CHAPTER 4
NONLINEAR STATIC AND DYNAMIC ANALYSIS OF TRANSMISSION LINE SYSTEM UNDER BROKEN WIRE CONDITION

4.1 INTRODUCTION

If cascading structural failures are to be prevented in overhead transmission line system, it is necessary for the support structures to be designed to withstand longitudinal unbalanced loads resulting from the sudden loss of tension in wires on one side of a support structure. On the other hand, with the introduction of Reliability-Based Design of transmission line structure, these unbalanced longitudinal forces will be a decisive factor for the load analysis of critical members to calculate the structural system reliability. The present design codes and specifications do not give adequate information to the line designer as to the nature and magnitude of the dynamic loads which might be expected during broken wire condition.

Nonlinear static and dynamic analysis is carried out to study the dynamic behaviour of transmission line systems when one or more wires suddenly break. The nonlinearity of the system results from the nonlinear sag-tension relationship of the wires and the nonlinear force-swing relationship. A lumped mass mathematical model is used and the nonlinear equations of motion are solved using numerical analysis techniques.

A typical 230 kV transmission tower system is analyzed for dynamic loads and the results compared with those obtained using IS:802 (1977) and IS:802 (1989) code of practice.
4.2 CASCADE COLLAPSES AND A STATE OF ART OF THE PRESENT CODAL PROVISIONS FOR LONGITUDINAL LOADS

The sudden breakage of wires in a transmission line system causes sudden unbalanced forces in the system resulting in dynamic movements in the system. The subsequent dynamic loads acting on the system can be very large in magnitude, and can result in the failure of one or more support structures. These dynamic loads may cause a cascading type of failure, such as that which occurred in 1991, when 25 km of a transmission line system in Chittagong, Bangladesh, collapsed (Daily News, 1991). Many similar failures of varying magnitude have also been reported (Ahluwalia, 1992). In January, a major structural failure occurred in an electric transmission line in Wisconsin, USA (Anon, 1975). Very recently, the CIGRE has made a survey on the ‘Recent transmission line failures’, (CIGRE-22, 1992) and found that the most of the failures reported all over the world comprise cascade failures of the line. An investigation showed that the failure was apparently initiated by the failure in a single tower during 1991 in Bangladesh. The collapse of the first tower resulted in suddenly applied high unbalanced forces in the longitudinal line direction on the adjacent towers, due to the tension in the wires in the adjacent spans. Although the towers were probably designed for a minimum unbalanced longitudinal load, in accordance with accepted practice, the magnitude of the resulting forces and the associated impact, in that situation, were obviously much greater than had been considered in the design of the line system thus resulting in a cascading type failure. This occurrence, and similar failures lead one to ask whether existing and proposed transmission lines are adequately designed for longitudinal impact loads due to broken wires or if additional catastrophic failures are waiting for a triggering action, either natural or man made, to set them off. Keeping this view in mind, series of model and full scale tests, were performed in the Electric Power Research Institute (EPRI), and it was shown that dynamic impact loads with magnitudes which are three to four times greater than the residual static loads, can be applied to the support structures when wires suddenly break (Peyrot et al. 1978). A review of many other experimental investigations have also shown that very high impact
loads can occur due to broken wires. In India, series of scale down model testing have been performed at the Central Power Research Institute (CPRI), Bangalore to get the dynamic responses under broken wires and it was shown that the impact factors varied up to five (Ramesh et al. 1985).

A study of present codes and design guides can serve as a measure of the state of the art since they represent an embodiment of past experiences and practices. The American Society of Civil Engineers (ASCE) has published its Manual as ‘Guidelines for Transmission line structural loading’ (1991) for the design and analysis of transmission line system. The recommendation made by this guide is that the longitudinal force on the structure will be the tension in the cable at the time of break plus the impact of the sudden application of this load on the tower. The guide also suggests the use of a computer analysis program to get the impact loads due to broken conductors. The Manual also gives an alarm to the line designer to use longitudinally strong suspension structures at intervals of ten to fifteen structures to contain cascade failures unless experience or local conditions justify their exclusion. The ASCE Manual-52 (1988) as ‘Guide for Design of steel Transmission Towers’ also specifies six basic loadings with the longitudinal loading cases to provide sufficient tower strengths to resist possible cascade failures, which is in line with the ANSI National Electricity Safety Code (NESC, 1984).

