CHAPTER 5

ROLE OF DECISION MAKERS ATTITUDE IN DECISION MAKING.4

5.0. Introduction.

Zhiping Chen and Wei Yang in [82] introduced an algorithm based on decision makers risk attitude, to rank alternatives in decision making problems for interval valued intuitionistic fuzzy sets. They applied this method in situations having exactly known and partly known criteria weight. If the weights of the criteria are only partially known, they introduced an optimization model to find these weights.

Usually in the decision process, the decision depends on the risk attitude of the decision maker. For example, the risk-averse decision maker would like to select low risk alternatives, while the risk

4Some of the results mentioned in this chapter have been published in the journal, International Journal of Fuzzy Mathematics and Systems, Vol. 3(2013), No.3, pp. 189-195, under the title “Multi attribute decision making using decision makers attitude in intuitionistic fuzzy context.”
seeking decision maker would like to select high risk alternatives [82]. Zhiping Chen and Wei Yang introduced a decision function based on the weighted score function and the weighted accuracy function. In their work, the decision maker can choose a proper decision function, according to his/her risk attitude, by selecting a suitable parameter value in the decision function. If criteria weights are known exactly, the evaluation values of the criteria can be aggregated directly by using the decision function. If criteria weights are not known exactly, a linear programming model is set up to determine criteria weights as in [82].

However, in this chapter we propose a decision function to solve MCDM models. Here the decision function consists of a score function and an accuracy function. This decision function can handle cases where, information about criteria weights are exactly known or partially known. It is illustrated with a simple example.

5.1. Weighted score function and weighted accuracy function.

In this section we recall a definition given by Zhiping Chen and Wei Yang, which is necessary for ranking of IFNs. For further details
one can refer to [66] and [82].

For interval valued intuitionistic fuzzy numbers,

\[ \alpha_1 = ([a_1, b_1], [c_1, d_1]), \]

\[ \alpha_2 = ([a_2, b_2], [c_2, d_2]), ..., \]

\[ \alpha_n = ([a_n, b_n], [c_n, d_n]) \]

and if

\[ w = (w_1, w_2, ..., w_n) \]

be the weight vector of these IVIFNs, then the weighted score function and weighted accuracy function are respectively defined by,

\[ S'_w(\alpha_1, \alpha_2, ..., \alpha_n) = w_1 S'(\alpha_1) + w_2 S'(\alpha_2) + \]

\[ ... + w_n S'(\alpha_n) \]

......(1)

and

\[ H'_w(\alpha_1, \alpha_2, ..., \alpha_n) = w_1 H'(\alpha_1) + w_2 H'(\alpha_2) + \]

\[ ... + w_n H'(\alpha_n) \]

......(2)

Here, \( S'(\alpha_i) \) and \( H'(\alpha_i) \) are score functions and accuracy functions as in the following note.

5.1.1. Note. If \( \alpha = ([a, b], [c, d]) \) be an IVIFN (alternatives), then the score function \( S' \) and accuracy function \( H' \) defined on the set of alternatives, are respectively defined as

\[ S'(\alpha) = \frac{1}{2}(a - c + b - d) \]

......(3)
and

\[ H'(\alpha) = \frac{1}{2}(a + b + c + d) \] ............(4)

5.1.2. Note. Greater the value of \( S'(\alpha) \) and \( H'(\alpha) \), higher will be
the rank for \( \alpha \).

5.1.3. Note. The accuracy function \( H'(\alpha) \) given in equation(4)
was detected wrong, as mentioned in [40]. In the next section we
define an accuracy function and a decision function as follows.

