CHAPTER 3

RESULTS AND DISCUSSIONS

3.1 PRESENTATION OF EXPERIMENTAL RESULTS

The results of the present investigation in rotating disc contactor (RDC) are presented in four parts. Part-A deals with the studies on dispersed phase hold-up. Part-B deals with the studies on slip velocity. Part-C deals with the studies on characteristic velocity. Part-D deals with the studies on overall mass transfer coefficient. The experimental details covering the column dimensions, the liquid-liquid systems, the range of rotor speed of the present investigation are listed in Table 3.1 while those considered from the literature for the purpose of analysis and correlation of data are given in Table 3.2.

The experimental data on dispersed phase hold-up obtained in the present study are analysed in Part-A for its dependency on the operating variables namely, phase velocities, rotor speed, column geometry and the physical properties of the liquid-liquid system. The present data are compared with the predictions using the earlier correlations of Kasatkin et al (1962), Murakami et al (1978), Jeffreys et al (1981) and Kumar and Hartland (1987, 1995). A correlation for the estimation of dispersed phase hold-up is developed in the present study in terms of the aforementioned operating variables. The performance of the present correlation is tested using available literature data (Table 3.2) covering the data of Kannappan (1973), Aravamudhan (1970), Venkataramana (1977), Ramesh (1980), Zhang et al (1985), Olney (1964), Bailes et al (1986), Vermijs and Kramers (1954), Korchinsky and Bastani
The slip velocity for the different operating conditions is calculated in Part-B with the knowledge of phase velocities as well as that of experimentally measured dispersed phase hold-up and is presented in Part-B. The effects of the operating variables like phase velocities, rotor speed, column geometry and physical properties of the liquid-liquid systems on slip velocity are analysed. The present slip velocity data (Table 3.1), along with those available in literature (Table 3.2) are compared with the predictions using the earlier correlation suggested by Kumar and Hartland (1989). A new correlation for the estimation of slip velocity is developed in terms of operating variables. The slip velocity values are estimated, with the calculated values of dispersed phase hold-up using the present proposed correlation in Part-A, for both the present as well as literature data (Table 3.1 and 3.2) (for no solute as well as solute transfer conditions namely, \( c \rightarrow d \) and \( d \rightarrow c \) directions of transfer) and are compared with the experimental slip velocity values.

The characteristic velocities are estimated in Part-C, using the experimentally measured dispersed phase hold-up values and phase velocities. The characteristic velocity values thus estimated (Table 3.1) are compared with the calculated values, based on the correlations proposed by earlier authors namely, Logsdail et al (1957), Kung and Beckmann (1961), Laddha et al (1978), Jeffreys et al (1981), Zhang et al (1981), Kamath and Subba Rao (1985) and Godfrey and Slater (1991). In addition, the correlations are again tested based on the available literature data (Table 3.2). Based on the analysis, a new correlation for the estimation of characteristic velocity is developed. The characteristic velocity values are estimated with the calculated values of dispersed phase hold-up using the present correlation (developed in Part-A) of Part-A and are compared with the characteristic velocity values obtained using...
experimental dispersed phase hold-up for both no solute and solute transfer (both c → d and d → c directions of solute transfer) conditions. The analysis is also extended to literature data (Table 3.2).

The data on overall mass transfer coefficients $K_{ocda}$, (both c → d and d → c directions of solute transfer conditions) obtained in the present investigation using Benzene(d)-Water(c) and n-Hexane(d)-Water(c) with Acetone as solute are analysed in Part-D. The present data on mass transfer coefficients ($K_{ocda}$) are tested with the correlation proposed by Laddha et al (1978). A correlation in terms of operating variables namely, phase velocities, rotor speed, column geometry, diffusivity of the solute and physical properties of the systems is proposed, for the two hydrodynamic regions identified for the operation of RDC. The present correlation is verified using the literature data for its applicability.

The present experimental data on dispersed phase hold-up ($\varepsilon$), slip velocity ($U_s$) and characteristic velocity ($U_0$) along with that of calculated values, according to the respective correlations are presented in Appendix -A for no solute transfer conditions, whereas for mass transfer conditions, the data are presented in Appendix - C.

The details of the literature data used in the present investigation under no solute transfer conditions are presented in Appendix - B where as Appendix - D gives the details of the literature data under mass transfer conditions. The details of the literature data on overall mass transfer coefficients are presented in Appendix - E. All Appendices B, D and E present the reported experimental results along with that calculated according to the present proposed correlations.
Table 3.1

Details of systems and data considered for the present analysis (Present data)

<table>
<thead>
<tr>
<th>Systems studied Present Work</th>
<th>Direction of solute transfer</th>
<th>$D_x \times 10^2$ m</th>
<th>$D_r \times 10^2$ m</th>
<th>$D_s \times 10^2$ m</th>
<th>$Z_c \times 10^2$ m</th>
<th>Speed, rps</th>
<th>No. of data points analysed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene(d)-Water(c)</td>
<td>no solute</td>
<td>10.00</td>
<td>4.70</td>
<td>6.25</td>
<td>3.50</td>
<td>3.33, 4.17, 5, 6.67, 10, 13.33, 16.67</td>
<td>140</td>
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<tr>
<td>Benzene(d)-Water(c)</td>
<td>no solute</td>
<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
<td>0, 6.5, 8.33, 10, 15.17, 16.67</td>
<td>106</td>
</tr>
<tr>
<td>Benzene(d)-Acetone-Water(c)</td>
<td>$c \rightarrow d$</td>
<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
<td>0, 2.5, 3.33, 6.67, 8.33, 10, 13.33, 16.67</td>
<td>90</td>
</tr>
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<td>$d \rightarrow c$</td>
<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
<td>2.5, 3.33, 4.17, 5, 8.33, 10, 11.67</td>
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<td>4.70</td>
<td>6.25</td>
<td>3.50</td>
<td>3.33, 4.17, 5, 6.67, 11.67, 13.33, 16.67</td>
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<td>Kerosene(d)-Water(c)</td>
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<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
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<td>MIBK(d)-Water(c)</td>
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<td>6.25</td>
<td>3.50</td>
<td>3.33, 4.17, 5, 8.33, 10, 13.33</td>
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<td>Isobutyl Alcohol(d)-Water(c)</td>
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<td>4.70</td>
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<td>3.50</td>
<td>0.83, 1.67, 2.5, 3.33, 4.17, 5</td>
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<tr>
<td>n-Hexane(d)-Acetone-Water(c)</td>
<td>$c \rightarrow d$</td>
<td>7.62</td>
<td>3.5</td>
<td>4.5</td>
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<td>2.54</td>
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Table 3.2
Details of systems and data considered for the present analysis (Literature data)

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<th>Author and systems studied</th>
<th>Direction of solute transfer</th>
<th>$D_t \times 10^2$ m</th>
<th>$D_r \times 10^2$ m</th>
<th>$D_s \times 10^2$ m</th>
<th>$Z_c \times 10^2$ m</th>
<th>Speed, rps</th>
<th>No. of data points analysed</th>
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<td>Kannappan (1973) Toluene(d)-Water(c)</td>
<td>no solute</td>
<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
<td>0, 6.5, 10, 10.17, 13.17, 15.17, 18.17</td>
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<td>4.70</td>
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<td>3.50</td>
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<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
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<td>4.70</td>
<td>6.25</td>
<td>3.50</td>
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<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
<td>6.33, 7.5, 10.33, 13.17</td>
<td>17</td>
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<tr>
<td>Toluene(d)-Acetone-Water(c)</td>
<td>d → c</td>
<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
<td>6.5, 7.5, 13.17, 17.33</td>
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<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
<td>13.17</td>
<td>8</td>
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<tr>
<td>n-Hexane(d)-Acetone-Water(c)</td>
<td>d → c</td>
<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
<td>19.5</td>
<td>15</td>
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<td>Aravamudhan (1970) Kerosene(d)-Water(c)</td>
<td>no solute</td>
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<td>3.56</td>
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<td>7.62</td>
<td>0, 3.83, 4.83, 5.83</td>
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<td>7.62</td>
<td>3.56</td>
<td>4.06</td>
<td>7.62</td>
<td>0, 3.83, 4.83, 5.83</td>
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<td>Isoamyl Alcohol(d)-Water(c)</td>
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<td>7.62</td>
<td>3.56</td>
<td>4.06</td>
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<td>0, 4.83, 5.83</td>
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<td>Venkataramana (1977) Toluene(d)-Water(c)</td>
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<td>10.00</td>
<td>4.70</td>
<td>6.25</td>
<td>3.50</td>
<td>8.17, 9.33, 11.25, 16.83, 17, 17.50, 26</td>
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<td>Kerosene(d)-Water(c)</td>
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<td>7.62</td>
<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
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<td>( D_2 \times 10^2 ) m</td>
<td>( D_3 \times 10^2 ) m</td>
<td>( Z_x \times 10^2 ) m</td>
<td>Speed, rps</td>
<td>No. of data points analysed</td>
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<td><strong>Zhang et al (1985)</strong></td>
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<td>10.00</td>
<td>5.00</td>
<td>6.75</td>
<td>2.97</td>
<td>9.17, 11.17</td>
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<tr>
<td>Kerosene(d)-n-Butyric acid-Water(c)</td>
<td>c ( \rightarrow ) d</td>
<td>10.00</td>
<td>5.00</td>
<td>6.75</td>
<td>2.97</td>
<td>6.67, 9.17, 8.33</td>
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<tr>
<td>Kerosene(d)-n-Butyric acid-Water(c)</td>
<td>d ( \rightarrow ) c</td>
<td>10.00</td>
<td>5.00</td>
<td>6.75</td>
<td>2.97</td>
<td>9.17, 10.83, 12.5</td>
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<tr>
<td>Toluene(d)-Water(c)</td>
<td>no solute</td>
<td>15.24</td>
<td>7.62</td>
<td>10.16</td>
<td>5.33</td>
<td>10.83</td>
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<td>Toluene(d)-60% Glycerol(c)</td>
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<td>15.24</td>
<td>7.62</td>
<td>10.16</td>
<td>5.33</td>
<td>5.83, 7.50</td>
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<td>White oil(d)-Water(c)</td>
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<td>7.62</td>
<td>10.16</td>
<td>5.33</td>
<td>18.33</td>
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<td>MIBK(d)-Acetic acid- Water(c)</td>
<td>c ( \rightarrow ) d</td>
<td>4.10</td>
<td>2.00</td>
<td>3.10</td>
<td>2.00</td>
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<td>n-Butanol(d)-Water(c)</td>
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<td>15.20</td>
<td>10.20</td>
<td>11.10</td>
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<td>0, 0.42, 0.83</td>
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<td>n-Butanol(d)-Succinic acid-Water(c)</td>
<td>c ( \rightarrow ) d</td>
<td>15.20</td>
<td>10.2</td>
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<td>0, 0.42</td>
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Table 3.2 (continued)

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<th>Author and Systems studied</th>
<th>Direction of solute transfer</th>
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<th>$D_v \times 10^2$ m</th>
<th>$D_a \times 10^2$ m</th>
<th>$Z_c \times 10^2$ m</th>
<th>Speed, rps</th>
<th>No of data points analysed</th>
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<tr>
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<td>2.50</td>
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<td>Toluene(d)-Acetone-Water (c)</td>
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<td>Toluene(d)-Acetone-Water (c)</td>
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<td>4.00</td>
<td>4.50</td>
<td>2.50</td>
<td>0, 1.91, 2.16</td>
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<tr>
<td>n-Butanol(d)-Succinic acid-Water (c)</td>
<td>c → d</td>
<td>21.90</td>
<td>11.00</td>
<td>13.20</td>
<td>7.20</td>
<td>0, 1, 1.33</td>
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<td>Toluene(d)-Acetone-Water(c)</td>
<td>d → c</td>
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<td>3.81</td>
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<td>15, 20</td>
<td>29</td>
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<td>3.81</td>
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<tr>
<td>Toluene(d)-Acetone-Water(c)</td>
<td>c → d</td>
<td>21.90</td>
<td>11.00</td>
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<tr>
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<td>c → d</td>
<td>7.60</td>
<td>4.00</td>
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<td>9.66, 10.17, 10.83, 11.83</td>
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<td>Author and Systems studied</td>
<td>Direction of solute transfer</td>
<td>$D_1 \times 10^2$ m</td>
<td>$D_2 \times 10^3$ m</td>
<td>$D_3 \times 10^2$ m</td>
<td>$Z_{xy} \times 10^2$ m</td>
<td>Speed, rps</td>
<td>No of data points analysed</td>
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<tr>
<td><strong>Endres (1993)</strong></td>
<td>c $\rightarrow$ d</td>
<td>29.50</td>
<td>15.20</td>
<td>18.40</td>
<td>8.90</td>
<td>0.83, 1</td>
<td>7</td>
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<td>n-Butanol(d)-Succinic acid-Water(c)</td>
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<tr>
<td>Toluene(d)-Acetone-Water(c)</td>
<td>c $\rightarrow$ d</td>
<td>29.50</td>
<td>15.20</td>
<td>18.40</td>
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<td>3.67, 4.17, 4.67</td>
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<td>c $\rightarrow$ d</td>
<td>29.50</td>
<td>15.20</td>
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<td>8.90</td>
<td>2.67, 3, 3.33</td>
<td>18</td>
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<td><strong>Ramesh (1980)</strong></td>
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<td>3.50</td>
<td>4.50</td>
<td>2.54</td>
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<td>Kerosene(d)-Water(c)</td>
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STUDIES ON DISPERSED PHASE HOLD-UP

3.2 DISPERSED PHASE HOLD-UP

The motion of drop swarms in rotating disc contactors is expected to depend on the tortuosity of the path offered by the stator compartments and the rotor and on the complex nature of the movement of both phases due to the rotating action of the rotor disc and further breakdown of the dispersed drops. These complex factors are difficult to evaluate, though their effect on total drop population (since the fractional hold-up of dispersed phase is a measure of the dispersed drop population per unit volume of dispersion) in the column could be estimated.

Many authors (Sarkar et al 1985, Kamath et al 1976) studied the axial and radial variation of hold-up in rotating disc contactors, and concluded that the radial variation of hold-up is not significant. The studies relating to the axial profiles of hold-up showed that the hold-up is low near the dispersed phase inlet, increases gradually to a maximum value, because a finite time is required for drop breakup. Due to the axial diffusion of drops in the contact zone and of drop discharge into the settler the hold-up decreases towards the dispersed phase outlet.

Dispersed phase hold-up by definition is the fractional volume of the dispersed phase (held up inside the rotating disc contactor at any instant) in the form of drops, which is controlled by the size and velocity of drops and hence their residence time. The energy spent on the rotor disc causes the breakup of the dispersed phase into drops of various sizes and the size distribution varies with the operating variables.
The hold-up of the dispersed phase increases markedly with increase in the flow rate of the dispersed phase. With the continuous phase moving in the opposite direction, the droplet motion is retarded through frictional drag. This tends to increase the dispersed phase hold-up. The hold-up along with drop size is necessary for the estimation of interfacial area per unit volume. To describe the motion of a droplet in a dispersion, relative to the continuous fluid, it is useful to define a term slip velocity $U_s$, which for counter-current flow is the sum of actual velocities of the dispersed phase droplets and the continuous phase:

$$U_s = \frac{U_d}{\varepsilon} + \frac{U_c}{1 - \varepsilon} = U_0 (1 - \varepsilon) \quad (1.2)$$

The characteristic velocity, $U_0$, defined as the mean relative velocity of the droplets extrapolated to zero flow rates, may be identified with the terminal velocity of a single drop in the equipment concerned. Estimation of hold-up using a correlation in terms of characteristic velocity $U_0$, generally involves the solution of a cubic equation and is often complex and difficult to handle. A problem may therefore arise when, for example Equation (1.2) may be re-written as:

$$\varepsilon^3 - 2\varepsilon^2 + \{1 + (U_d / U_0) -(U_c / U_0)\} \varepsilon - (U_d / U_0) = 0 \quad (3.1)$$

and Equation (3.1) is to be solved for its three roots, of which only one represents a realistic value of hold-up.