The EPRI report on ‘Reliability-Based Design of Transmission line structures’ (EPRI Report-1352, 1987) recommends two design philosophies to prevent extensive cascade failures. One design philosophy that is to specify strain structures capable of arresting the cascade at fixed short intervals. Such a philosophy has been used in California, USA where the code of that state specifies that ‘the line as a whole shall be designed so that the failure of an individual support structure shall not cause successive failures of more than ten additional structures’. Another design philosophy is to provide the structures with enough longitudinal strength so that a cascade would be arrested after the failure of a very few structures, say one to three. The above EPRI report also suggests that the
broken wire loads equal to 1.5 times the static residual tension of the bare wire after a nearby line breaks, be used in design.

The IEEE has also published a series of reports to serve as aids to designers (Ghannoum 1983, 1984; Peyrot et al. 1983). There are no specific criteria for determining longitudinal loads on tangent or suspension structures, but recommends the NESC loads as minimum load requirements.

For failure containment loads, the IEC-TC/11 (1991) proposes longitudinal loads equal to the imbalance produced by bare wires on one side of the structure and wires twice as heavy on the other side.

In India, IS:802 (1977) specifies a load factor of 1.5 under broken wire condition for all directional loadings. But the proposed IS:802 (1989) specifies a load factor of 0.8 on the tension in the conductor or conductor bundle in the case of broken conductor with a suspension string and 1.6 in the case of broken conductor with a tension insulator string. It also specifies a load factor of 1.6 on the tension of the ground wire in the case of ground wire broken condition with suspension or tension clamps.

The study of the above mentioned codes and guides reveals their obvious limitations with respect to proper determination of unbalanced longitudinal loads. There are no specific requirements with respect to the ability of the structures to sustain minimum longitudinal loads. The overload factors specified for suspension structures are much lower than the impact factors obtained from experimental data for the case of sudden wire breakage in the line system. There are also no provisions specifically designed to limit or prevent failures due to such loads. Thus the designer is left to himself to establish and justify the criteria for appropriate longitudinal loading for each individual case of transmission line design.

A number of investigators in the past few decades have studied the problem of determining longitudinal unbalanced loads on transmission support structures
caused either by unbalanced ice loads or by broken wires. The analytical investigations were concerned with providing mathematical models for the line system or else consisted mainly of the development of computer programs. Essentially all of the analytical analyses have been directed toward the determination of the residual static unbalanced longitudinal loads in the system after all dynamic effects have ceased.

Peyrot et al. (1978a, 1978b) described an iterative numerical procedure which can be used to analyze flexible transmission lines for static loads. A general purpose static analysis computer program has been developed by Fleming et al. (1978) under EPRI project. The program is completely general so that line systems with variable stiffness support structures and span lengths may be analyzed for any broken wire or differential ice load combination. Recently, Siddiqui et al. (1984) have proposed a general purpose procedure for computing the dynamic response of electric transmission line systems when one or more wires suddenly break. Alam et al. (1993d) have proposed a dynamic nonlinear analysis procedure for the transmission line system under broken wire conditions, to get the dynamic amplification factors and made a comparison with the relevant Indian Standards with respect to the overload factor specified.

If transmission line systems are to be designed to properly withstand the peak dynamic stresses, (in order to reduce the probability of a cascading type collapse) a method must be available for analyzing the dynamic behaviour of transmission line systems. The following discussion describes an analysis procedure and a corresponding computer program, which can be used to analyze a transmission line system for the dynamic effects due to any combination of broken wires.

4.3 NONLINEAR BEHAVIOUR OF THE SYSTEM

During the analysis, the displacements of the system are related to the unbalanced forces in the system by the structural stiffness. Unfortunately, the
structural stiffness cannot be considered to remain constant during the analysis, as is the case in more conventional structural system, since the force-deformation relationship for the system is highly nonlinear. This relationship has shown that as the displacement changes, the stiffness changes in a nonlinear manner.

The nonlinearity in the force-displacement relationship results from two factors. First, the individual wires exhibit a nonlinear sag-tension relationship. Since the axial stiffness of a wire varies with the sag, the axial stiffness varies in a nonlinear manner with the tension in the wire. Second, the change in the angle of inclination of a suspension insulator varies nonlinearly with the horizontal force component applied at the bottom of the insulator. The actual angle of inclination depends upon the difference in conductor tensions in the spans on either side of the insulator. These conductor tensions also change with the swing of the insulator.