5.2. Accuracy function and Decision function

Let \( A = ([a,b], [c,d]) \) be an interval valued intuitionistic fuzzy
number (alternative), then the new accuracy function \( S \) defined from
the set of alternatives to \([-1,1]\) denoted by \( S(A) \) is defined as

\[ S(A) = \frac{(a + b - cd)}{2} \] ............(5)

5.2.1. Definition. Let \( \alpha_1 = ([a_1, b_1], [c_1, d_1]) \), \( \alpha_2 = ([a_2, b_2], [c_2, d_2]) \), ..., 
\( \alpha_n = ([a_n, b_n], [c_n, d_n]) \) be n IVIFNs with weights \( w = (w_1, w_2, ..., w_n) \),
then the weighted accuracy function is defined as

\[ T_w(\alpha_1, \alpha_2, ..., \alpha_n) = w_1S(\alpha_1) + w_2S(\alpha_2) + ... + w_nS(\alpha_n) \] ......(6)

5.2.2. Proposition. For any interval valued intuitionistic fuzzy
number \( A = ([a,b],[c,d]) \), the value of the new proposed accuracy function \( S(A) \in [-1,1] \)

**5.2.3. Theorem.** For any two comparable IVIFS \( A \) and \( B \),

\[
\text{if } A \subset B, \text{ then } S(A) < S(B)
\]

**Proof.** Let \( A = ([a_1,b_1],[c_1,d_1]) \) and \( B = ([a_2,b_2],[c_2,d_2]) \) be two comparable IVIFS such that \( A \subset B \), i.e., if \( a_2 > a_1, b_2 > b_1, c_1 > c_2 \) and \( d_1 > d_2 \) then,

\[
S(B) - S(A) = \frac{(a_2-a_1)}{2} + \frac{(b_2-b_1)}{2} + \frac{(c_1-d_1 - c_2d_2)}{2} > 0
\]

**5.2.4. Definition.**

For alternatives \( A_1, A_2,\ldots,A_m \), based on attributes \( T_1, T_2,\ldots,T_n \), let us denote \([a_{ij}, b_{ij}]\) to represent the degree of the belonginess where the alternative \( A_i \) satisfy the attribute \( T_j \) and \([c_{ij}, d_{ij}]\) to represent the degree of the non belonginess where the alternative \( A_i \) does not satisfy the attribute \( T_j \).

Now we can represent the situation as in the following table.
The decision function $R$ on the set of alternatives denoted by $R(A_i)$ can be defined as follows

$$R(A_i) = S_w'(A_i) + \lambda T_w(A_i)$$

using equations (1),(3),(5) and (6) we can rewrite decision function as

$$R(A_i) = \sum_{j=1}^{n} w_j \left( \frac{a_{ij} + b_{ij} - c_{ij} - d_{ij}}{2} \right) + \lambda \sum_{j=1}^{n} w_j \left( \frac{a_{ij} + b_{ij} - c_{ij} d_{ij}}{2} \right) \ldots \ldots (7)$$

Where $\lambda = 0$ or 1 according as the decision maker is risk neutral or risk seeking. However, if the decision maker is risk averse the decision function is

$$R(A_i) = S_w'(A_i) - \lambda T_w(A_i)$$
i.e.,
\[ R(A_i) = \sum_{j=1}^{n} w_j \left( \frac{a_{ij} + b_{ij} - c_{ij} - d_{ij}}{2} \right) - \lambda \sum_{j=1}^{n} w_j \left( \frac{a_{ij} + b_{ij} - c_{ij}d_{ij}}{2} \right) \] .........(8)

Where \( \lambda = -1 \)

5.2.5. **Theorem.** If two IVIF alternatives A and B are such that \( A \subset B \) then \( R(A) < R(B) \)

**Proof:** Without loss of generality we can consider A and B as

\[ A = ([a_1, b_1], [c_1, d_1]) \text{ and } B = ([a_2, b_2], [c_2, d_2]) \]

as \( A \subset B \) we have \( a_1 < a_2, b_1 < b_2 \) also \( c_2 < c_1, d_2 < d_1 \)

Then,

\[ R(B) - R(A) = \frac{(a_2 + b_2 - c_2 - d_2)}{2} - \lambda \frac{(a_2 + b_2 - c_2d_2)}{2} - \frac{(a_1 + b_1 - c_1 - d_1)}{2} + \lambda \frac{(a_1 + b_1 - c_1d_1)}{2} \]

\> 0 \text{ when } \lambda = 0 \text{ or } 1 \text{ (using (7))}

Also,

\[ R(B) - R(A) = \frac{(a_2 + b_2 - c_2 - d_2)}{2} - \lambda \frac{(a_2 + b_2 - c_2d_2)}{2} - \frac{(a_1 + b_1 - c_1 - d_1)}{2} - \lambda \frac{(a_1 + b_1 - c_1d_1)}{2} \]

\> 0 \text{ when } \lambda = -1 \text{ ( Using (8))}

Now we can illustrate the application of the new decision function by an example.
5.3. Illustration.