Similarly, estimation of hold-up using a correlation in terms of slip velocity $U_s$, also involves the solution of quadratic equation of the following form:

$$\varepsilon^2 - \{1 + (U_d / U_s) - (U_c / U_s)\} \varepsilon + (U_d / U_s) = 0 \quad (3.2)$$
Estimation of hold-up using Equations (3.1 and 3.2) requires prior knowledge of characteristic velocity and slip velocity respectively. Further, the same have to be calculated with the actual experimental value of hold-up using Equation (1.2). An explicit correlation for hold-up in terms of operating conditions, column geometry and physical properties of liquid-liquid systems, should therefore be more useful than that based on the characteristic velocity or slip velocity approach.

An attempt has been made in this work to measure the overall dispersed phase hold-up in rotating disc contactor and to study the effect of the operating parameters on dispersed phase hold-up. Towards this, the experimentally measured dispersed phase hold-up are plotted against the operating variables.

3.2.1 Effect of phase velocities on dispersed phase hold-up

The motion of dispersed drops with counter current flow of continuous phase is controlled both by the size of the drops as well as their velocities in the dispersion. For a given column configuration (internal geometry) and rotor speed, the motion of a drop of a given (mean) size, is dependent upon the velocities of continuous as well as dispersed phase and also the drop crowding. To study the effect of phase velocities on dispersed phase hold-up, the experimentally measured hold-up values are plotted against dispersed phase velocity for constant (nearly) continuous phase velocities for (four) different rotor speeds. The effect of dispersed phase velocity on hold-up is shown for Benzene(d)-Water, Kerosene(d)-Water(c), MIBK(d)-Water(c) and Isobutyl Alcohol(d)-Water(c) systems in Figures 3.1. to 3.4. From the plots, it can be observed that hold-up increases with dispersed phase velocity (as observed by Sarkar et al (1985), Kung and Beckmann (1961) and Kamath et al (1976)); this is attributed to the fact that large number of dispersed phase droplets are produced as the dispersed phase velocity is increased; this also agrees with the drift flux theory (Wallis
Figure 3.1  Effect of dispersed phase velocity on dispersed phase hold-up
[Benzene(d)-Water(c); D_t = 0.10m]
Figure 3.2  Effect of dispersed phase velocity on dispersed phase hold-up  
[Kerosene(d)-Water(c); D_t = 0.10m]
Figure 3.3 Effect of dispersed phase velocity on dispersed phase hold-up

[MIBK(d)-Water(c); $D_t = 0.10m$]
Figure 3.4 Effect of dispersed phase velocity on dispersed phase hold-up
[Isobutyl Alcohol(d)-Water(c); D_t = 0.10m]
(1969)), that the hold-up increases with increase in dispersed phase velocity. It is observed from Figure 3.5 that the increase in continuous phase velocity increases the dispersed phase hold-up which is due to the hindrance offered by the continuous phase for the flow of the dispersed phase droplets thereby increasing the residence time of these droplets inside the column. It is also observed from the figures that the increase in dispersed phase velocity increases the hold-up to a greater extent compared to the extent with the same increment in the continuous phase velocity. This observation suggests that the flow behaviour of the dispersed phase tends towards plug flow with increasing dispersed phase velocity whereas it tends to become backmixed with increasing continuous phase velocity.

3.2.2 Effect of rotor speed on dispersed phase hold-up

The motion of dispersed drops counter current to a continuous phase depends on the size of the drops and their velocities in the dispersion. The size of the dispersed phase droplets in the rotating disc contactor is controlled by the rotor speed. To study the effect of the rotor speed on dispersed phase hold-up, the measured dispersed phase hold-up are plotted against dispersed phase velocities as shown in Figures 3.1. to 3.4. for different rotor speeds, for four different systems (Benzene(d)-Water(c), Kerosene(d)-Water(c), MIBK(d)-Water(c) and Isobutyl Alcohol(d)-Water(c)). From the Figures 3.1., 3.2., 3.3., 3.4. and 3.6., it is observed that the dispersed phase hold-up increases with rotor speed, this is consistent with the observation due to Murakami et al (1978), Jeffreys et al (1981), Laddha et al (1978) and Kamath et al (1976). This indicates that the residence time and drop population are increased by increasing the rotor speed. With the increase in rotor speed, the larger drops are broken into smaller ones, reducing considerably their rise velocity thereby increasing the residence time. With the increase in rotor speed, the drops are thrown radially outward and are retained for longer duration in the compartment which causes a considerable increase in hold-up.
Figure 3.5 Effect of continuous phase velocity on dispersed phase hold-up
Figure 3.6  Effect of rotor speed on dispersed phase hold-up
3.2.3 Effect of system properties on dispersed phase hold-up

Figure 3.7 shows the plot of dispersed phase hold-up versus dispersed phase velocity for three rotor speeds ($N = 3.33$ rps, 5 rps and 13.3 rps) for three different liquid-liquid systems namely, Kerosene(d)-Water(c), Benzene(d)-Water(c) and MIBK(d)-Water(c). It is observed that the dispersed phase hold-up is higher for MIBK(d)-Water(c) system (Morton Number, $\mu_c g/\gamma \Delta \rho = 6.93 \times 10^7$) than for Benzene(d)-Water(c) system (Morton Number, $\mu_c g/\gamma \Delta \rho = 1.41 \times 10^9$) which is higher than for Kerosene(d)-Water(c) system (Morton Number, $\mu_c g/\gamma \Delta \rho = 4.49 \times 10^9$). This indicates the dependency of dispersed phase hold-up on physical properties (in particular the interfacial tension) of the liquid-liquid system (Korchinsky and Bastani (1993)).

3.2.4 Effect of column diameter on dispersed phase hold-up

Figure 3.8 shows the plot of dispersed phase hold-up versus dispersed phase velocity for two different column diameters ($D_t = 0.10$ m and 0.0762 m) for a continuous phase velocity of $0.365 \times 10^3$ m/sec. The system is Benzene(d)-Water(c) ($N = 10$ rps and 16.67 rps). The ratios ($D_s/D_t$, $D_r/D_t$) and ($Z_c/D_t$) are almost same for both cases. Increase in column diameter considerably increases the dispersed phase hold-up. The increase in column diameter for the same length, increases the dispersion volume, and hence is the increase in dispersed phase hold-up.

3.2.5 Effect of solute transfer on hold-up

Figure 3.9 shows the plot of dispersed phase hold-up versus dispersed phase velocity for the systems Benzene(d)-Acetone-Water(c) ($N = 8.33$ and 10 rps) and n-Hexane(d)-Acetone-Water(c) ($N = 13.33$ and 16.67 rps) for the column with a diameter of ($D_t$) 0.0762 m. The continuous phase velocity (approximately $0.138 \times 10^2$ m/sec) is kept constant for both
Figure 3.7 Effect of system properties on dispersed phase hold-up ($D_t = 0.10$ m)

- $N = 3.33 \text{ rps}$, $U_c = 0.2760 \times 10^2 \text{ m/s}$

- Kerosene (d) - Water (c)
- Benzene (d) - Water (c)
- MIBK (d) - Water (c)

- $N = 5.00 \text{ rps}$, $U_c = 0.2760 \times 10^2 \text{ m/s}$
Figure 3.8 Effect of column diameter on dispersed phase hold-up.

Velocity, $U_d \times 10^2$ m/s

Hold-up, $E$

N = 15.67 rpm, $U_c = 0.365 \times 10^{-2}$ m/s

Benzene (d) - Water (c)

For columns of diameter 0.10 m, the hold-up is lower compared to columns of diameter 0.16 m. The velocity also decreases with increasing column diameter.

Figure 3.8a

Figure 3.8b
Figure 3.9 Effect of direction of solute transfer on dispersed phase hold-up.
conditions of column operation namely, solute transfer from continuous phase to dispersed phase, c → d; and from dispersed phase to continuous phase, d → c. Hold-up is higher for d → c direction of solute transfer than for c → d direction of solute transfer for both Benzene(d)-Acetone-Water(c) system and n-Hexane(d)-Acetone-Water(c) system. A shift in hold-up 'ε' is observed in both cases of solute transfer conditions from those of no solute transfer case, and also found to vary with different individual systems, (For the cases where dy /dC < 0, the hold-up is expected to be higher for d → c direction of transfer and when dy /dC > 0, the effect will be reverse) as well as with direction of solute transfer as observed by Zhang et al (1985). The shift in the dispersed phase hold-up is attributed to the properties of the liquid systems being influenced by the concentration induced turbulence caused by the rotor kinetic energy as well as Marangoni surface instability.

3.2.6 Correlation for dispersed phase hold-up


3.2.6.1 Applicability of dispersed phase hold-up correlation proposed by Kasatkin et al (1962) to present data

The correlation proposed by Kasatkin et al (1962), for the prediction of dispersed phase hold-up in terms of rotor speed, column geometry and phase flow rates is of the following form:

\[
\varepsilon = 158 \left\{ \frac{N D_r}{U_c} \right\} \left\{ \frac{(U_c(D_s - D_t)\rho_c)}{\mu_c} \right\}^{0.13} \left\{ \frac{\gamma}{(U_c^2 \rho_c(D_s - D_t))} \right\}^{-0.245} \\
\left\{ \frac{(gZ_c)}{U_c^2} \right\}^{-0.96} \left\{ \frac{\Delta p/\rho_c}{U_c^2} \right\}^{1.31} \left\{ \frac{Z_c}{Z_T} \right\}^{-0.426} \\
\left\{ \frac{(D_s^2 - D_t^2)/D_t^2} \right\}^{-0.7} \left\{ \frac{U_c}{U_d} \right\}^{-0.96}
\]

(1.3)
where $Z_T$ is the contact / dispersion height. The above correlation when tested with the present experimental dispersed phase hold-up values, showed a RMS error of 56.14% based on no solute transfer condition (740 data points). The calculated values of dispersed phase hold-up, based on Equation (1.3) are compared with that of the present experimental values and sample data are shown in Figure 3.10. In all the cases, the predicted values are lower than that of the experimental values, indicating the inadequacy of the correlation, for data with wider range of variables.

3.2.6.2 Applicability of dispersed phase hold-up correlation proposed by Murakami et al (1978) to present data

Murakami et al (1978) have suggested a correlation of the following form.

$$
\varepsilon = 3.3 \left( \Delta \rho / \rho_c \right)^{0.13} \left( ND_r / U_c \right)^{0.55} \left( U_d / U_c \right)^{0.8} \left( (U_c^2 D_r) / \eta \right)^{0.18} \left( \frac{U_c^2 D_t}{gD_t} \right)^{0.6} \left( D_r / D_t \right)^{0.4} \left( \frac{Z_c / D_t}{D_r} \right)^{0.66} \left( (D_r^2 D_t^2) / D^2 \right)^{0.3} \tag{1.4}
$$

Dispersed phase hold-up data of the present study (1052 data points) for all the three conditions of column operation (i.e, no solute, solute transfer from continuous to dispersed phase, (c $\rightarrow$ d) and dispersed phase to continuous phase, (d $\rightarrow$ c)) are analysed and the corresponding RMS errors are found to be 52.85%, 32.26% and 38.67% respectively and sample data are shown plotted according to the correlation proposed by Murakami et al (1978) in Figure 3.11. The correlation is in terms of operating variables as well as physical properties of the systems.

Eventhough the correlation proposed by Murakami et al (1978) involves a number of operating variables along with the physical properties of the liquid-liquid systems, the correlation does not make any difference in hold-up estimation for the case of no solute from that of solute transfer. The hold-up for the solute transfer condition is expected to be different from that
Figure 3.10 Present data on hold-up (no solute) according to the correlation proposed by Kasatkin et al (1962)
Figure 3.11 Present data on hold-up according to the correlation proposed by Murakami et al. (1987)
of no solute transfer conditions, even when other parameters remain unaltered (Laddha et al (1978), Kamath et al (1985) and Logsdail et al (1957)). Figure 3.11 shows wide scatter (for both no solute and solute transfer conditions) indicating the inadequacy of the correlation to the present experimental data.

3.2.6.3 Application of dispersed phase hold-up correlation proposed by Jeffreys et al (1981) to present data

Jeffreys et al (1981) gave the following correlation for the estimation of dispersed phase hold-up in RDC.

\[
\varepsilon = 1.05 \times 10^{14} \left[ \frac{\Delta \rho}{\rho_c} \right]^{4.843} \left[ \frac{N D_r U_c}{U_c} \right]^{0.521} \left[ \frac{U_0}{U_c} \right]^{0.775} \left( \frac{U_c D_r \rho_c}{\mu_c} \right)^{2.367} \\
\left[ \frac{U_c^2}{g D_t} \right]^{0.892} \left( \frac{(U_c^2 - D_r \rho_c \gamma)}{\gamma} \right)^{1.082} \left( \frac{D_r}{D_t} \right)^{0.201} \\
\left[ \frac{Z_c}{D_t} \right]^{0.873} \left( \frac{(D_t^2 - D_r^2)}{D_t^2} \right)^{0.187}
\]  

(1.5)

The above equation is a modified form of the correlation suggested by Murakami et al (1978). The present data, when analysed using the above equation shows a RMS error of more than 100%. The correlation is basically applicable for the condition where there is no solute transfer. The correlation constant (1.05 x 10^{14}) is a very high value and hence minor changes in the group value are insignificant and also the power on Δρ (i.e., the density difference) is 4.843 which is again an unrealistic value, since the large difference in density tends to promote higher hold-up and in turn lower drop velocities.
3.2.6.4 Applicability of dispersed phase hold-up correlations proposed by Kumar and Hartland (1987, 1995) to present data

Based on the available literature data, Kumar and Hartland (1987) proposed a dimensionless correlation for the estimation of dispersed phase hold-up in rotating disc contactor.

\[
\varepsilon = [k_1 + k_2 (N^2 D_r/g)^n_1] \left\{ \frac{(D_r^2 Z_c^2 \rho_c g)/(D_r^2 \gamma)}{(\Delta \rho/\rho_c)^n_3} \right\} \frac{((\mu_c^2 \rho_c^2 \gamma^2))/(\mu_c^2 \rho_c^2 \gamma^2))^{n_4}}{((U_d^4 \rho_c^4)/(g \gamma))^{0.22}} \left\{ 1 + (U_c/U_d) \right\}^{0.35} \tag{1.6}
\]

and the constants and indices for Equation (1.6) are given below;

For no solute transfer condition;
\[k_1 = 570.53, \quad k_2 = 747.78, \quad n_1 = 1.28, \quad n_2 = -0.45, \quad n_3 = -0.58, \quad n_4 = 0.85\]

For c \rightarrow d direction of solute transfer;
\[k_1 = 6.40, \quad k_2 = 14.08, \quad n_1 = 1.28, \quad n_2 = -0.16, \quad n_3 = -0.68, \quad n_4 = 0.29\]

For d \rightarrow c direction of solute transfer
\[k_1 = 19.68, \quad k_2 = 60.00, \quad n_1 = 0.68, \quad n_2 = -0.17, \quad n_3 = -0.78, \quad n_4 = 0.64\]

Eventhough the authors claim that the correlation has a wide range of applicability, analysis of present data according to Equation (1.6), shows a RMS error of 48.19%, 49.30% and 47.96%, for the cases of no solute, c \rightarrow d direction of solute transfer and d \rightarrow c direction of solute transfer respectively. Most of the cited data are not available in open literature. The correlation constants \(k_1 (= 570.33)\) and \(k_2 (= 747.78)\) are very high and a slight change in the rotor kinetic energy in the form of Froude number is negligible when compared with the constant \(k_1 (= 570.53)\) making the correlation relatively insensitive to the changes in rotor speed. The calculated dispersed phase hold-up values according to Kumar and Hartland
(1987) are compared with the experimental hold-up and sample data are shown in Figure 3.12.