Another possible source of nonlinearity is in the support structures themselves since it is possible for very flexible support structures to undergo relatively large longitudinal displacements due to unbalanced longitudinal loads in the line system. The additional moment due to the vertical loads, at the wire attachment points, acting through the moment arms corresponding to the horizontal displacements can cause additional displacements. If an accurate analysis procedure is to be developed it must consider the above nonlinear effects.

4.4 MATHEMATICAL MODEL OF THE TRANSMISSION LINE SYSTEM

The breakage of a wire in a transmission line system disturbs the system equilibrium and causes those points no longer in equilibrium to accelerate. These movements rapidly transit through the entire system until all points in the system are moving. In order to determine the displacements, velocities and accelerations in the system at any time, it is necessary to perform a dynamic analysis of the total system.
In an actual structural system the mass and corresponding inertia forces are distributed continuously throughout the system. The development of an exact dynamic solution for this condition usually results in considerable mathematical difficulty. For many structural systems the analytical problem can be simplified by considering a mathematical model in which the distributed mass is concentrated at discrete points, in which case inertia forces are only considered at these concentrated mass points. For this type of mathematical model it is only necessary to define the displacements and accelerations at these discrete mass points in order to define the deformed shape of the system. The number of displacement components which must be considered, in order to represent the effects of all significant inertia forces, is defined as the number of dynamic degrees of freedom of the structure.

To simplify the analysis procedure used this investigation, certain restrictions are placed upon the transmission line system which will be analyzed. It is assumed that the wires are either rigidly clamped to the support structures or attached by a single insulator string. Dead-end insulators are considered to merely be an extension of the wire, therefore, for this case the wire is considered to be clamped to the support structure. All attachment points, for any particular wire, are assumed to be at the same elevation and the line is assumed to be essentially straight with no large horizontal angles. It is also assumed that no external dynamic loads, such as wind or seismic excitations, are present. For a system of this type the primary unbalanced forces, due to one or more broken wires, are in the longitudinal direction. The most significant motions of the support structures occur in the longitudinal line direction. However, the suspension insulator ends and the conductors undergo movements in both the vertical and longitudinal line directions due to the swinging of the insulators. Consequently, mainly longitudinal inertial forces act on the support structures, whereas both vertical and longitudinal inertia forces on the conductors and insulators. The forces acting on the conductors and insulators are not independent of each other and depend on the angle of inclination of the insulator. In the mathematical model used in this investigation, only longitudinal degrees of freedom will be considered. The mass of the system
will be considered to be lumped at the top and bottom of the suspension insulators and at as many additional points on the support structures as desired in order to accurately represent their dynamic properties. The form of the lumped mass mathematical model for a typical system, composed of five intermediate support structures, with two conductors, supported by suspension insulators, and a ground wire were clamped directly to the support structures, is shown in Figure 4.1. It is assumed that the end support structures are designed as dead-end structures and can be considered to be rigid during the analysis. Therefore, the ends of the wires, at these and support points, can be assumed to be rigidly supported. It is also assumed that the material in the support structures and wires remain elastic under the loads applied to the system.

4.5 STATIC ANALYSIS FORMULATION OF THE TRANSMISSION LINE SYSTEM

In order to determine the static equilibrium position of the system for the broken wire condition, a static analysis must be performed. The object of the static analysis is to find a deformed position of the system in which no unbalanced internal forces exist at any point in the system.

The stiffness method is applied in the static analysis of the broken wire structural system. This method consists of determining the individual stiffness of each of the elements of the system and then combining these stiffnesses to obtain the overall stiffness of the system. The displacements are related to the loads and the overall stiffness of the system by the equilibrium equations:

$$[K] \{D\} = \{W\}$$  \hspace{1cm} (4.1)

where $[K]$ is the overall stiffness matrix of the system, $\{D\}$ are the displacements corresponding to the degrees of freedom of the system being considered in the analysis and $\{W\}$ are the unbalanced loads due to the broken wires. For a transmission line system the force-deformation relationships for the wires, suspension insulators, structures are not linear, as described previously, therefore,
FIGURE 4.1 LUMPED MASS MATHEMATICAL MODEL OF THE TRANSMISSION LINE SYSTEM
the system stiffness matrix \([K]\), obtained by combining these element stiffnesses, will not remain constant as the system deforms. This nonlinearity requires a modification in the manner in which the analysis is performed.

