5} = 0.733 \quad (3.35)$$

Using the firing rules given in Table 3, corresponding to $\mu_{x_{12}}(x_1)$ and $\mu_{x_{22}}(x_2)$ the value of $b_2$ is obtained. Now the output CLR value denoted as $y$ is computed as

$$y = \frac{b_1\mu_{x_{12}}(x_1) + b_2\mu_{x_{22}}(x_2)}{\mu_{x_{12}}(x_1) + \mu_{x_{22}}(x_2)} \quad (3.36)$$

In this example $b_1 = b_2$ and therefore $y = b_2 = 12.235$. Accordingly, the predicted CLR value is $12.235 \times 10^4$ for the given input, which is comparable to the theoretical CLR value of $12.46 \times 10^4$

3.6 RESULTS AND DISCUSSIONS

For the self-similar traffic in ATM networks considered here the buffer size is varied from 0 to 10000 and arrival rate is varied from 0.1 to 0.9 to obtain the theoretical CLR values. It is observed that in self-similar traffic model, the CLR varies in a linear manner with small buffers in the cell region, whereas at larger buffers the CLR does not decrease significantly in the burst region. Further, it is also observed that when the occupancy of the buffer is in
the range of 1000 to 1300, there is a drastic change in the fuzzy estimated CLR value. To overcome this, the membership functions are derived separately for the buffer size less than 1300 and greater than 1300. But, it is noticed that the estimation of CLR using the fuzzy algorithm leads to a sharp increase in the percentage of deviation during the transition from cell region to burst region. This is due to the change in the control algorithm from one set of fuzzy rule base to another. Figure 3.8 shows the predicted and the theoretical CLR values for arrival rates $\lambda = 0.2$. Comparison of the CLR values estimated theoretically and using the fuzzy system for arrival rates and buffer size chosen at random is given in Table 3.4. Table 3.5 gives the mean square error of the output CLR for different arrival rates. From these tables, it is clear that the novel fuzzy algorithm estimates CLR values accurately.
Figure 3.8 Comparison of theoretical and novel fuzzy predicted CLR values for Self-similar traffic

\[
\text{Log CLR} \times 10^{-3}
\]

Buffer size in cells

- Theoretical
- Novel Fuzzy predicted

\[
\text{Lambda} = 0.3 \\
\text{Hurst factor} = 0.6
\]
### Table 3.1 Training Data set D

<table>
<thead>
<tr>
<th>$X_1$ (Arrival rate)</th>
<th>$X_2$ (Buffer size)</th>
<th>$y$ (Output CLR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ 0.8</td>
<td>1200</td>
<td>14.52</td>
</tr>
<tr>
<td>0.7</td>
<td>1200</td>
<td>14.29</td>
</tr>
<tr>
<td>0.5</td>
<td>1400</td>
<td>12.01</td>
</tr>
<tr>
<td>0.2</td>
<td>500</td>
<td>10.74</td>
</tr>
<tr>
<td>0.1</td>
<td>120</td>
<td>10.44</td>
</tr>
<tr>
<td>0.5</td>
<td>1300</td>
<td>12.46</td>
</tr>
<tr>
<td>0.6</td>
<td>1100</td>
<td>14.42</td>
</tr>
<tr>
<td>0.4</td>
<td>1100</td>
<td>12.18</td>
</tr>
</tbody>
</table>

$D = \begin{align*}
0.1 & \quad 120 & \quad 10.44 \\
0.2 & \quad 500 & \quad 10.74 \\
0.5 & \quad 1400 & \quad 12.01 \\
0.4 & \quad 1100 & \quad 12.18 \\
0.5 & \quad 1300 & \quad 12.46 \\
0.7 & \quad 1200 & \quad 14.29 \\
0.6 & \quad 1100 & \quad 14.42 \\
0.8 & \quad 1200 & \quad 14.52 
\end{align*}$