Recently Kumar and Hartland (1995) proposed a more generalised correlation in terms of power consumed per unit volume of dispersion, which is applicable for all two phase extraction columns.

\[
\varepsilon = [0.27 + \left( \frac{x}{g} \left( \frac{\rho_c}{g} \right)^{0.25} \frac{U_d}{U_c} \left( \frac{\rho_c}{g} \right)^{0.25} \frac{\Delta p}{\rho_c} \right)^{0.58} \left( \frac{\mu_d}{\mu_c} \right)^{0.18} \exp \left[ 3.34 U_c \left( \frac{\rho_c}{g} \gamma \right)^{0.25} \right] \]  

\[
\]  

The present data when tested with Equation (1.8) gave a RMS errors of 33.03%, 26.04% and 20.05% for the cases of no solute, c \( \rightarrow \) d direction of solute transfer and d \( \rightarrow \) c direction of solute transfer respectively. Though the authors (Kumar and Hartland (1995)) claim that the suggested correlation is applicable to all the three conditions, (i.e, no solute, c \( \rightarrow \) d direction of solute transfer as well as d \( \rightarrow \) c direction of solute transfer), the hold-up values under solute transfer are expected to be different from that of no solute transfer. The calculated values of dispersed phase hold-up according to the correlation proposed by Kumar and Hartland (1995) are compared with that of the present measured experimental hold-up and are plotted in Figure 3.13., which shows a high scatter.

3.2.6.5 Applicability of literature correlations for the published data on dispersed phase hold-up

The available literature data consisting of 985 reported experimental measurements using 13 sources with 10 different liquid-liquid systems as given in Table 3.2, when tested with the literature correlations namely, Equations (1.3) to (1.6) and (1.8) gave an average deviation of more than 35% and the details are given in Table 3.3. However, literature data on dispersed phase hold-up are not tested according to the Equation (1.3) proposed by Kasatkin et al (1962), since this correlation involves a
Figure 3.12 Present data on hold-up according to the correlation proposed by Kumar and Hartland (1987)
Figure 3.13 Present data on hold-up according to the correlation proposed by Kumar and Hartland (1995)
Table 3.3

RMS errors for literature data on dispersed phase hold-up according to literature correlations

<table>
<thead>
<tr>
<th>S.No</th>
<th>Author</th>
<th>Solute transfer</th>
<th>No. of Points</th>
<th>RMS Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Murakami et al' (1978)</td>
<td>No solute</td>
<td>490</td>
<td>45.79%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c → d</td>
<td>255</td>
<td>38.97%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d → c</td>
<td>95</td>
<td>51.97%</td>
</tr>
<tr>
<td>2</td>
<td>Jeffrey's et al' (1981)</td>
<td>No solute</td>
<td>490</td>
<td>&gt; 100%</td>
</tr>
<tr>
<td>3</td>
<td>Kumar and Hartland (1987)</td>
<td>No solute</td>
<td>626</td>
<td>55.07%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c → d</td>
<td>264</td>
<td>39.67%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d → c</td>
<td>95</td>
<td>61.87%</td>
</tr>
<tr>
<td>4</td>
<td>Kumar and Hartland (1995)</td>
<td>No solute</td>
<td>626</td>
<td>31.35%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c → d</td>
<td>264</td>
<td>33.72%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d → c</td>
<td>95</td>
<td>32.42%</td>
</tr>
</tbody>
</table>

* - not applicable for the cases of N (rotor speed) = 0 and Uc (continuous phase velocity) = 0

Thus, the analysis of the available literature correlations for their applicability to data of present investigation as well as those of published work, proved that most of the available correlations are developed using a limited range of variables, and hence these correlations can be used only for the particular liquid-liquid systems or column geometry for which the equations are developed. Further, the analysis indicates a need for better understanding of the effect of individual parameters on dispersed phase hold-up in rotating disc
contactors. Eventhough a vast amount of literature is available for the estimation of dispersed phase hold-up in rotating disc contactors, all the available correlations except that of Kumar and Hartland (1987, 1995) do not involve phase flow rates as variables. The effect of flow rates is more pronounced at low rotor speeds. Hence, a fresh analysis of dispersed phase hold-up data is necessary to develop a satisfactory correlation for better estimation of dispersed phase hold-up in terms of operating variables.

3.2.7 Derivation of generalised correlation for dispersed phase hold-up

An attempt is made in the present work to develop an expression for the direct estimation of dispersed phase hold-up in terms of operating variables namely, phase flow rates, column geometry, rotor speed and also the physical properties of liquid-liquid systems used. The present data on dispersed phase hold-up are graphically analysed for its dependency on these variables like phase velocities (Figures 3.1., 3.2., 3.3., 3.4., and 3.5.), rotor speed (Figures 3.1., 3.2., 3.3., 3.4. and 3.6.), physical properties of the liquid system (Figure 3.7.) and the column geometry (Figure 3.8.). Based on the results of the graphical analysis, it is observed that the variation in the dispersed phase hold-up $e$, is due to the effect of variables namely, phase velocities, rotor speed, column geometry, and the physical properties of the liquid system. It is proposed to develop an equation for the estimation of dispersed phase hold-up in RDC, through dimensional analysis. The results of the analysis leads to the functional relationship between the dispersed phase hold-up $e$, and the variables:

$$e \propto f (N, U_d, U_c, D_r, D_s, Z_c, \rho_c, D_p, \gamma, \mu_c, \mu_d, g)$$

(3.3)

Different combinations of the physical properties are considered for the formulation of the dimensionless groups. The continuous phase density $\rho_c$, the gravitational constant $g$, the density difference between the phases $\Delta \rho$ along with the interfacial tension $\gamma$ are used to form a group $(\gamma \Delta \rho g / \rho_c^2)^{0.25}$ with the dimension of velocity, which could be related to the
terminal velocities of the drops (Laddha et al (1978)). Based on dimensional analysis all the independent variables of Equation (3.3) are rearranged in terms of the dimensionless groups to give a general form:

\[
\varepsilon = C_1 (N^2 D_r / g)^{a_1} (\mu_c g / \gamma^2 D_r)^{a_2} \left( \frac{U_d}{(\gamma \Delta \rho g / \rho_c)^{2.25}} \right)^{a_3} \left( \frac{U_c}{U_d} \right)^{a_4} \\
(\mu_c / \mu_d)^{a_5} (\rho_c / \Delta \rho)^{a_5} (D_r / D_c)^{a_7} \left( \frac{D_r}{D_c} \right)^{a_8} \left( D_r / Z_c \right)^{a_9} \quad (3.4)
\]

However, at zero rpm (N = 0) RDC may be considered similar to spray column, but with a set of column internals, consisting of stator rings and rotor discs, hindering the free rise of droplets. Based on the analysis of the available data for zero speed conditions (i.e. N = 0), for a set of constant velocities (both continuous as well as dispersed phases) the hold-up is represented by modifying Equation (3.4), (taking into account, the tortuosity of the path for the droplets due to the presence of column internals, by involving the geometry group G_f), to give the following form of equation.

\[
\varepsilon / G_f = C_1 (\mu_c g / \gamma^2 D_r)^{a_2} \left( \frac{U_d}{(\gamma \Delta \rho g / \rho_c)^{2.25}} \right)^{a_3} \left( \frac{U_c}{U_d} \right)^{a_4} \\
(\rho_c / \Delta \rho)^{a_5} (\mu_c / \mu_d)^{a_5} \quad (3.5)
\]

where

\[
G_f = (D_r / D_c)^{2.1} (D_r / D_c)^{2.5} (D_r / Z_c)^{0.75}
\]

representing the effect of the system geometry. The calculated indices on the geometrical parameters agree well with that of the reported values of Laddha et al (1978) and Kamath and Subba Rao (1985). A data bowl consisting of 101 measurements (8 present data along with 93 data drawn from literature) with zero rpm is used for regressing Equation (3.5). The analysis showed that the value of indices a2 (= -0.001) and a6 (= 0.005) are close to zero and are statistically insignificant. Hence, the group representing the ratio of viscosities (both continuous phase as well as dispersed phase) i.e. \( \mu_c / \mu_d \) is omitted for the subsequent analysis. The group \( (U_c / U_d) \) which gives the effect of phase flow rate on dispersed phase hold-up is modified, as
(1 + (U_c/U_d)) to suit the stagnant condition of the continuous phase. By using further regression analysis, a correlation of the following form is obtained.

$$\epsilon / G_f = 0.56 \left( U_d^* \right)^{1.0} \left( 1 + (U_c/U_d) \right)^{0.15} \left( \rho_c / \Delta \rho \right)^{0.2}$$

$$= 0.56 \left( \psi_1 \right) \left( \rho_c / \Delta \rho \right)^{0.2}$$

where

$$U_d^* = \left[ U_d / \left( \gamma \Delta \rho g / \rho_c \right)^{0.25} \right]$$

$$\psi_1 = \left[ \left( U_d^* \right)^{1.0} \left( 1 + (U_c/U_d) \right)^{0.15} \right]$$

representing the effect of phase velocities on dispersed phase hold-up. The RMS error for the predicted values of hold-up, $\epsilon$, from the experimental points (for no rotor speed i.e., $N = 0$) is found to be 6.40% and Figure 3.14 shows the comparison of experimentally measured dispersed phase hold-up with the predicted values using Equation (3.6) for zero rps condition.

With rotor in operation, the following approach is made for further development of the correlation. When the rotor is in operation i.e., $N > 0$, the effect of rotor speed, (in terms of Froude Number), and also the effect of physical properties (in terms of Morton Number) are to be included in the above Equation (3.6); (which could also be obtained by dimensional analysis of Equation (3.3)) to yield an expression in the form of Equation (3.4). On rearrangement,

$$\frac{\epsilon}{[G_f \psi_1]} \propto (N^2 D_r g)^{b_1} (\mu_c^4 g / \gamma^3 \Delta \rho)^{b_2} (\rho_c / \Delta \rho)^{b_3}$$

$$\propto [Fr]^{b_1} [Mo]^{b_2} [\rho_c / \Delta \rho]^{b_3}$$

The following values of the indices $b_1$, $b_2$ and $b_3$ of Equation (3.7) are obtained using a regression analysis with 740 experimental values.

$$\psi_2 = [Fr]^{0.33} [Mo]^{0.07} [\rho_c / \Delta \rho]^{0.2}$$
Figure 3.14  Data on hold-up at N = 0 rps, according to the present correlation (Equation (3.6))
Based on the above indices, the predicted values of dispersed phase hold-up showed a high error and a large deviation at both extremes i.e., at very low as well as very high speeds, distinctly showing two regions of operation. To have a better understanding of the dependency of dispersed phase hold-up data on rotor speed, a plot of rotor peripheral velocity versus hold-up is made in Figure 3.15. From the graph, it is observed that a change in slope occurs at different peripheral velocity, for different system, indicating two specific hydrodynamic regions of operation. From the graph, it is also observed that this difference in operating regions varies with the liquid-liquid systems and also with the column dimensions of the rotating disc contactor, i.e., for Benzene(d)-Water(c) system the change occurs at 451 rpm whereas for Kerosene(d)-Water(c) the change occurs at 579 rpm, indicating a change in hydrodynamic behaviour after the critical point. Based on the observations, a reanalysis is made to establish the different regions of operation in terms of critical speed. The general form of equation for the estimation of dispersed phase hold-up is given as:

\[
\frac{\varepsilon}{\left[\frac{D_r}{D_i}\right]^{2.1} \left(\frac{D_r}{D_i}\right)^{2.5} \left(\frac{D_r}{Z_c}\right)^{0.75} \left[\frac{U_d}{(\gamma \Delta \rho g / \rho_c)^{0.25}}\right]^{1.0} (1+(U_c/U_d))^{0.15}}
\]

\[= C_2 \left[ (N^2 D_r/g)^{0.33} (\mu_c g/\gamma^3 \Delta \rho)^{0.07} (\rho_c/\Delta \rho)^{0.2} \right]^{n^1} \tag{3.8} \]

i.e., \[\varepsilon/\left[ G_f \psi_1 \right] = C_2 [ \psi_2 ]^{n^1} \tag{3.8} \]

where

\[\psi_1 = \left[\frac{U_d}{(\gamma \Delta \rho g / \rho_c)^{0.25}}\right]^{1.0} (1+(U_c/U_d))^{0.15} \]

\[\psi_2 = \left(\frac{N^2 D_r/g}{\mu_c g/\gamma^3 \Delta \rho}\right)^{0.07} (\rho_c/\Delta \rho)^{0.2} \]

\[G_f = \left(\frac{D_r}{D_i}\right)^{2.1} \left(\frac{D_r}{D_i}\right)^{2.5} \left(\frac{D_r}{Z_c}\right)^{0.75} \]

The term \(\psi_1\) in Equation (3.8), represents the effect of phase velocities (both dispersed as well as continuous phases) on dispersed phase hold-up, whereas the term \(\psi_2\) represents the effects of rotor speed in terms of rotor kinetic energy (Froude number, \(N^2 D_r/g\)), the physical properties of the dispersion (in terms of Morton’s Number) and the density difference of the phases. For geometrically different systems, the effect of the internal
Figure 3.15 Effect of rotor peripheral velocity on dispersed phase hold-up

<table>
<thead>
<tr>
<th>System</th>
<th>Symbol</th>
<th>D(m) (m)</th>
<th>U_{i} \times 10^2 (m/s)</th>
<th>U_{f} \times 10^2 (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kerosene(d)-Water(c)</td>
<td>ε</td>
<td>0.0762</td>
<td>0.276</td>
<td>0.0276</td>
</tr>
<tr>
<td>Benzene(d)-Water(c)</td>
<td>x</td>
<td>0.1000</td>
<td>0.276</td>
<td>0.0276</td>
</tr>
<tr>
<td>Isobutyl alcohol(c)-Water(c)</td>
<td>♦</td>
<td>0.1000</td>
<td>0.1061</td>
<td>0.0212</td>
</tr>
<tr>
<td>Benzene(d)-Water(c)</td>
<td>■</td>
<td>0.0762</td>
<td>0.2193</td>
<td>0.0365</td>
</tr>
</tbody>
</table>
geometry of the column to provide tortuosity for drop movement, is represented in terms of geometry group, Gf representing the ratios of rotor diameter to stator ring opening (Dr/Ds), column diameter to rotor diameter (Dt/Dr) and rotor diameter to compartment height (Dr/Zc).

Mathematical analysis of data, comprising of 740 present experimental measurements for no solute transfer condition, resulted in the following constants and indices for the two hydrodynamic regions of operation.