In the static analysis procedure used in this investigation, a combined incremental and iterative approach is used to solve the nonlinear equilibrium equations, in which the unbalanced loads are applied incrementally during each iteration cycle. The system stiffness matrix is recomputed after the application of each load increment. Iteration is continued until satisfactory equilibrium is achieved. In order to use the analysis procedure just described, it is necessary to compute the system stiffness matrix, \([K]\), corresponding to the deformed shape of the system after each load increment is applied. The expression for the stiffness of the wires, the suspension insulators and support structures are similar to those originally presented by Campbell (1970).

4.6 DYNAMIC ANALYSIS FORMULATION OF THE TRANSMISSION LINE SYSTEM

The equations of motion for a lumped mass system, subjected to time varying loads, can be expressed in the form:

\[
[M]{A} + [C]{V} + [K]{D} = \{W(t)\} \tag{4.2}
\]

where \([M]\), \([C]\), \([K]\) are the mass matrix, the viscous damping matrix, and the stiffness matrix of the system respectively; \(\{W(t)\}\) are the externally applied time varying loads at the mass points; and \([D]\), \({V}\) and \({A}\) are the displacements, velocities and accelerations respectively, corresponding to each degree of freedom of the system. These equations can be formulated by expressing the equilibrium of the internal forces and external loads associated with each of the individual degrees of freedom of the system. The order of the matrices will be equal to the number of degrees of freedom of the system.

It is assumed that damping is of a viscous nature. Since a direct integration technique is to be used for solving the equations of motion, the terms in the
Damping matrix \([C]\) must be defined explicitly. The degree of damping in an electric transmission line system is unknown and no quantitative data is available except for rough estimates that have been made from some experimental data. For the present study, only damping of the support structures will be considered. Since the support structures are linear, the damping matrix can be expressed in terms of specified percentages of critical damping for each of their natural mode of the system, by a procedure described by Biggs (1964).

### 4.6.1 Solution of Equations of Motion

As stated previously, the response of a transmission line system, when one or more wires break, actually corresponds to free vibration about the broken wire equilibrium position since no external excitations are being considered. For this situation, the term \(\{W(t)\}\) in Equation (4.2) is zero. The displacement \(\{D\}\) is referenced to the broken wire equilibrium position. Therefore, the initial displacements, which must be used for the start of the free vibration analysis, are the displacements from the equilibrium position, with broken wires, but in the opposite sense. These displacements can be determined using the static analysis procedure mentioned earlier.

Since the nonlinear dynamic analysis is to be approximated as a sequence of linear analyses, it is convenient to rewrite Equation (4.2) in the incremental form:

\[
[M] \{\Delta A\} + [C] \{\Delta V\} + [K] \{\Delta D\} = 0 \tag{4.3}
\]

where \(\{\Delta D\}\), \(\{\Delta V\}\) and \(\{\Delta A\}\) are the displacement, velocity and acceleration increments at the mass points, which occur over a time increment \(\Delta t\).

The Newmark Beta Method (Newmark, 1959) and Average Acceleration Method (Fleming, 1989) have been incorporated to obtain a step-by-step approximate solution of Equation (4.2). In each of these methods, expressions for the change in the accelerations and velocities over any time step are computed in...
terms of known displacements, velocities and accelerations, at the end of the previous time step. These expressions can then be substituted into Equation (4.3) to obtain a modified set of equations of motion of the form:

\[ [K] \{ \Delta D \} = \{ W \} \tag{4.4} \]

where \([K]\) is an effective stiffness matrix, \(\{W\}\) is the effective loads and \(\{\Delta D\}\) is the displacement increments which occur over the time step. This system of equations can be solved for the displacement increments, after which the accelerations at the end of the step can be computed using the unbalanced internal forces corresponding to the deformed shape of the system at that instant. The solution can then be advanced one step, and the process can be repeated using the new stiffness corresponding to the computed deformed shape of the system.