### Table 3.2 Sorted training data set $D'$

<table>
<thead>
<tr>
<th>$x_1$ (Arrival rate)</th>
<th>$x_2$ (Buffer size)</th>
<th>$y$ (Output CLR x $10^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ 0.1</td>
<td>120</td>
<td>10.44</td>
</tr>
<tr>
<td>0.2</td>
<td>500</td>
<td>10.74</td>
</tr>
<tr>
<td>0.5</td>
<td>1400</td>
<td>12.01</td>
</tr>
<tr>
<td>0.4</td>
<td>1100</td>
<td>12.18</td>
</tr>
<tr>
<td>0.5</td>
<td>1300</td>
<td>12.46</td>
</tr>
<tr>
<td>0.7</td>
<td>1200</td>
<td>14.29</td>
</tr>
<tr>
<td>0.6</td>
<td>1100</td>
<td>14.42</td>
</tr>
<tr>
<td>0.8</td>
<td>1200</td>
<td>14.52</td>
</tr>
</tbody>
</table>
Table 3.3 Input and output ranges for defuzzification

<table>
<thead>
<tr>
<th></th>
<th>72.5 ≤ x₂ ≤ 237.5</th>
<th>1062.5 ≤ x₂ ≤ 1250</th>
<th>1087.5 ≤ x₂ ≤ 1150</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₂</td>
<td>X₂₁</td>
<td>X₂₂</td>
<td>X₂₃</td>
</tr>
<tr>
<td>x₁</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.08 ≤ x₁ ≤ 0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15 ≤ x₁ ≤ 0.212</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.39 ≤ x₁ ≤ 0.45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.45 ≤ x₁ ≤ 0.513</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.515 ≤ x₁ ≤ 0.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7 ≤ x₁ ≤ 0.825</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b₁, 10.59, b₂, 12.235, b₃, 14.405
Table 3.4 Sample comparison of theoretical CLR and fuzzy predicted CLR

<table>
<thead>
<tr>
<th>Arrival Rate</th>
<th>Buffer Size</th>
<th>Output CLR x 10^4</th>
<th>Percentage deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Theoretical</td>
<td>Fuzzy Predicted</td>
</tr>
<tr>
<td>0.2</td>
<td>500</td>
<td>10.77</td>
<td>11</td>
</tr>
<tr>
<td>0.3</td>
<td>1500</td>
<td>8.68</td>
<td>8.38</td>
</tr>
<tr>
<td>0.4</td>
<td>1000</td>
<td>12.77</td>
<td>13</td>
</tr>
<tr>
<td>0.5</td>
<td>3000</td>
<td>8.21</td>
<td>8.21</td>
</tr>
<tr>
<td>0.6</td>
<td>5000</td>
<td>6.77</td>
<td>6.8</td>
</tr>
<tr>
<td>0.7</td>
<td>1000</td>
<td>15.65</td>
<td>14</td>
</tr>
<tr>
<td>0.8</td>
<td>5000</td>
<td>7.12</td>
<td>6.84</td>
</tr>
<tr>
<td>0.9</td>
<td>7000</td>
<td>6.04</td>
<td>6.62</td>
</tr>
</tbody>
</table>

Table 3.5 Mean square errors for buffer occupancy from 100 – 10000

<table>
<thead>
<tr>
<th>Arrival Rate</th>
<th>MSE = Σ(y_{ith} - y_{ipr})^2 / N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3.1453 x 10^{-7}</td>
</tr>
<tr>
<td>0.2</td>
<td>1.3702 x 10^{-7}</td>
</tr>
<tr>
<td>0.3</td>
<td>2.5719 x 10^{-7}</td>
</tr>
<tr>
<td>0.4</td>
<td>4.0593 x 10^{-7}</td>
</tr>
<tr>
<td>0.5</td>
<td>5.8665 x 10^{-7}</td>
</tr>
<tr>
<td>0.6</td>
<td>7.2922 x 10^{-7}</td>
</tr>
<tr>
<td>0.7</td>
<td>6.5463 x 10^{-7}</td>
</tr>
<tr>
<td>0.8</td>
<td>4.7555 x 10^{-8}</td>
</tr>
<tr>
<td>0.9</td>
<td>4.7745 x 10^{-8}</td>
</tr>
</tbody>
</table>

n – Number of samples
y_{ith} – Theoretical CLR
y_{ipr} – Fuzzy predicted CLR
MSE – Mean Square Error