**Region I**

$\psi_2 < 0.225 \quad C_2 = 0.79 \quad n1 = 0.14$

**Region II**

$\psi_2 > 0.225 \quad C_2 = 0.52 \times 10^{-2} \quad n1 = 3.2$

It is of interest to observe from the above analysis that Region I covers the operation at low rotor speeds. In this region, the dispersed phase hold-up is nearly independent of rotor speed. The drop size remains constant at lower rotor speeds indicating the absence of shearing action on dispersed phase droplets by the rotor discs. The moving dispersed drops are deflected by the rotating discs without causing any appreciable breakup of the drops as the difference in dynamic pressure created may not be sufficient to overcome the interfacial tension. The drop terminal velocities are slightly affected due to the deflection and also the tortuosity provided thereby for the drop movement. Region II covers the operation at higher rotor speeds, where the dependency of dispersed phase hold-up, $'\varepsilon'$, on the rotor speed $N$ is more. In this region, as the rotor speed increases, the energy spent on the rotor disc exceeds a critical value; the interfacial forces are overcome, thereby resulting in drop breakup in addition to enhancing the tortuous path of drops. Thus the dispersed phase hold-up shows a marked increase with the rotor speed. With the estimated constants and indices for
Equation (3.8), the RMS error for 740 experimental measurements is found to be 10.46% with a maximum deviation of 17.23%. The calculated values of dispersed phase hold-up according to the present correlation (Equation (3.8)) are compared with those of present experimental values (Appendix - A1 -A6), and sample data are shown plotted in Figure 3.16. The data bowl consisting of 626 measurements based on the reported literature (Table 3.2) are used for validating the present correlation (Equation (3.8) and the RMS error is found to be 15.50% with a maximum deviation of 27.50% (Appendix B-1).

3.2.7.1 Effect of solute transfer on hold-up

An attempt is made in the present investigation to investigate the effect of solute and its direction of transfer on dispersed phase hold-up, based on the present experimental data under solute transfer conditions namely, \( c \rightarrow d \) and \( d \rightarrow c \) direction of solute transfer. It is proposed to develop a generalised correlation for satisfactory prediction of dispersed phase hold-up \( \varepsilon \) under solute transfer conditions. Analysis of data on solute transfer indicated that a shift in hold-up \( \varepsilon \) correlation resulted in both cases of solute transfer condition from those of no solute transfer case, and is also found to vary with the individual liquid system as well as with direction of solute transfer. The shift in the dispersed phase hold-up may be attributed to the properties of the liquid system being influenced by the concentration induced turbulence caused by the rotor kinetic energy as well as Marangoni surface instability. However, the graphical analysis of the present data, gave a satisfactory fit of Equation (3.8), (with two breaks in the correlation) with different constants and indices for \( c \rightarrow d \) and \( d \rightarrow c \) directions of solute transfer. The present experimental data (156 measurements) on \( c \rightarrow d \) direction of solute transfer are used for regressing Equation (3.8) and

\[
\frac{\varepsilon}{[G_f \cdot \psi_1]} = C_2 [\psi_2]^n\]  \hspace{1cm} (3.8)
Figure 3.16 Present and literature data on hold-up (no solute) according to the present correlation (Equation (3.8))
the calculated values for the constants and the indices for Region I and II are given below:

Region I

\[ \psi_2 < 0.17 \quad C_2 = 0.45 \quad n_1 = 0.05 \]

Region II

\[ \psi_2 > 0.17 \quad C_2 = 5.4 \quad n_1 = 1.5 \]

With the above calculated constants for Equation (3.8), the estimated hold-up values for \( c \rightarrow d \) direction of solute transfer are compared with those of the experiment (Appendix C-2 and C-3) and the RMS error is found to be 10.55%. The sample data are shown plotted in Figure 3.17.a.

The present correlation (Equation (3.8)) with the estimated constants, when tested with the reported literature data for \( c \rightarrow d \) direction of transfer, consisting of 262 measurements (Table 3.2.) gave a RMS error of 16.37% (Appendix D-1). The sample data are shown in Figure 3.17.b.

The approach is extended for \( d \rightarrow c \) direction of solute transfer conditions, using 156 present experimental measurements and the following values of constants and indices are obtained.

Region I

\[ \psi_2 < 0.2 \quad C_2 = 0.57 \quad n_1 = 0.04 \]

Region II

\[ \psi_2 > 0.2 \quad C_2 = 8.1 \quad n_1 = 1.7 \]

The dispersed phase hold-up values, under \( d \rightarrow c \) direction of mass transfer is estimated, with the above established constants and indices for the Equation (3.8) and the RMS error for 156 experimental
Figure 3.17 Present and literature data on hold-up (c → d) according to the present correlation (Equation (3.8))
measurements is found to be 10.53% (Appendix C-3 and C-4). The sample data are represented in Figure 3.18.a.

The applicability of the present correlation (Equation (3.8)) to the reported literature data on \( d \rightarrow c \) direction of solute transfer is tested with the data bowl consisting of 95 measurements (Table 3.2.) and the RMS error is found to be 26.54% (Appendix D-2). The sample data are shown plotted in Figure 3.18.b.

It is clear from the above analysis that the dispersed phase hold-up could be estimated with a generalised correlation of the following form in terms of the operating variables, namely, phase velocities, rotor speed, column geometry, and the physical properties of the liquid systems used:

\[
\varepsilon / \left[ G_f \cdot \psi_1 \right] = C_2 \left[ \psi_2 \right]^{n_1}
\]  

(3.8)

The constants and indices for different operating conditions are compiled in Table 3.4 and the error analysis are presented in Table 3.5.

**Table 3.4**

**Summary of constants and indices for hold-up correlation (Equation (3.8))**

<table>
<thead>
<tr>
<th>Region of operation</th>
<th>no solute</th>
<th>c ( \rightarrow ) d direction of solute transfer</th>
<th>d ( \rightarrow ) c direction of solute transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \psi_2 )</td>
<td>( C_2 )</td>
<td>( n_1 )</td>
</tr>
<tr>
<td>Region I</td>
<td>&lt;0.22</td>
<td>0.79</td>
<td>0.14</td>
</tr>
<tr>
<td>Region II</td>
<td>&gt;0.22</td>
<td>0.52x10^1</td>
<td>3.20</td>
</tr>
</tbody>
</table>
Figure 3.18 Present and literature data on hold-up (d → c) according to the present correlation (Equation (3.8))
Table 3.5

Error analysis of present data and literature data on dispersed phase hold-up according to the proposed correlation (Equation (3.8))

<table>
<thead>
<tr>
<th>Region of operation</th>
<th>Number of data points</th>
<th>RMS Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>no solute</td>
</tr>
<tr>
<td>Based on present data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region I</td>
<td>400</td>
<td>60</td>
</tr>
<tr>
<td>Region II</td>
<td>340</td>
<td>96</td>
</tr>
<tr>
<td>Based on Literature data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region I</td>
<td>234</td>
<td>29</td>
</tr>
<tr>
<td>Region II</td>
<td>299</td>
<td>226</td>
</tr>
<tr>
<td>Overall data</td>
<td>1273</td>
<td>411</td>
</tr>
</tbody>
</table>

It is of interest to note that the dispersed phase hold-up could be well correlated with the fundamental variables in the form of Equation (3.8). The estimated RMS error for no solute transfer conditions involving 1273 data (740 data of present work and 533 data of published literature) is 12.8% with a maximum deviation of ±25%. For c→d direction of mass transfer for 411 data points (156 data of present work and 255 data of published literature), the RMS error is found to be 14.6%, with a maximum deviation of ±27%. The RMS error for d→c direction of mass transfer involving 251 data (156 data of present work along with 95 data of published literature) is 18.8%. The details of the present as well as the literature data used for the development and validation of Equation (3.8) are given in Table 3.1. and 3.2. respectively.

The range of variable used for the development of the Equation (3.8) are given in Table 3.6.
Table 3.6

Range of variables covered in the present investigation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol and unit</th>
<th>Present study</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Column geometry</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column diameter</td>
<td>$D_i \times 10^2$ m</td>
<td>7.62 - 10.00</td>
<td>4.10 - 29.50</td>
</tr>
<tr>
<td>Stator ring opening</td>
<td>$D_s \times 10^2$ m</td>
<td>4.50 - 6.25</td>
<td>3.10 - 18.40</td>
</tr>
<tr>
<td>Rotor disc diameter</td>
<td>$D_r \times 10^2$ m</td>
<td>3.50 - 4.70</td>
<td>2.00 - 15.20</td>
</tr>
<tr>
<td>Compartment height</td>
<td>$Z_c \times 10^2$ m</td>
<td>2.54 - 3.50</td>
<td>2.00 - 8.90</td>
</tr>
<tr>
<td><strong>Physical properties of the systems</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interfacial tension</td>
<td>$\gamma$ mN/m</td>
<td>2.12 - 50</td>
<td>1.40 - 60</td>
</tr>
<tr>
<td><strong>Continuous phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_c \times 10^3$ kg/m$^3$</td>
<td>0.9875 - 1.0</td>
<td>0.798 - 1.141</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\mu_c \times 10^3$ Pa.s</td>
<td>0.74 - 1.14</td>
<td>0.67 - 9.1</td>
</tr>
<tr>
<td><strong>Dispersed phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_d \times 10^3$ kg/m$^3$</td>
<td>0.664 - 0.8643</td>
<td>0.597 - 0.881</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\mu_d \times 10^3$ Pa.s</td>
<td>0.29 - 3</td>
<td>0.29 - 26</td>
</tr>
<tr>
<td><strong>Operating Variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuous phase velocity</td>
<td>$U_c \times 10^2$ m/s</td>
<td>0.0106 - 0.3608</td>
<td>0.0212 - 0.9486</td>
</tr>
<tr>
<td>Dispersed phase velocity</td>
<td>$U_d \times 10^2$ m/s</td>
<td>0.0053 - 0.3608</td>
<td>0.0212 - 0.7330</td>
</tr>
<tr>
<td>Rotor speed</td>
<td>$N$, rps</td>
<td>0.0000 - 16.67</td>
<td>0.0000 - 32.50</td>
</tr>
</tbody>
</table>
3.3 CONCEPT OF SLIP VELOCITY

The motion of a droplet in a dispersion, relative to the continuous fluid medium, is conveniently described using 'slip velocity, \( U_s \)', which is expected to control the mass transfer coefficient when solute is transferred between the two phases. The slip velocity or relative velocity can be related to some function of the volumetric fractional hold-up of dispersed phase drop, the motion of which is determined by buoyancy, gravitational force, interfacial forces and local drag forces that depend on agitation and column geometry (Godfrey and Slater 1991). The slip velocity for counter-current flow of two immiscible phases in an extraction column without packing is defined as:

\[
U_s = \frac{U_d}{\varepsilon} + \frac{U_c}{(1-\varepsilon)} \quad (1.2)
\]

The slip velocity values thus calculated using Equation (1.2) are analysed for its dependency on the operating variables namely, phase velocities, rotor speed, column geometry and physical properties of the liquid systems. For the purpose, slip velocity calculated using the experimentally measured hold-up value is plotted against the operating variables, and is discussed below.

3.3.1 Effect of phase velocities on slip velocity

Figures 3.19 to 3.22 show the plot of slip velocity data versus dispersed phase velocities for nearly constant continuous phase velocities for different rotor speeds. The systems used are Benzene(d)-Water(c).
Figure 3.19 Effect of dispersed phase velocity on slip velocity
[Benzene(d)-Water(c); D_t = 0.10m]
Figure 3.20  Effect of dispersed phase velocity on slip velocity
[Kerosene(d)-Water(c); D_t = 0.10m]
Figure 3.21 Effect of dispersed phase velocity on slip velocity

[MIBK(d)-Water(c); D_t = 0.10m]
Figure 3.22 Effect of dispersed phase velocity on slip velocity
[Isobutyl Alcohol(d)-Water(c); D, = 0.10m]
Kerosene(d)-Water(c), MIBK(d)-Water(c) and Isobutyl Alcohol(d)-Water(c). It is observed that the increase in dispersed phase velocity increases the slip velocity initially to a certain extent, beyond which the slip velocity remains constant. This initial increase in slip velocity may be attributed to the increase in rise velocity of the dispersed phase droplets due to the formation of larger drops at higher dispersed phase velocities. From the figures it is also observed that the effect of dispersed phase velocity on slip velocity is more pronounced at very low rotor speeds. At low rotor speed, the size of the drops depends more on the phase velocity rather than on rotor speed. When the rotor speed increases, the dispersion will have uniform size of droplets with nearly constant rise velocity (mean size of drops are dependent on rotor speed) and hence the slip velocity remains (nearly) constant for a given rotor speed, irrespective of the phase velocity. Figure 3.23 shows the effect of continuous phase velocity on slip velocity i.e., increase in continuous phase velocity decreases the slip velocity. An increase in continuous phase velocity offers resistance to the upflow of dispersed phase droplets thereby decreasing the rise velocity of the droplets and hence increasing the dispersed phase hold-up, which in turn reduces the slip velocity.

3.3.2 Effect of rotor speed on slip velocity

Slip velocities calculated using experimentally measured hold-up are plotted against dispersed phase velocities at four different rotor speeds for the systems namely, Benzene(d)-Water, Kerosene(d)-Water(c), MIBK(d)-Water(c) and Isobutyl alcohol(d)-Water(c) and are shown in Figures 3.19 to 3.22. Figure 3.24 shows the plot of rotor speed versus slip velocity for two different columns namely, 0.0762 m and 0.10 m diameter. It is observed that the slip velocity decreases with the increase in rotor speed; this implies that increase in rotor speed increases the hold-up which in turn decreases the slip velocity, which is consistent with the observation reported by Kannappan (1973) and Logsdail et al (1981) for characteristic velocity.
Figure 3.23 Effect of continuous phase velocity on slip velocity

- Kerosene (d) - Water (c); N=1.67 ppm
- Dt = 0.10 m
- Velocity, U_d x 10^2 m/s
- Slip velocity, U_s x 10^2 m/s

- Benzene (d) - Water (c); N=0.00 ppm
- Dt = 0.10 m
- Velocity, U_d x 10^2 m/s
- Slip velocity, U_s x 10^2 m/s
Figure 3.24  Effect of rotor speed on slip velocity
As the rotor speed increases, the larger drops are broken into smaller ones and the drops are thrown radially outward with a greater force. The rise velocity of the fine drops reduces considerably; they are retained for longer duration in the compartment which causes a considerable decrease in slip velocity with the increase in rotor speed.

3.3.3 Effect of system properties on slip velocity

Slip velocity data are plotted against dispersed phase velocities for two different rotor speeds namely, 5 rps and 13.3 rps and at nearly constant continuous phase velocity. The plots for three different liquid-liquid systems namely Kerosene(d)-Water(c), Benzene(d)-Water(c) and MIBK(d)-Water(c) are shown in Figure 3.25. It is observed that the slip velocity is higher for Kerosene(d)-Water(c) system (Morton Number, \( \mu_c^4 g/\gamma^3 \Delta \rho = 4.49 \times 10^9 \)). The slip velocity for Benzene(d)-Water(c) system (Morton Number, \( \mu_c^4 g/\gamma^3 \Delta \rho = 1.41 \times 10^9 \)) is lower than for Kerosene(d)-Water(c) and is higher than for MIBK(d)-Water(c) (Morton Number, \( \mu_c^4 g/\gamma^3 \Delta \rho = 6.93 \times 10^7 \)) system, indicating the dependency of slip velocity on the physical properties namely, interfacial tension, density difference and viscosity, of both continuous as well as the dispersed phases. A similar phenomena has already been observed by Korchinsky and Bastani (1993).