A decision maker has to select one among the five candidates (say) $A_1, A_2, A_3, A_4, A_5$ for a teaching post, based on three criteria, $C_1$ (Basic Qualification), $C_2$ (Teaching Experience) and $C_3$ (Highly Qualified) with weight vector $w = (0.3, 0.4, 0.3)$. The interval valued intuitionistic fuzzy decision matrix is given below

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>[.35, .65], [.2, .3]</td>
<td>[.5, .6], [.3, .4]</td>
<td>[.4, .5], [.1, .3]</td>
</tr>
<tr>
<td>$A_2$</td>
<td>[.7, .8], [.1, .2]</td>
<td>[.6, .7], [.1, .3]</td>
<td>[.2, .3], [.4, .5]</td>
</tr>
<tr>
<td>$A_3$</td>
<td>[.2, .4], [.4, .5]</td>
<td>[.4, .5], [.1, .2]</td>
<td>[.3, .4], [.1, .2]</td>
</tr>
<tr>
<td>$A_4$</td>
<td>[.6, .7], [.2, .3]</td>
<td>[.6, .7], [.2, .3]</td>
<td>[.4, .8], [.1, .2]</td>
</tr>
<tr>
<td>$A_5$</td>
<td>[.3, .5], [.2, .4]</td>
<td>[.1, .3], [.6, .7]</td>
<td>[.3, .4], [.5, .6]</td>
</tr>
</tbody>
</table>

Suppose the decision maker is risk neutral, then take $\lambda = 0$.

Then by the decision function in equation (1), we get

$R(A_1) = 0.23$,

$R(A_2) = 0.3$,

$R(A_3) = 0.135$,  

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\[ R(A_4) = 0.415, \]
\[ R(A_5) = -0.21. \]

The rank is

\[ R(A_5) < R(A_3) < R(A_1) < R(A_2) < R(A_4). \]

Which gives

\[ A_5 \prec A_3 \prec A_1 \prec A_2 \prec A_4. \]

The best alternative is thus \( A_4 \).

If the decision maker is risk seeking, then take \( \lambda = 1 \), then

\[ R(A_1) = 0.698, \]
\[ R(A_2) = 0.821, \]
\[ R(A_3) = 0.473, \]
\[ R(A_4) = 1.026, \]
\[ R(A_5) = -0.046. \]

Here also the rank is

\[ R(A_5) < R(A_3) < R(A_1) < R(A_2) < R(A_4). \]

Which gives

\[ A_5 \prec A_3 \prec A_1 \prec A_2 \prec A_4. \]
Thus the best alternative is again $A_4$.

If the decision maker is risk averse, by taking $\lambda = -1$, the decision function in equation (2) gives the same ranking as when $\lambda = 1$. Therefore, in all the cases the rank for the alternatives is

$$R(A_5) < R(A_3) < R(A_1) < R(A_2) < R(A_4)$$

5.3.1. Note.

In this chapter we introduced a multiple criteria decision making algorithm to rank the alternatives, and is based on the weighted score function, weighted accuracy function and a decision function which reflects the decision maker’s attitude. Also, we propose another accuracy function and a decision function to rank the alternatives. The advantage of this new decision function is that it not only count the attitude of the decision maker but also it avoid a lengthy aggregation operator.

In chapter 2 we have discussed, some special IFS developed from $(\alpha, \beta)$ cuts and derived theorems to decompose IFSs. Now we are going to derive some theorems for the set of $(\alpha, \beta)$ level generated by an IVIFS.