4.7 COMPUTER PROGRAM

The program, DYNATL, consists of a short main program, which calls two primary subroutines STATIC and DYNAMC, to perform the actual analysis.

The subroutine module STATIC, which is called by the main program, performs a static analysis of the line system to determine the static equilibrium position for the broken wire condition. These static displacements are then used to establish the initial displacements for the free vibration dynamic analysis, to be performed by the subroutine DYNAMC.

The subroutine module DYNAMC, which is called by the main program, performs a time-history free vibration dynamic analysis of the lumped mass mathematical model of the line system assuming the system starts at rest, in the unbroken wire equilibrium position, with one or more broken wires.

The structure of the main program and subroutine modules used in DYNATL are reproduced in Figure 4.2 in the form of flow diagram.
FIGURE 4.2 FLOW STRUCTURE OF DYNATL

(a) Main Program

(b) Subroutine Static

(c) Subroutine Dynamic
4.8 TYPICAL EXAMPLE OF A TRANSMISSION LINE SYSTEM

A double circuit 230 kV transmission line system consisting of the following:

- No. of tower structures = 5
- No. of suspension towers = 4
- No. of dead end tower = 1
- No. of conductor/tower = 6
- No. of earth wire/tower = 1
- No. of modal points at the wire attachment = 7
- No. of modal point for tower body = 4
- Span length = 254 m
- Insulator length = 2.54 m
- Initial tension of ground wire = 14.23 kN
- Initial tension of conductors = 34.69 kN

4.8.1 Broken wire condition

A single conductor was broken in the first span.

Output

- Impact factor for cross arm load = 2.25
- Impact factor for ground line moment = 3.57

4.8.2 IS Codal Provision (IS:802; 1977, 1989)

(i) The over load factor due to broken conductor in the case of supports with suspension strings = 0.80.
(ii) The over load factor for the ground wire broken condition with suspension or tension clamps = 1.60.
(iii) The over load factor for the conductor broken wire condition in the case of supports with tension insulator string = 1.60.
4.9 EXPERIMENTAL INVESTIGATION AT CENTRAL POWER RESEARCH INSTITUTE (CPRI), BANGALORE

In India, practically, the tower design is based on static forces rather than dynamic forces whereas most of the calamitous cases of failures had been due to dynamic forces. The practice of design and testing towers for dynamic forces is over-due and the present IS:802 (1977) and IS:802 (1989) need a revision.

In consequence with this thought the Central Power Research Institute (CPRI), Bangalore had carried out experimental investigation on two exact scaled-down models of transmission towers with simulated broken wire conditions to assess the impact factor to account for the dynamic forces on a tangent tower. The details of the test were discussed in reference the CPRI Technical Report 131 (1985).

Impact factors obtained from tests were observed to have variation from 1.36 to 4.9. Based on data collected from the tests frequency curve for impact factor was drawn and a mean value of 2.42 with a standard deviation of 0.37 was obtained. If the mean is adopted as the impact factor the safety factor is obviously 50%. To ensure 100% safety, the tower may have to be designed for an impact factor of 4.9 as against a load factor of 1.5 specified in IS: 802 (1977) and a maximum of 1.6 specified in IS:802 (1989). Evidently, 100% safety or security will not be economically justifiable. A more appropriate value of 67% has been chosen to strike a balance between cost and safety. The corresponding impact factor from probability curve is 2.9. This value is nearly 200% of that followed in the present practice.

The load factor presently employed for broken wire condition is 1.5 for all three types of loading i.e. vertical, longitudinal and transverse. However, in the event of adoption of rational loads, the impact factor of 2.9 recommended need be applied only for longitudinal loads. Other loads on towers need not be applied
with any other load factor. Thus a rationalization of longitudinal loads for tower design is recommended.

4.10 IMPACT OF LONGITUDINAL LOADS ON RELIABILITY

The statistical load analysis described in the Chapter 2 does not include unbalanced longitudinal loading cases that may result from broken wire as described in the previous sections. This is because it is very difficult to ascertain the occurrence rates of these load case. Consequently, the unbalanced longitudinal load analysis for any line must be performed separately from the meteorological load analysis. The computer program, as explained above can be used to calculate the attachment point loads which results from such a break. These attachment point loads are then used to compute the load effects on critical members and thus to get structure reliability.