3.3.4 Effect of solute transfer on slip velocity

Plots of calculated values of slip velocity using the measured hold-up data versus dispersed phase velocities for both c \( \rightarrow \) d and d \( \rightarrow \) c directions of solute transfer for the systems Benzene(d)-Acetone-Water(c) and n-Hexane(d)-Acetone-Water(c) systems are shown in Figure 3.26. In all these cases nearly constant continuous phase velocities have been maintained. Slip velocity is higher for c \( \rightarrow \) d direction of solute transfer than that of d \( \rightarrow \) c direction of solute transfer for both Benzene(d)-Acetone-Water(c) and n-Hexane(d)-Acetone-Water(c) systems. This change may be
Figure 3.25 Effect of system properties on slip velocity.

- Figure 3.25a: Benzene (d) - Water (c)
- Figure 3.25b: MIBK (d) - Water (c)

**Figure 3.25a**: Velocity, $U_d \times 10^2$ m/s

**Figure 3.25b**: Velocity, $U_d \times 10^2$ m/s

- **Figure 3.25a**:
  - $D_t = 0.10 m$; $N = 1333$ rps;
  - $U_c = 2.76 \times 10^{-2}$ m/s

- **Figure 3.25b**:
  - $D_t = 0.10 m$; $N = 500$ rps;
  - $U_c = 0.276 \times 10^{-2}$ m/s
Figure 3.26 Effect of direction of solute transfer on slip velocity
attributed to the properties of the liquid systems being influenced by the concentration induced turbulence caused by the rotor kinetic energy as well as Marangoni surface instability as in the case of dispersed phase hold-up.

3.3.5 Correlation for slip velocity

The earlier correlations suggested and available for the estimation of slip velocity, involve dispersed phase hold-up as a parameter, which itself depends on other operating variables. The recent correlation suggested by Kumar and Hartland (1989) may be considered as important, (since it gives slip velocity as a function of operating variables) and can be used for further comparison of present as well as literature data.

3.3.5.1 Applicability of slip velocity correlation proposed by Kumar and Hartland (1989) to the present data as well as literature data

Kumar and Hartland (1989) developed a correlation in terms of the operating variables as given below.

\[
U_s \rho_c^{1/4} / (g^{1/4} \gamma^{1/4}) = [k_6 + k_7 \exp (k_5 N^2 D_d/g)] x (\Delta \rho/\rho_c)^{0.52} \\
\left[ \mu_c g^{1/4} / \rho_c^{1/4} \gamma^{1/4} \right]^{0.45} \left( D_t^2 \rho_c g / \gamma \right)^{0.08} (Z_d/D_t)^{1.03} \\
(D_d/D_t)^{0.51} \left[ (U_d \rho_c^{1/4}) / (g^{1/4} \gamma^{1/4}) \right]^{0.28}
\]  

(1.18)

The values for \( k_6 \) and \( k_7 \) for no mass transfer and both directions of mass transfer are,

For no mass transfer: \( k_6 = 4.84 \times 10^{-2}; k_7 = 0.22 \)

For \( c \rightarrow d \) direction of solute transfer: \( k_6 = -3.25 \times 10^{-2}; k_7 = 0.30 \)

For \( d \rightarrow c \) direction of solute transfer: \( k_6 = 5.78 \times 10^{-2}; k_7 = 0.20 \)
The above correlation (Equation (1.22)) suggested by Kumar and Hartland (1989) is tested for its applicability to the present experimental values (Table 3.1) as well as that of the literature data (Table 3.2) and are shown in Figure 3.27. RMS errors for all the cases are found to be more than 25% and the details are given in Table 3.7.

### Table 3.7

RMS errors for present and literature data on slip velocity according to the correlation proposed by Kumar and Hartland (1989)

<table>
<thead>
<tr>
<th>S.No</th>
<th>Data</th>
<th>Solute transfer</th>
<th>No. of Points</th>
<th>RMS error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Present data</td>
<td>No solute</td>
<td>748</td>
<td>32.97%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c → d</td>
<td>172</td>
<td>37.35%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d → c</td>
<td>165</td>
<td>24.87%</td>
</tr>
<tr>
<td>2</td>
<td>Literature data</td>
<td>No solute</td>
<td>626</td>
<td>38.12%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c → d</td>
<td>264</td>
<td>31.27%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d → c</td>
<td>95</td>
<td>32.48%</td>
</tr>
</tbody>
</table>

### 3.3.6 Development of generalised correlation for slip velocity

The slip velocity is a function of both dispersed as well as continuous phase velocities and the dispersed phase hold-up as given in Equation (1.2), which can be written as,

\[ U_s = f(U_d, U_c, \epsilon) \]  

(3.9)

It is concluded from the earlier chapter on the analysis of dispersed phase hold-up that the hold-up is a function of all operating variables (Equation (3.3)).
Figure 3.27 Present data on slip velocity according to the correlation proposed by Kumar and Hartland (1989)

- **Figure 3.27a**
  - No solute
  - System Symbols:
    - Benzene(d)-Water(c)
    - MIBK(d)-Water(c)
    - Kerosene(d)-Water(c)

- **Figure 3.27b**
  - Solute transfer
  - System Symbols:
    - Benzene(d)-Acetone-Water(c) d→c
    - Benzene(d)-Acetone-Water(c) c→d
    - n-Hexane(d)-Acetone-Water(c) d→c
    - n-Hexane(d)-Acetone-Water(c) c→d
Based on the analysis of slip velocity data, it is observed that the slip velocity itself is a function of all operating variables as in the case of dispersed phase hold-up and hence incorporating Equation (3.3) into Equation (3.9):

\[ U_s \propto f (N, U_d, U_c, D_t, D_s, Z_c, \rho_c, \Delta \rho, \gamma, \mu_c, \mu_d, g) \]  

(3.3)

It is always possible to develop a correlation for the estimation of \( U_s \), (Kalaichelvi and Murugesan (1997)) similar to the one developed for dispersed phase hold-up (Equation (3.8)) in the earlier chapter in terms of dimensionless groups, using \( \pi \) theorem. The data bowl consisting of the same number of data points (740) used for hold-up studies is used for optimizing the indices on the groups representing phase flow rates as,

\[ \psi_3 = \left( \frac{(U_d)}{(\gamma \Delta \rho g / \rho_c)} \right)^{0.25} \left( 1 + \frac{(U_c/U_d)}{0.07} \right) \]  

(3.11)

Column geometry ‘Gf’ is given by

\[ G_f = \left( \frac{D_t}{D_s} \right)^{2.1} \left( \frac{D_t}{D_r} \right)^{2.5} \left( \frac{D_t}{Z_c} \right)^{0.75} \]  

(3.12)

‘\( \psi_2 \)’ represents the Froude number along with physical properties of the liquid-liquid systems as

\[ \psi_2 = \left( \frac{(N^2 D_t g)}{0.33} \left( \mu_c g / \gamma^3 \Delta \rho \right)^{0.07} (\rho_c / \Delta \rho)^{0.2} \right) \]  

(3.13)

After rearranging the groups, the general form of correlation for the estimation of slip velocity in two regions of operation (nearly constant and decreasing slip velocity) is represented with the following general form of correlation.
Regression analysis of data using 1052 present experimental measurements (Table 3.1) resulted in the following constants and indices for the two regions of operation as given in Table 3.8.

Table 3.8

Values of constants and indices for slip velocity correlation (Equation (3.14))

<table>
<thead>
<tr>
<th>Region of operation</th>
<th>No solute</th>
<th>c → d direction of solute transfer</th>
<th>d → c direction of solute transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\psi_1$, $C_3$, $n_3$</td>
<td>$\psi_1$, $C_3$, $n_3$</td>
<td>$\psi_2$, $C_3$, $n_3$</td>
</tr>
<tr>
<td>Region I</td>
<td>&lt;0.22 3.2 -0.06</td>
<td>&lt;0.17 4.2 -0.09</td>
<td>&lt;0.2 3.3 -0.15</td>
</tr>
<tr>
<td>Region II</td>
<td>&gt;0.22 0.1 -2.6</td>
<td>&gt;0.17 0.62 -1.2</td>
<td>&gt;0.2 0.36 -1.45</td>
</tr>
</tbody>
</table>

The slip velocity values thus estimated using the constants and indices for Equation (3.14) as given in Table 3.8, are compared with 1052 present experimental measurements and 883 published literature values and the corresponding RMS errors are given in Table 3.9.
Table 3.9

RMS errors for present and literature data on slip velocity according to the Equation (3.14)

<table>
<thead>
<tr>
<th>S.No</th>
<th>Data</th>
<th>Direction of solute transfer</th>
<th>No. of Points</th>
<th>RMS error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Present data</td>
<td>No solute</td>
<td>740</td>
<td>14.29%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c → d</td>
<td>156</td>
<td>13.97%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d → c</td>
<td>156</td>
<td>14.39%</td>
</tr>
<tr>
<td>2</td>
<td>Literature data</td>
<td>No solute</td>
<td>533</td>
<td>21.59%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c → d</td>
<td>255</td>
<td>22.38%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d → c</td>
<td>95</td>
<td>28.57%</td>
</tr>
</tbody>
</table>

Sample values from present data as well as literature data (for no solute as well as solute transfer conditions) are plotted according to the correlation (Equation (3.14)) and are shown in Figures 3.28, 3.29 and 3.30. From these graphs, the two hydrodynamic regions with negative slopes have been observed, resulting in the same critical rotor speeds as in the case of hold-up correlation.

3.3.7 Applicability of the present proposed hold-up correlation for the estimation of slip velocity to the present data as well as literature data

The slip velocity $U_s$, which is defined by the superficial velocities of the contacting phases and fractional dispersed phase hold-up of drops moving through the column as defined earlier in this chapter, is given in the form of Equation (1.2),

$$U_s = \frac{U_d}{\varepsilon} + \frac{U_c}{(1-\varepsilon)}$$  \hspace{1cm} (1.2)

The estimation of slip velocity involves the knowledge of dispersed as well as continuous phase velocities (which are the operating
Figure 3.28 Present and literature data (no solute) according to the present correlation for slip velocity (Equation (3.14))

**Figure 3.28a**

<table>
<thead>
<tr>
<th>System</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toluene(d)-Water(c)</td>
<td>x</td>
</tr>
<tr>
<td>n-Hexane(d)-Water(c)</td>
<td>□</td>
</tr>
<tr>
<td>MiBK(d)-Water(c)</td>
<td>◇</td>
</tr>
<tr>
<td>Isoamyl alcohol(d)-Water(c)</td>
<td>△</td>
</tr>
<tr>
<td>White oil(d)-Water(c)</td>
<td>O</td>
</tr>
<tr>
<td>n-Butanol(d)-Water(c)</td>
<td>□</td>
</tr>
</tbody>
</table>

**Figure 3.28b**

<table>
<thead>
<tr>
<th>System</th>
<th>Author</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toluene(d)-Water(c)</td>
<td>Venkataramana</td>
<td>x</td>
</tr>
<tr>
<td>n-Hexane(d)-Water(c)</td>
<td>Kannappan</td>
<td>*</td>
</tr>
<tr>
<td>Kerosene(d)-Water(c)</td>
<td>Zhang et al</td>
<td>○</td>
</tr>
<tr>
<td>Kerosene(d)-50% Glycol(c)</td>
<td>Aravamudhan</td>
<td>◇</td>
</tr>
<tr>
<td>Isobutyl alcohol(d)-Water(c)</td>
<td>Aravamudhan</td>
<td>△</td>
</tr>
<tr>
<td>White oil(d)-Water(c)</td>
<td>O'ney</td>
<td>●</td>
</tr>
<tr>
<td>n-Butanol(d)-Water(c)</td>
<td>Bailes et al</td>
<td>□</td>
</tr>
</tbody>
</table>
Figure 3.29 Present and literature data (c → d) according to the present correlation for slip velocity (Equation (3.14))
Figure 3.30 Present and literature data (d → c) according to the present correlation for slip velocity (Equation (3.14))
variables) and the dispersed phase hold-up (which is a measured variable). Since a better correlation with a wide range of applicability has already been developed in Part-A for the estimation of dispersed phase hold-up (Equation (3.8)) with minimum error limits, it is possible to estimate the slip velocity values with the prior knowledge of operating phase velocities along with the estimated dispersed phase hold-up calculated using Equation (3.8). The slip velocity values thus calculated are compared with the present experimental values (Appendix A-1 to A-6 (no solute), C-1 to C-4 (c → d and d → c direction of solute transfer)) and the corresponding RMS errors are given in Table 3.9. Sample data are shown in Figures 3.31(a), 3.32(a) and 3.33(a). The same analysis has been done for 883 published literature data (Appendix B-1 (no solute), D-1 and D-2 (c → d and d → c directions of solute transfer)) and the RMS errors are given in Table 3.10 and sample data are shown in Figures 3.31(b), 3.32(b) and 3.33(b).

Table 3.10

Error analysis for the estimation of slip velocity based on present correlation for dispersed phase hold-up (both literature and present data)

<table>
<thead>
<tr>
<th>Region of operation</th>
<th>Number of data points</th>
<th>RMS Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>no solute</td>
</tr>
<tr>
<td>Based on present data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region I</td>
<td>400</td>
<td>60</td>
</tr>
<tr>
<td>Region II</td>
<td>340</td>
<td>96</td>
</tr>
<tr>
<td>Based on Literature data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region I</td>
<td>234</td>
<td>29</td>
</tr>
<tr>
<td>Region II</td>
<td>299</td>
<td>226</td>
</tr>
<tr>
<td>Overall Data</td>
<td>1273</td>
<td>411</td>
</tr>
</tbody>
</table>
Figure 3.31 Comparison of data on slip velocity (no solute), calculated using the present hold-up correlation.
Figure 3.32 Comparison of data on slip velocity (c \rightarrow d), calculated using the present hold-up correlation.
Figure 3.33 Comparison of data on slip velocity (d → c), calculated using the present hold-up correlation
3.4 CONCEPT OF CHARACTERISTIC VELOCITY

Under the condition of counter current flow, the slip velocity is defined as the sum of the individual linear velocities of the continuous and dispersed phases, and then,

\[ U_s = \left( \frac{U_d}{\varepsilon} \right) + \left( \frac{U_c}{(1-\varepsilon)} \right) \]  

The slip velocity under the limiting conditions of negligible or zero hold-up is known as characteristic velocity, \( U_0 \), which could be defined as the mean rise velocity of a single drop moving under the constraints due to the internal design of the equipment. The slip velocity \( U_s \), is a function of hold-up and at zero hold-up, the slip velocity is equal to the characteristic velocity. Godfrey and Slater (1991) in their extensive review on two phase liquid-liquid systems discussed the equations relating characteristic velocity to slip velocity with continuous phase hold-up, \((1-\varepsilon)\). The equations suggested by Gayler and Pratt (1957), Misek (1970) and Godfrey and Slater (1991) for rotating disc contactors are considered basically important and are discussed further. Misek (1963) proposed an empirical relationship of the following form, based on the coalescence behavior of the liquid systems:

\[ U_s = U_0 (1-\varepsilon) \exp(-b\varepsilon) \]  

where 'b' is suggested to be a measure of drop coalescence and the values of 'b' are not readily available. Recently Godfrey and Slater (1991) suggested a relationship, in the form of power law model, based on the work of Richardson and Zaki (1954) on liquid-solid fluidization,
with the limits of $0 < \varepsilon < 0.3$. The index ‘m’ has been related to the drop Reynolds number. An approximate correlation for the estimation of ‘m’ is also suggested by Godfrey and Slater (1991).

$$m = 0.19 \, \text{Re}^{0.5}$$

Godfrey and Slater (1991) reported negative values for ‘m’ at very low rotor speeds and zero at very high speeds. For any given rotor speed, it is not possible to obtain either negative value or zero for ‘m’ using Equation (1.20). In the present work, an attempt is made to reanalyse these values. Rearranging Equation (1.19) to yield;

$$\log(U_s) = \log(U_0) + m \log(1-\varepsilon)$$

Figure 3.34 shows the logarithmic plot of estimated slip velocity ($U_s$) versus (1-$\varepsilon$) and the slopes of the lines for various systems and different rotor speeds are found to vary between 0.8 to 1.2, for the range of variables covered in the present investigation. Hence an average value for ‘m’ as unity is assumed for further analysis. Equation (1.19) becomes

$$U_s = U_0 \, (1-\varepsilon) \text{ and}$$

$$U_0 = U_s / (1-\varepsilon), \text{ i.e.,}$$

i.e.,

$$U_0 = [1 / (1-\varepsilon)] \{ (U_d / \varepsilon) + (U_c / (1-\varepsilon)) \}$$

which is a linear form of equation originally suggested by Logsdail et al (1957) for liquid-liquid extraction in spray columns.
Figure 3.34 Logarithmic plot of slip velocity versus continuous phase hold-up
3.4.1 Correlation for characteristic velocity

The characteristic velocity in RDC is studied by many authors namely, Laddha et al (1978), Zhang et al (1981), Kamath and Subba Rao (1985), Kung and Beckmann (1961), Logsdail et al (1957), Jeffreys et al (1981), Godfrey and Slater (1991) and Misek and Marek (1970, 1983) and correlations for the estimation of characteristic velocity are also proposed by them. Even though a number of correlations exist, the correlations involving operating variables alone are considered for further testing of their applicability to the present (Table 3.1) as well as literature data (Table 3.2) and are discussed below.

3.4.1.1 Applicability of the correlation proposed by Logsdail et al (1957) for the prediction of characteristic velocity.

Based on dimensional analysis approach, Logsdail et al (1957) have formulated a simple correlation of the following form, for the estimation of characteristic velocity.

\[ [U_0 \mu \gamma] = 0.012 \left( \frac{\Delta \rho}{\rho_c} \right)^{0.90} \left( \frac{g}{N^2} D_t \right)^{1.0} \left( \frac{D_{x}/D_r}{Z_{o}/D_t} \right)^{2.3} \left( \frac{D_r/D_t}{} \right)^{2.7} \] (1.21)

Though the correlation (Equation (1.21)) involves most of the operating variables namely, rotor speed, column geometry and the physical properties of the liquid-liquid systems, is applicable only for no solute transfer conditions. Equation (1.21) is tested for its applicability to the present experimental data (Table 3.1) along with those of literature data (Table 3.2) under no solute transfer conditions, gave RMS errors of 74.4% (based on 740 data point) and 73.19% (based on 533 data point) respectively, indicating a very high deviation, limiting its range of applicability. Predicted values using Equation (1.21) are compared with those of experimental results and sample data are shown in Figure 3.35.
Figure 3.35 Present data on characteristic velocity (no solute) according to the correlation of Logsdail et al (1957)
3.4.1.2 Applicability of the correlation proposed by Kung and Beckmann (1961) for the prediction of characteristic velocity

Using the dispersed phase hold-up data obtained in a RDC with a diameter of 0.1524 m, Kung and Beckmann (1961) gave a relation between the characteristic velocity, phase velocities and hold-up. They also used various combinations of column heights with different rotor speeds.

\[ U_0(1-\varepsilon) = U_d/\varepsilon + K \left( U_c/(1-\varepsilon) \right) \]  
(1.22)

The characteristic velocity values thus calculated using Equation (1.22), are correlated in terms of column dimensions, rotor speed and physical properties of the systems.

\[ [U_0/\mu/\gamma] = K_1 \left[ \Delta \rho/\rho_c \right]^{0.90} \left[ g/\pi^2 D_t \right]^{1.0} \left[ D_o/D_t \right]^{2.3} \left[ Z_c/D_t \right]^{0.9} \left[ D_o/D_t \right]^{2.6} \]  
(1.23)

Kung and Beckmann (1961) observed a change in hydrodynamic region beyond a rotor peripheral speed of 1.52 m/s. The change in hydrodynamic region is defined using the column geometry but not based on rotor peripheral speed. The critical value for column geometry \((D_s - D_t) / (D_t)\), is reported as 1/24 and based on these values, the constants \(K\) and \(K_1\) are established by Kung and Beckmann (1961).

For \((D_s - D_t) / (D_t) < 1/24\)  
\[ K = 2.1 \quad \text{and} \quad K_1 = 0.0225 \]

for \((D_s - D_t) / (D_t) > 1/24\)  
\[ K = 1.0 \quad \text{and} \quad K_1 = 0.012 \]

The correlation (Equation (1.23)) suggested by Kung and Beckmann (1961), which is similar to that of Logsdail et al (1957) (Equation (1.21)) is tested for its applicability to the present data (Table 3.1) as well as literature data (Table 3.2) and the corresponding RMS errors are found to be 74.66% (based on 740 data points) and 73.55% (based on 533 data points) respectively and are found to be similar to that of Logsdail et al
Characteristic velocity values calculated using Equation (1.23) are compared with the experimental results in Figure 3.36 (no solute). Equation (1.23) is applicable only for systems with no mass transfer. The high error may be attributed to the fact that the Equation (1.23) is developed using a single system (Toluene(d)-Water(c)) in a column of 0.1524 m diameter.

3.4.1.3 Applicability of the correlation proposed by Laddha et al (1978) for the prediction of characteristic velocity

Laddha et al (1978) analysed their experimental data on characteristic velocity along with those of published literature and confirmed two hydrodynamic regions of operation. Based on the critical rotor speed, Region I, where the characteristic velocity remains constant (at low rotor speed) and Region II, where the characteristic velocity decreases with increase in rotor speed, were identified by Laddha et al (1978) and the following empirical correlations were developed by them.

\[
U_0 / \left\{ \left[ \gamma \Delta \rho \ g / \rho_c \right]^{0.25} \left[ D_s / D_t \right]^{2.1} \left[ D_r / D_t \right]^{2.4} \left[ Z_c / D_t \right]^{0.9} \right\} = K_2 \left\{ \left[ g / N^2 D_t \right]^{1.0} \left[ \gamma \rho_c / \mu c \right]^{0.25} \left[ \Delta \rho / \rho_c \right]^{0.6} \right\}^{n_1} = K_2 \{X_1\}^{n_1}
\]

(1.25)

For no solute,

\[
X_1 < 180; \quad K_2 = 0.01, \quad n_1 = 1.0 \text{ and } \]

\[
X_1 > 180; \quad K_2 = 1.08, \quad n_1 = 0.08
\]

\[
U_0 / \left\{ \left[ \gamma \Delta \rho g / \rho_c \right]^{0.25} \left[ D_s / D_t \right]^{2.1} \left[ D_r / D_t \right]^{2.4} \left[ Z_c / D_t \right]^{0.9} \right\} = K_3 \left\{ \left[ g / N^2 D_t \right]^{1.0} \left( \left[ \gamma \rho_c / \mu c \right]^{0.25} \left[ \Delta \rho / \rho_c \right]^{0.6} \right)^{0.5} \right\}^{n_2} = K_3 \{X_2\}^{n_2}
\]

(1.26)

For c -> d direction of transfer,

\[
X_2 > 25; \quad K_3 = 1.46, \quad n_2 = 0.08 \text{ and } \]

\[
X_2 < 25; \quad K_3 = 0.077, \quad n_2 = 1.0
\]
Figure 3.36 Present data on characteristic velocity (no solute) according to the correlation of Kung and Beckmann (1961)
For $d \rightarrow c$ direction of transfer,

$X_2 > 16; \quad K_3 = 1.46, \quad n_2 = 0.08$ and

$X_2 < 16; \quad K_3 = 0.11, \quad n_2 = 1.0$

Equations (1.25 and 1.26) when tested for its applicability to present data (Table 3.1) as well as for literature data (Table 3.2), gave RMS errors of 33.51% and 33.82% (details listed in Table 3.11) for no solute and solute transfer ($c \rightarrow d$ and $d \rightarrow c$) conditions respectively. The values thus calculated according to the correlation of Laddha et al (1978) are compared with the characteristic velocity values estimated (Equations 1.30 and 1.31) using experimentally measured hold-up and phase velocities and are shown in Figure 3.37. Figure 3.37a. shows a plot of data on no solute transfer whereas Figure 3.37b. shows the data on solute transfer (both $c \rightarrow d$ and $d \rightarrow c$) conditions.

3.4.1.4 Applicability of the correlation proposed by Zhang et al (1981) for the prediction of characteristic velocity

The correlation of Laddha et al (1978) for the estimation of characteristic velocity is reanalysed by Zhang et al (1981) and three hydrodynamic regions of operation are identified by them. Zhang et al (1981) however suggested correlations only for two regions. The suggested correlations are similar to the correlations of Laddha et al (1978), with different constants and indices. The correlations of Zhang et al (1981) for the estimation of characteristic velocity are given below:

$$U_0 / \left[ \gamma \Delta \rho \frac{g}{\rho_c} \right]^{0.25} \left[ \frac{D_r / D_i}{D_i / D_r} \right]^{2.1} \left[ \frac{Z_r / D_r}{Z_i / D_i} \right]^{0.9} \left[ \frac{D_r / D_i}{\rho_i / \rho_r} \right]^{0.3} = K_4 \left[ \frac{N^2}{D_i / g} \right] \left[ \left[ \gamma \rho_c / \mu \rho_c / g \right]^{0.25} \left[ \Delta \rho / \rho_c \right]^{0.6} \right]^{n_3}$$

$$= K_4 \left[ X_3 \right]^{n_3} \quad (1.27)$$
Figure 3.37: Present data on characteristic velocity according to the correlation of Laddha et al. (1978)
For no solute,

\[ X_3 < 19 \text{ (Region II); } K_4 = 0.092, \quad n_3 = 1.0 \text{ and } \]
\[ X_3 > 19 \text{ (Region I); } K_4 = 1.08, \quad n_3 = 0.08. \]

For \( c \rightarrow d \) direction of solute transfer,

\[ X_3 < 25 \text{ (Region II); } K_4 = 0.077, \quad n_3 = 1.0 \text{ and } \]
\[ X_3 > 25 \text{ (Region I); } K_4 = 1.46, \quad n_3 = 0.08. \]

For \( d \rightarrow c \) direction of solute transfer,

\[ X_3 < 16 \text{ (Region II); } K_4 = 0.11, \quad n_3 = 1.0 \text{ and } \]
\[ X_3 > 16 \text{ (Region I); } K_4 = 1.46, \quad n_3 = 0.08. \]

Zhang et al (1981), observed that the correlation proposed by Laddha et al (1978) for the estimation of characteristic velocity is applicable only for low levels of agitation. The above mentioned correlations along with the conditions / limits are analysed for its applicability to the present data (Table 3.1) and literature data (Table 3.2) on no solute as well as solute transfer conditions (both \( c \rightarrow d \) and \( d \rightarrow c \) direction of mass transfer) and the corresponding RMS errors are given in Table 3.11. Figures 3.38a and 3.38b show the comparison between the experimental and calculated (using Equation (1.27)) characteristic velocity values for no solute and solute transfer conditions respectively.

3.4.1.5 Applicability of the correlation proposed by Jeffreys et al (1981) for the prediction of characteristic velocity

Jeffreys et al (1981) proposed a correlation for the prediction of characteristic velocity using the method of dimensional analysis and the suggested correlation given below is applicable only for no solute transfer conditions of RDC operation.
Figure 3.38 Present data on characteristic velocity according to the correlation of Zhang et al. (1981)
Equation (1.28), when analysed using the data obtained in the present investigation (Table 3.1) along with those available in literature (Table 3.2) under no solute transfer condition gave RMS errors of more than 100% for both the cases. The correlation constant is very small, when compared with the group values, indicating the inadequacy of the groups defined in Equation (1.28), to represent the data on characteristic velocity.

3.4.1.6 Applicability of the correlation proposed by Kamath and Subba Rao (1985) for the prediction of characteristic velocity

Kamath and Subba Rao (1985) reanalysed the correlation of Laddha et al (1978) and confirmed the existence of three regions of operation, based on rotor speed namely, nearly constant region, gradually decreasing region and rapidly decreasing region. The following empirical equations for no solute and solute transfer (c → d and d → c directions of transfer) are suggested by them.

For no solute
\[
\exp \left[ \frac{U_0}{\left[ \frac{\gamma \Delta p}{\rho c} \right]^{0.25} \left[ \frac{D_r}{D_i} \right]^{1.25} \left[ \frac{D_i}{D_r} \right]^{2.4} \left[ \frac{Z_c}{D_i} \right]^{0.9}} \right] = K_5 \left[ \frac{\gamma \rho c}{\rho c} \right]^{0.35} \left[ \Delta p / \rho c \right]^{0.5} n_4
\]
\[K_5 \{ X_4 \}^{n_4} \quad (1.29)\]

180 > X4 > 90; \quad K_5 = 3.12 \times 10^{-6}, \quad n_4 = 3.24 \quad \text{and} \quad n_4 = 1.205

For solute transfer
\[
\exp \left[ \frac{U_0}{\left[ \frac{\gamma \Delta p}{\rho c} \right]^{0.25} \left[ \frac{D_r}{D_i} \right]^{2.1} \left[ \frac{D_i}{D_r} \right]^{2.4} \left[ \frac{Z_c}{D_i} \right]^{0.9}} \right] = K_6 \left[ \frac{\gamma \rho c}{\rho c} \right]^{0.35} \left[ \Delta p / \rho c \right]^{0.5} n_5
\]
\[K_6 \{ X_5 \}^{n_5} \quad (1.30)\]
For c → d direction of solute transfer

\[
\begin{align*}
X_5 > 25; & \quad K_6 = 79.5, \quad n_5 = 0.167 \\
25 > X_5 > 9; & \quad K_6 = 0.0308, \quad n_5 = 2.37 \\
X_5 < 9; & \quad K_6 = 0.224, \quad n_5 = 1.744
\end{align*}
\]

For d → c direction of solute transfer

\[
\begin{align*}
X_5 > 16; & \quad K_6 = 34.56, \quad n_5 = 0.187 \\
16 > X_5 > 8; & \quad K_6 = 0.0124, \quad n_5 = 2.65 \\
X_5 < 8; & \quad K_6 = 0.502, \quad n_5 = 0.97
\end{align*}
\]

The suggested correlations of Kamath and Subba Rao (1985) (Equations 1.29 and 1.30) are tested for its applicability to the present data (Table 3.1) as well as literature data (Table 3.2) on both no solute and solute transfer conditions. The RMS errors are given in Table 3.11. Present data on characteristic velocities are compared with the calculated values using Equations (1.35) and (1.36) and sample data are shown in Figures 3.39a and 3.39b.

### 3.4.1.7 Applicability of the correlation proposed by Godfrey and Slater (1991) for the prediction of characteristic velocity

Recently Godfrey and Slater (1991) suggested a correlation for the prediction of characteristic velocity in terms of drop size \(D_p\) and drop terminal velocity \(U_t\) as given below:

\[
\frac{U_0}{U_t} = 1.0 - 1.443 \left\{ N^3 D_t^5 \right\}^{0.3} - 0.494 \left\{ \frac{D_p}{(D_s - D_d)} \right\}^{0.77} \tag{1.31}
\]

Godfrey and Slater (1991) recommended the correlations of Chang-Kakoti et al (1985) and Kumar and Hartland (1986) for the estimation of drop size \(D_p\). For the estimation of characteristic velocity using Equation (1.31), a knowledge of terminal velocity of the drops is needed. The terminal velocity of the drop in RDC is expected to be the
Figure 3.39 Present data on characteristic velocity according to the correlation of Kamath and Subba Rao (1985)
function of rotor speed, internal geometry of the column and also the phase flow rates, but a method for the estimation of drop terminal velocity \((U_t)\) is not suggested by Godfrey and Slater (1991).

The important available correlations for the estimation of characteristic velocity as suggested by the authors namely, Logsdail et al (1957), Kung and Beckmann (1961), Laddha et al (1978), Zhang et al (1981), Jeffreys et al (1981) and Kamath and Subba Rao (1985) are tested for their applicability to the present experimental results along with those available in the literature and the resulting RMS errors are summarised in Table 3.11.

3.4.2 Development of generalised correlation for characteristic velocity

From the foregoing analysis, it is observed that many of the correlations suggested (Logsdail et al (1957), Kung and Beckmann (1961) and Jeffreys et al (1981)) are based on no solute transfer condition and the suggested correlations could not be used for mass transfer condition. Since, the characteristic velocity under mass transfer conditions \((c \rightarrow d \text{ and } d \rightarrow c)\) is expected to be different from that of no solute transfer condition, due to the change in physical properties, a better understanding of the dependency of the characteristic velocity (for both no solute and solute transfer conditions) on the operating variables is necessary. With this view, to develop a correlation for the estimation of characteristic velocity in terms of operating variables, the following approach is made.

The characteristic velocity \(U_0\) as defined in Equation (1.2),

\[ U_0 = \left[ \frac{1}{1-\epsilon} \right] \left\{ \frac{(U_d / \epsilon)}{U_c / (1-\epsilon)} \right\} \]  

(1.2)
Table 3.11

Error analysis of present and literature data on characteristic velocity according to literature correlations

<table>
<thead>
<tr>
<th>Author and year</th>
<th>Data</th>
<th>Solute transfer</th>
<th>No. of Points</th>
<th>RMS Error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Present data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Logsdail et al (1957)</td>
<td>No solute</td>
<td>c → d</td>
<td>740</td>
<td>74.4%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>74.4%</td>
</tr>
<tr>
<td></td>
<td>d → c</td>
<td></td>
<td>740</td>
<td>74.4%</td>
</tr>
<tr>
<td><strong>Literature data</strong></td>
<td>No solute</td>
<td>c → d</td>
<td>533</td>
<td>73.19%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>533</td>
<td>73.19%</td>
</tr>
<tr>
<td>Kung and Beckmann (1961)</td>
<td>No solute</td>
<td>c → d</td>
<td>740</td>
<td>74.66%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>74.66%</td>
</tr>
<tr>
<td></td>
<td>d → c</td>
<td></td>
<td>740</td>
<td>74.66%</td>
</tr>
<tr>
<td><strong>Literature data</strong></td>
<td>No solute</td>
<td>c → d</td>
<td>533</td>
<td>73.55%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>533</td>
<td>73.55%</td>
</tr>
<tr>
<td>Laddha et al (1978)</td>
<td>No solute</td>
<td>c → d</td>
<td>740</td>
<td>36.00%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>36.00%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>36.00%</td>
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<td></td>
<td>d → c</td>
<td></td>
<td>740</td>
<td>36.00%</td>
</tr>
<tr>
<td><strong>Literature data</strong></td>
<td>No solute</td>
<td>c → d</td>
<td>533</td>
<td>24.76%</td>
</tr>
<tr>
<td></td>
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<td>533</td>
<td>24.76%</td>
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<td></td>
<td>c → d</td>
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<td>533</td>
<td>24.76%</td>
</tr>
<tr>
<td></td>
<td>d → c</td>
<td></td>
<td>533</td>
<td>24.76%</td>
</tr>
<tr>
<td>Jeffreys et al (1981)</td>
<td>No solute</td>
<td>c → d</td>
<td>740</td>
<td>120.6%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>120.6%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>120.6%</td>
</tr>
<tr>
<td></td>
<td>d → c</td>
<td></td>
<td>740</td>
<td>120.6%</td>
</tr>
<tr>
<td><strong>Literature data</strong></td>
<td>No solute</td>
<td>c → d</td>
<td>533</td>
<td>106.8%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>533</td>
<td>106.8%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>533</td>
<td>106.8%</td>
</tr>
<tr>
<td>Zang et al (1981)</td>
<td>No solute</td>
<td>c → d</td>
<td>740</td>
<td>50.78%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>50.78%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>50.78%</td>
</tr>
<tr>
<td></td>
<td>d → c</td>
<td></td>
<td>740</td>
<td>50.78%</td>
</tr>
<tr>
<td><strong>Literature data</strong></td>
<td>No solute</td>
<td>c → d</td>
<td>533</td>
<td>63.01%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>533</td>
<td>63.01%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>533</td>
<td>63.01%</td>
</tr>
<tr>
<td>Kamath and Subba Rao (1985)</td>
<td>No solute</td>
<td>c → d</td>
<td>740</td>
<td>43.74%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>43.74%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>740</td>
<td>43.74%</td>
</tr>
<tr>
<td></td>
<td>d → c</td>
<td></td>
<td>740</td>
<td>43.74%</td>
</tr>
<tr>
<td><strong>Literature data</strong></td>
<td>No solute</td>
<td>c → d</td>
<td>533</td>
<td>44.24%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>533</td>
<td>44.24%</td>
</tr>
<tr>
<td></td>
<td>c → d</td>
<td></td>
<td>533</td>
<td>44.24%</td>
</tr>
</tbody>
</table>


is a function of phase flow rates (both dispersed, $U_d$ and continuous phases, $U_c$) and the dispersed phase hold-up $\varepsilon$, which can be represented in a simple form,

$$U_0 = f(U_d, U_c, \varepsilon) \quad (3.17)$$

As discussed earlier in Part-A and Part-B of the present investigation, the dispersed phase hold-up depends on operating variables, the physical properties and the column geometry and Equation (3.17) can be modified as:

$$U_0 \approx f(N, U_d, U_c, D_t, D_r, Z_c, \rho_c, \Delta \rho, \gamma, \nu_c, \mu_d, g) \quad (3.18)$$

Using dimensional analysis, the following form of equation (P.Kalaichelvi and T.Murugesan (1998)), which is similar to that of Equation (3.8) (for dispersed phase hold-up) and Equation (3.14) (for slip velocity) is developed to represent characteristic velocity data.

$$U_0^*/[Gf^{1-\psi_4}] = C_4 [\psi_2]^{n_4} \quad (3.19)$$

where

$$U_0^* = ((U_0) / (\gamma \Delta \rho g (\rho_c)^{0.25}))$$

$$\psi_4 = ((U_d) / (\gamma \Delta \rho g (\rho_c)^{0.25})^{0.3} (1 + (U_c / U_d))^{0.02}$$

$$G_f = (D_r / D_t)^{2.7} (D_t / D_r)^{2.5} (D_r / Z_c)^{0.75}$$

$$\psi_2 = [(N^2 D_t / g)^{0.33} (\mu_c g^{4/3} \Delta \rho)^{0.07}(\rho_c / \Delta \rho)^{0.2}]$$

A data bowl consisting of 740 present data on no solute transfer conditions and 156 measurements each on $c \rightarrow d$ and $d \rightarrow c$ conditions of mass transfer are used for the estimation of constants and indices of Equation (3.19). The constants and indices, thus estimated for both the regions of operation (nearly constant and decreasing characteristic velocity) are given in Table 3.12 and the corresponding RMS errors for all the available data for both regions of operation are calculated and are given in Table 3.13.
Table 3.12

Values of constants and indices for characteristic velocity correlation (Equation (3.19))

<table>
<thead>
<tr>
<th>Region of operation</th>
<th>no solute</th>
<th>c → d direction of solute transfer</th>
<th>d → c direction of solute transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\psi_2$</td>
<td>$C_4$</td>
<td>$n_4$</td>
</tr>
<tr>
<td>Region I &lt;0.22</td>
<td>3.85</td>
<td>-0.01</td>
<td></td>
</tr>
<tr>
<td>Region II &gt;0.22</td>
<td>0.2</td>
<td>-2.16</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.13

Error analysis of present and literature data on characteristic velocity according to the present correlation (Equation (3.19))

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of data points</th>
<th>RMS error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Literature</td>
<td>Present work</td>
</tr>
<tr>
<td>No solute</td>
<td>533</td>
<td>740</td>
</tr>
<tr>
<td>c → d</td>
<td>255</td>
<td>156</td>
</tr>
<tr>
<td>d → c</td>
<td>95</td>
<td>156</td>
</tr>
</tbody>
</table>
3.4.3 Applicability of the present proposed hold-up correlation for the estimation of characteristic velocity to the present data as well as literature data

The characteristic velocity $U_0$, is defined as the hypothetical terminal velocity of a drop, when the continuous phase velocity is zero and the dispersed phase velocity tending to zero; which has been represented in terms of slip velocity $U_s$ and the continuous phase hold-up $(1-\varepsilon)$, i.e.,

$$U_0 = U_s / (1-\varepsilon) = (1 / (1-\varepsilon)) + [(U_d / \varepsilon) + (U_c / (1-\varepsilon))]$$ \hspace{1cm} (3.16)

It is shown in Part-A of the present investigation, that the dispersed phase hold-up could be estimated with better accuracy from the knowledge of operating variables and a correlation is presented (Equation (3.8), using a wide range of variables. Hence, it is proposed to estimate characteristic velocity using Equation (3.16) with the prior knowledge of phase flow rates as well as the predicted values of dispersed phase hold-up using Equation (3.8). The data bowl used in Part-A and Part-B of the present investigation, consisting of 1273 data points on no solute transfer condition, 411 data points under $c \rightarrow d$ direction of solute transfer and 251 data points under $d \rightarrow c$ direction of solute transfer gave RMS errors of 12.2%, 13.7% and 14.9% respectively and the details are given in Table 3.14. Figure 3.40 shows the comparison of characteristic velocity data with the predicted values according to Equation (3.16) under no solute transfer conditions, whereas Figures 3.41 and 3.42 show the comparison of data under $c \rightarrow d$ and $d \rightarrow c$ directions of solute transfer respectively.
Figure 3.40  Comparison of data on characteristic velocity (no solute), calculated using the present hold-up correlation.
Figure 3.41 Comparison of data on characteristic velocity \((c \rightarrow d)\), calculated using the present hold-up correlation.
Figure 3.42 Comparison of data on characteristic velocity (d → c), calculated using the present hold-up correlation.
Table 3.14

Error analysis for the estimation of characteristic velocity based on present correlation for dispersed phase hold-up (both literature and present data)

<table>
<thead>
<tr>
<th>Region of operation</th>
<th>Number of data points</th>
<th>RMS Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no solute</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c→d</td>
<td>d→c</td>
</tr>
<tr>
<td></td>
<td>no solute</td>
<td>c→d</td>
</tr>
<tr>
<td>Based on present data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region I</td>
<td>400</td>
<td>60</td>
</tr>
<tr>
<td>Region II</td>
<td>340</td>
<td>96</td>
</tr>
<tr>
<td>Based on Literature data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region I</td>
<td>234</td>
<td>29</td>
</tr>
<tr>
<td>Region II</td>
<td>299</td>
<td>226</td>
</tr>
<tr>
<td>Overall data</td>
<td>1273</td>
<td>411</td>
</tr>
</tbody>
</table>
PART-D

STUDIES ON OVERALL MASS TRANSFER COEFFICIENTS

3.5 OVERALL MASS TRANSFER COEFFICIENTS

For the design and scale up of RDC, it is essential to explore the hydrodynamic behavior (namely, slip velocity, characteristic velocity, hold-up, drop size) along with mass transfer coefficients. The dispersed phase hold-up together with drop size determines the interfacial area while slip velocity controls the mass transfer coefficients so that the total mass transfer could be estimated. Most of the cases the mass transfer coefficients are reported as a function of dispersed phase hold-up / drop size / characteristic velocity and also the available correlations are restricted to a limited range of applications; many of the correlation are pertaining to the prediction of film coefficients based on either continuous or dispersed phase. Since the overall mass transfer coefficients are important for the design of the column (i.e, for the estimation of column height), in the present investigation, an attempt is made to develop a generalised correlation for the estimation of overall mass transfer coefficients in terms of operating variables namely, phase velocities, rotor speed, column geometry and physical properties of the liquid-liquid systems.

3.5.1 Analysis of data on overall mass transfer coefficients

In the present investigation, experimentally measured overall mass transfer coefficient (K_{\text{oa}}) data are analysed graphically to study the influence of the various operating variables namely, dispersed phase velocity, continuous phase velocity, rotor speed, physical properties of the liquid-liquid systems and the direction of solute transfer and are discussed below.
3.5.1.1 Effect of dispersed phase velocity on overall mass transfer coefficients

Experimental values of overall mass transfer coefficients are plotted against dispersed phase velocity at nearly constant continuous phase velocity for four different rotor speeds. The systems studied are Benzene(d)-Acetone-Water(c) and n-Hexane(d)-Acetone-Water(c). Data on overall mass transfer coefficients (K_{oa}) for both c \rightarrow d and d \rightarrow c directions of solute transfer conditions are shown in Figure 3.43. It is observed from the graphs that the increase in dispersed phase velocity has increased the overall mass transfer coefficients irrespective of the liquid systems and direction of solute transfer. This could be attributed to the fact that increase in dispersed phase velocity increases the dispersed phase hold-up (Part -A), which in turn increases the overall mass transfer coefficients.

3.5.1.2 Effect of continuous phase velocity on overall mass transfer coefficients

Figure 3.44 shows the plot of overall mass transfer coefficients (K_{oa}) versus dispersed phase velocity at constant rotor speed and for two different continuous phase velocities. The systems used for the present investigation involve, Benzene(d)-Acetone-Water(c) and n-Hexane(d)-Acetone-Water(c) for both c \rightarrow d and d \rightarrow c directions of solute transfer conditions. From the graphs, it is observed that for any fixed dispersed phase velocity, the overall mass transfer coefficient increases slightly with the increase in continuous phase velocity. The slight increase in overall mass transfer coefficient may be attributed to the fact that the residence time of dispersed droplets are increased slightly with the increase in continuous phase velocity.
Figure 3.43 Effect of dispersed phase velocity on mass transfer coefficients
Figure 3.44 Effect of continuous phase velocity on mass transfer coefficients.
3.5.1.3 Effect of system properties on overall mass transfer coefficients

Experimental data on overall mass transfer coefficients are plotted in Figure 3.45 against dispersed phase velocities for Benzene(d)-Acetone-Water(c) and n-Hexane(d)-Acetone-Water(c) (for both c → d and d → c directions of solute transfer conditions) at constant rotor speeds (16.67 rps and 11.67 rps). The overall mass transfer coefficient values are found to be more for Benzene(d)-Acetone-Water(c) (Morton Number, $\mu_c^4 g/\gamma \rho_c = 1.41 \times 10^9$), than that of n-Hexane(d)-Acetone-Water(c) (Morton Number, $\mu_c^4 g/\gamma \rho_c = 7.8 \times 10^9$) for both directions (c → d and d → c) of solute transfer conditions, the reason could be attributed to the change in physical properties (in particular the interfacial tension, Korchinsky W.J and Bastani D.(1993)) of liquid-liquid systems.

3.5.1.4 Effect of direction of solute transfer on overall mass transfer coefficients

The present data on overall mass transfer coefficients ($K_{oa}$) measured experimentally are plotted against rotor speed, at nearly constant phase velocities for both directions of solute transfer conditions as shown in Figure 3.46 and the systems used are Benzene(d)-Acetone-Water(c) and n-Hexane(d)-Acetone-Water(c). In both cases mass transfer coefficients are more for d → c direction of solute transfer than that for c → d direction of solute transfer. The shift in overall mass transfer coefficient values may be attributed to the properties of the liquid systems being influenced by the concentration induced turbulence caused by the rotor kinetic energy as well as Marangoni surface instability.

3.5.1.5 Effect of rotor speed on overall mass transfer coefficients

The size of the dispersed phase droplets in the rotating disc contactor is controlled by the rotor speed and the effect of the rotor speed on overall mass
Figure 3.45 Effect of system properties on mass transfer coefficients
Figure 3.46 Effect of solute transfer and rotor speed on mass transfer coefficients
transfer coefficients can be observed from the Figure 3.43. From the Figure 3.43, it is observed that increase in rotor speed increases the overall mass transfer coefficients in all four cases, namely, Benzene(d)-Acetone-Water(c) (c → d and d → c) and n-Hexane(d)-Acetone-Water(c) (c → d and d → c). At higher rotor speeds, the larger drops are broken into smaller ones, thereby increasing the interfacial area available for mass transfer resulting in increased overall mass transfer coefficient values.

3.5.2 Correlation for overall mass transfer coefficients


Among the available correlations, the correlation due to Laddha et al (1978) gives the mass transfer coefficients in terms of operating variables. Hence the correlation of Laddha et al (1978) is considered for further analysis.

3.5.2.1 Applicability of the correlation proposed by Laddha et al (1978) for the estimation of overall mass transfer coefficients

Laddha et al (1978) have proposed a generalised correlation for the prediction of overall mass transfer coefficients in terms of operating variables namely, rotor speed, column geometry, physical properties of the liquid-liquid systems and diffusivity of the solute along with dispersed phase hold-up. The suggested correlations for two hydrodynamic regions of operation are given below:
Region I (below critical rotor speed) for both $c \to d$ and $d \to c$ directions of solute transfer

$$K_{oda} = 0.068 \varepsilon (1-\varepsilon) \left( \frac{g^3}{\Delta \rho c^3} \gamma \right)^{1/4} \left[ (Sc)^{1/2} + m (Sc)^{1/2} \right]^{-1} \quad (1.41)$$

Region II (above critical rotor speed) for both $c \to d$ and $d \to c$ directions of solute transfer

$$K_{oda} = 0.95 \varepsilon (1-\varepsilon) \left( \frac{g^3}{\Delta \rho c^3} \gamma \right)^{1/4} \left( \frac{g}{N^2 D_r} \right)^{1/2} \left[ (\gamma^3 \rho_d / \mu_c^4) g^{1/4} (\Delta \rho / \rho_c)^{0.6} \right]^{1/2} \left[ (Sc)^{1/2} + m (Sc)^{1/2} \right]^{-1} \quad (1.42)$$

and the conditions for the estimation of critical speed as suggested by Laddha et al (1978) are given below;

For $c \to d$ direction of solute transfer

$$\left( \frac{g}{N^2 D_r} \right) \left\{ \left( \frac{g^3}{\Delta \rho c^3} \gamma \right)^{1/4} \left( \frac{g}{N^2 D_r} \right)^{1/2} \left( \frac{\gamma^3 \rho_d / \mu_c^4}{\gamma^4 g} \right)^{1/4} \left( \Delta \rho / \rho_c \right)^{0.6} \right\}^{1/2} = 16 \quad (1.43)$$

For $d \to c$ direction of solute transfer

$$\left( \frac{g}{N^2 D_r} \right) \left\{ \left( \frac{g^3}{\Delta \rho c^3} \gamma \right)^{1/4} \left( \frac{g}{N^2 D_r} \right)^{1/2} \left( \frac{\gamma^3 \rho_d / \mu_c^4}{\gamma^4 g} \right)^{1/4} \left( \Delta \rho / \rho_c \right)^{0.6} \right\}^{1/2} = 25 \quad (1.44)$$

Equations (1.41) and (1.42) are tested for its applicability, using the present data on overall mass transfer coefficients along with literature data and the estimated RMS errors are 80.57% ($c \to d$), 41.42% ($d \to c$) and 35.73% ($c \to d$), 27.72% ($d \to c$) respectively, indicating a shift in the critical speed. The correlation proposed by Laddha et al (1978) includes dispersed phase hold-up ($\varepsilon$), which in turn depends upon phase flow rates, rotor speed, column geometry and physical properties of the systems. Figure 3.47 compares the present as well as literature data on overall mass transfer coefficients with those calculated according to the correlation of Laddha et al (1978).
Figure 3.47 Data on mass transfer coefficients according to the correlation of Laddha et al (1978)
3.5.3 Development of correlation for the estimation of overall mass transfer coefficients (K_{o,dl})

A similar approach, which has been applied for the development of dispersed phase hold-up correlation is extended to establish a correlation for the estimation of overall mass transfer coefficient (K_{o,dl}) in terms of operating variables namely, phase velocities, rotor speed, column geometry, diffusivity of the solute and physical properties of the liquid-liquid systems.

Based on the assumption, that the probability of a surface element being replaced by another element is independent of time and ‘s’ is the fractional rate of replacement of elements, Danckwerts (1951) proposed an equation for the average rate of solute transfer,

\[ N_{av} = (\Delta C)\sqrt{Ds} \quad (3.20) \]

Based on this surface renewal theory, the mass transfer coefficient for any phase can be defined as

\[ k = \sqrt{Ds} \quad (3.21) \]

Equation (3.21) could be rearranged to give

\[ k = (\mu s/\rho)^{1/2} (\mu /\rho D)^{1/2} \quad (3.22) \]

where the group \((\mu s/\rho)^{1/2}\) has the dimensions of velocity, and hence may be identified with the interfacial velocity proportional to the slip velocity \(U_s\), therefore,

\[ k = U_s (\mu /\rho D)^{1/2} \quad (3.23) \]
The rate of mass transfer could be represented in terms of area-based and volumetric mass transfer coefficients.

\[ N_A = k \cdot A_d (\Delta c)_m = ka V (\Delta c)_m \] (3.24)

Hence

\[ ka = \frac{(A_d \cdot k)}{V} \] (3.25)

Introducing the dispersed phase holdup \( \varepsilon \) (= \( V_d/V \)) into Equation (3.25),

\[ ka \propto \frac{(A_d / V_d)}{k \cdot \varepsilon} \] (3.26)

Considering the mean volume to surface ratio of the droplets \( d_{32} \) to represent for \( A_d / V_d \), and combining Equations (3.23) and (3.26) to represent mass transfer coefficient

\[ ka \propto \frac{(\varepsilon /d_{32}) U_s (\mu /\rho D)^{1/2}}{1} \] (3.27)

The slip velocity \( U_s \) and characteristic velocity \( U_0 \) are related as (Thronton et al. (1957),

\[ U_s = (1-\varepsilon) U_0 \] (3.28)

Substituting Equation (3.28) into Equation (3.27);

\[ ka \propto \varepsilon (1-\varepsilon) (U_0 / d_{32}) (\mu /\rho D)^{1/2} \] (3.29)

The individual mass transfer coefficients for continuous phase \( 'k_a' \) and that for the dispersed phase \( 'k_d' \) may be defined as:
The overall mass transfer coefficient can be obtained by using the additivity principle and combining Equations (3.30) and (3.31) to yield;

$$K_{\text{oa}} \propto \varepsilon (1-\varepsilon) \left( \frac{U_0}{d_{32}} \right) \left( \frac{\mu_c}{\rho_c D_c} \right)^{1/2}$$

Equation (3.32) is a generalised model for representing the overall coefficient for interface mass transport. In the operation of RDC, the rotor speed considerably influences the drop size distribution and it is always possible that the droplet swarm will have a large variation of drop size. The smaller drops may act as rigid spheres and the larger ones may show internal circulation or oscillation. In case of RDC, the average drop size in the normal operating conditions is expected to be considerably less than the peak diameter and the drops do not show appreciable circulation or oscillation. If the drop size and characteristic velocity of the swarms can be estimated, it is possible to predict the mass transfer coefficient using Equation (3.32).

Rearranging Equation (3.32) in terms of dimensionless form:

$$\frac{K_{\text{oa}} \rho_c^2 \Delta \rho^3}{k_{\text{g}} \Delta \rho^3 / \rho_c^2 \gamma} = f(\varepsilon, U_0, d_{32})$$

Under the normal operating conditions of RDC, the rotor speed and other operating variables considerably influence dispersed phase hold-up, characteristic velocity and also the drop size distribution. In the previous section, the effect of variables on dispersed phase hold-up (Part-A) and characteristic velocity (Part-C), is discussed in detail and correlated in terms
of operating variables and hence replacing \( e, U_0 \) and \( d_{32} \) in terms of operating variables namely, phase velocities, rotor speed, column geometry, and the physical properties of the liquid systems, Equation (3.33) becomes

\[
\frac{K_{oda}^*}{\psi_5} = f(U_c, U_d, N, D_c, D_r, D_s, Z_c, \rho_c, \rho_d, \gamma, \mu_c, \mu_d, g)
\] (3.34)

where,

\[
K_{oda}^* = \frac{K_{oda}}{(g^{2.5} \Delta \rho^3 / \rho_c^2 \gamma)^{1/4}}
\]

\[
\psi_5 = [(Sc)^{1/2} + m(Sc_c)^{1/2}]^{-1}
\]

The variables are grouped to form dimensionless groups and this leads to the following form of equation similar to the form developed for dispersed phase hold-up correlation in the earlier section (Equation (3.8)),

\[
\frac{K_{oda}^*}{\psi_5} = C (U_d^*)^{n1} (U_c/U_d)^{n2} (\mu_c/\mu_d)^{n3} (D_c/D_r)^{n4} (D_r/D_c)^{n5} (Z_c/D_c)^{n6} \left( \frac{N^2 D_c g}{N^2 D_c g} \right)^{n7} (\mu_c^4 g / \gamma^2 \Delta \rho)^{n8} (\rho_c / \Delta \rho)^{n9}
\] (3.35)

where

\[
U_d^* = \left\{ U_d / (\gamma \Delta \rho g / \rho_c)^{0.25} \right\}
\]

The term representing the ratio of phase velocities i.e., \( U_c / U_d \), is modified as \( (1 + (U_c / U_d)) \) to represent the stagnant conditions of continuous phase. Based on the regression analysis performed for the present data on overall mass transfer coefficient for both directions of solute transfer condition, (namely, solute transfer from continuous phase to the dispersed phase (c \( \rightarrow \) d) and dispersed phase to continuous phase (d \( \rightarrow \) c)), the values of all the constants and indices are established. The value of \( a_3 \) is found to be insignificant and hence it is been omitted for further analysis. Using the estimated constants and indices Equation (3.35) is redefined as;

\[
\frac{K_{oda}^*}{Gr \cdot \psi_1 \cdot \psi_5} = C_5 [\psi_2]^{n5}
\] (3.36)
where \( G_f = \left( \frac{D_s}{D_r} \right)^{2.1} \left( \frac{D_r}{D_\infty} \right)^{2.5} \left( \frac{Z_c}{D_r} \right)^{0.75} \)

\( \psi_1 = \left( \frac{U_d}{\psi_1} \right)^{1.1} \left( 1 + \frac{U_c}{U_d} \right)^{0.15} \)

\( \psi_2 = \left( \frac{N^2 D_s}{g^3} \right)^{0.33} \left( \frac{\mu_s}{\gamma^3 \Delta \rho} \right)^{0.07} \left( \frac{\rho_s}{\Delta \rho} \right)^{0.2} \)

The terms \( G_f \) and \( \psi_1 \) represent the effect of column geometry and phase velocities on the overall mass transfer coefficients respectively, whereas \( \psi_2 \) represents the effect of rotor speed along with the physical properties of the liquid-liquid systems. From the analysis of data, it is also observed that the rotor speed has a pronounced effect on the overall mass transfer coefficients. Based on the graphical analysis and observation made in the case of dispersed phase hold-up estimation, a reanalysis is done for the estimation of constants and indices for two hydrodynamic regions of operation (Region I and Region II) for both cases of solute transfer conditions (i.e., \( c \rightarrow d \) and \( d \rightarrow c \) directions of solute transfer) and the estimated constants and indices for Equation (3.36) are given below;

For \( c \rightarrow d \) direction of solute transfer

**Region I**

\( \psi_2 < 0.17; \quad C_5 = 0.02; \quad n_5 = 0.37 \) and

**Region II**

\( \psi_2 > 0.17; \quad C_5 = 0.58; \quad n_5 = 2.50 \)

For \( d \rightarrow c \) direction of solute transfer

**Region I**

\( \psi_2 < 0.2; \quad C_5 = 0.014; \quad n_5 = 0.2 \) and

**Region II**

\( \psi_2 > 0.2; \quad C_5 = 0.25; \quad n_5 = 1.92. \)
The two regions of operation observed in the case of dispersed phase hold-up are confirmed with the present data on overall mass transfer coefficients ($K_{\text{ata}}$), namely, Region I indicating nearly constant mass transfer coefficients and Region II representing the increasing mass transfer coefficients. In Region I, the drop size is nearly independent of the rotor speed and under this condition the moving dispersed phase drops are only deflected by the rotating disc without causing any appreciable breakup which restricts mass transfer rate and nearly constant. At high rotor speed (Region II), the kinetic energy produced at the tip of the rotor is mainly spent for the breakup of drops into smaller ones in addition to enhancement of tortuous path of drops and with these smaller drops, the interfacial area for mass transfer increases, and hence the increase in mass transfer coefficients.

Figure 3.48 shows the plot of present data and literature data plotted according to the Equation (3.36). Using the above estimated constants and indices, for the correlation (Equation (3.36)), the RMS errors for $c \rightarrow d$ (222 data points) and $d \rightarrow c$ (241 data points) directions of mass transfer based on present and literature data are found to be 10.35% and 12.35% respectively and the details are given in Table 3.15. The comparison of experimental data on overall mass transfer coefficients ($K_{\text{ata}}$) (for $c \rightarrow d$ and $d \rightarrow c$ directions of mass transfer) with that of calculated values according to Equation (3.36) are shown plotted in Figure 3.49.

Table 3.15
RMS errors based on present correlation for the prediction of overall mass transfer coefficients

<table>
<thead>
<tr>
<th>Direction of solute transfer</th>
<th>Present data points</th>
<th>Literature data points</th>
<th>Total data points</th>
<th>RMS error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c \rightarrow d$</td>
<td>156</td>
<td>66</td>
<td>222</td>
<td>10.35%</td>
</tr>
<tr>
<td>$d \rightarrow c$</td>
<td>156</td>
<td>85</td>
<td>241</td>
<td>12.35%</td>
</tr>
</tbody>
</table>
Figure 3.48 Data on mass transfer coefficients (c→d and d→c) according to the present correlation (Equation (3.36))
Figure 3.49 Comparison of data on mass transfer coefficients (c→d and d→c) with calculated values (Equation (3.36))

System  | Author | Symbol | Data
--- | --- | --- | ---
Benzene(c)-Acetone-Water(c) | present data | x | I03, I/S
n-Hexane(d)-Acetone-Water(c) | present data | A | L
Toluene(d)-Acetone-Water(c) | Kannappan (1973) | ° | O
n-Hexane(d)-Acetone-Water(c) | Kannappan (1973) | a | X

Figure 3.49b