CHAPTER - 2

\(\tau_1 \tau_2^*\) CLOSED SETS

In this chapter, we introduce \(\tau_1 \tau_2^*\) closed sets, \(\tau_1 \tau_2^*\) derived sets and star continuous map.

2.1 \(\tau_1 \tau_2^*\) closed sets

Definition: 2.1.1

Let \((X, \tau_1, \tau_2)\) be bigeneralized topological space. A subset \(A\) of \(X\) is said to be \(\tau_1 \tau_2^*\) closed if

\[
\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A = A
\]

A is said to be \(\tau_1 \tau_2^*\) open if its complement is \(\tau_1 \tau_2^*\) closed.

Example: 2.1.2

Let \(X = \mathbb{R}\). Let \(\tau_1 = \{(-\infty, a) / a \in \mathbb{R}\} \cup \emptyset \cup X\) and \(\tau_2 = \{(b, \infty) / b \in \mathbb{R}\} \cup \emptyset \cup X\). Here, \((X, \tau_1, \tau_2)\) is pairwise \(T_2\) space. Also every finite closed interval is \(\tau_1 \tau_2^*\) closed set but neither \(\tau_1\) closed set nor \(\tau_2\) closed set.

Example: 2.1.3

Let \(X = \mathbb{R}\). Let \(\tau_1\) be Discrete topology and \(\tau_2\) be Indiscrete topology. Then every subset of \(X\) is \(\tau_1 \tau_2^*\) closed sets.
Example: 2.1.4

Let X be an uncountable set.

Let $\tau_1$ be co-finite topology and $\tau_2$ be co-countable topology. Clearly $\tau_1 \subseteq \tau_2$. Every countable set is $\tau_1 \tau_2^*$ closed sets.

The $\tau_1 \tau_2^*$ closed sets can be defined in bitopological space also in a similar way. But a bitopological space is a particular case of bigeneralized topological space. Therefore we prove theorems in general case.

Theorem: 2.1.5

The concept of $\tau_1 \tau_2^*$ closed sets in bigeneralized topological space $(X, \tau_1, \tau_2)$ is an extension of the concept of closed sets in a generalized topological space.

Proof:

Let $\tau_1 = \tau_2$ and $A \subseteq X$.

Let A be $\tau_1 \tau_2^*$ closed set. Then $\tau_1 \text{cl} A \cap \tau_2 \text{cl} A = A$. This implies, $\tau_1 \text{cl} A \cap \tau_1 \text{cl} A = A$. Hence $\tau_1 \text{cl} A = A$. Therefore, A is closed. Conversely, let A is closed in generalized topology. Then $\tau_1 \text{cl} A = A$. This implies, $\tau_1 \text{cl} A \cap \tau_1 \text{cl} A = A$. Hence, $\tau_1 \text{cl} A \cap \tau_2 \text{cl} A = A$. Therefore, A is $\tau_1 \tau_2^*$ closed set.

Definition: 2.1.6

The smallest $\tau_1 \tau_2^*$ closed set containing A is called the $\tau_1 \tau_2^*$ closure of A and is denoted by $\tau_1 \tau_2^* \text{Cl} (A)$. 
**Theorem: 2.1.7**

Let \((X, \tau_1, \tau_2)\) be a bigeneralized topological space and let \(A\) be a subset of \(X\). Then
\[
\tau_1 \tau_2^* \text{Cl} (A) = \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} (A).
\]

**Proof:**

Since \(\tau_1 \text{Cl} A\) and \(\tau_2 \text{Cl} A\) are the \(\tau_1 \tau_2^*\) closed set containing \(A\). Therefore, \(\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} (A)\) is the \(\tau_1 \tau_2^*\) closed set containing \(A\). Suppose \(B\) is any other \(\tau_1 \tau_2^*\) closed set containing \(A\). Then \(\tau_1 \text{Cl} B \cap \tau_2 \text{Cl} B = B\) and \(A \subset B\). This implies, \(\tau_1 \text{Cl} A \subset \tau_1 \text{Cl} B\) and \(\tau_2 \text{Cl} A \subset \tau_2 \text{Cl} B\).

This implies, \(\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} (A) \subset \tau_1 \text{Cl} B \cap \tau_2 \text{Cl} B\). Hence, \(\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A\) is the smallest \(\tau_1 \tau_2^*\) closed set containing \(A\). Hence \(\tau_1 \tau_2^* \text{Cl} (A) = \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} (A)\).

**Theorem: 2.1.8**

Intersection of two \(\tau_1 \tau_2^*\) closed set is \(\tau_1 \tau_2^*\) closed.

**Proof:**

Let \(A\) and \(B\) are \(\tau_1 \tau_2^*\) closed sets.

Then \(\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A = A\) and \(\tau_1 \text{Cl} B \cap \tau_2 \text{Cl} B = B\)

Now, 
\[
\tau_1 \text{Cl} (A \cap B) \cap \tau_2 \text{Cl} (A \cap B) \subset (\tau_1 \text{Cl} A \cap \tau_1 \text{Cl} B) \cap (\tau_2 \text{Cl} A \cap \tau_2 \text{Cl} B) = (\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A) \cap (\tau_1 \text{Cl} B \cap \tau_2 \text{Cl} B) = A \cap B
\]

Also, \(A \cap B \subset \tau_1 \text{Cl} (A \cap B) \cap \tau_2 \text{Cl} (A \cap B)\)

Therefore, \(A \cap B\) is \(\tau_1 \tau_2^*\) closed set.
Remark: 2.1.9

In general arbitrary intersection of \( \tau_1 \tau_2^* \) closed set is \( \tau_1 \tau_2^* \) closed.

Remark: 2.1.10

Union of two \( \tau_1 \tau_2^* \) closed set is not \( \tau_1 \tau_2^* \) closed.

For example,

Let \( X = \{a, b, c, d\} \)

\( \tau_1 = \{\phi, X, \{a, b\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\} \)

\( \tau_2 = \{\phi, X, \{a, b, c\}, \{b, c, d\}\} \)

The \( \tau_1 \) closed sets are \( \{\phi, X, \{c, d\}, \{a, b\}, \{b, d\}, \{d\}, \{b\}\} \)

The \( \tau_2 \) closed sets are \( \{\phi, X, \{d\}, \{a\}\} \)

\( \{a, b\} \) and \( \{b, d\} \) are \( \tau_1 \tau_2^* \) closed sets but \( \{a, b, d\} \) is not \( \tau_1 \tau_2^* \) closed.

Theorem: 2.1.11

Arbitrary union of \( \tau_1 \tau_2^* \) open set is \( \tau_1 \tau_2^* \) open.

Proof:

Let \( A = \bigcup_i \{A_i/A_i \text{ is } \tau_1 \tau_2^* \text{ open}\} \) Since each \( A_i^c \) is \( \tau_1 \tau_2^* \) closed, \( \bigcap_i A_i^c \) is \( \tau_1 \tau_2^* \) closed.

\[ \Rightarrow \left( \bigcup_i A_i \right)^c \text{ is } \tau_1 \tau_2^* \text{ closed.} \]

\[ \Rightarrow \bigcup_i A_i \text{ is } \tau_1 \tau_2^* \text{ open.} \]

Therefore, \( A \) is \( \tau_1 \tau_2^* \) open.
Remark: 2.1.12

Intersection of two $\tau_1 \tau_2$ * open set is not $\tau_1 \tau_2$ * open

For example, let $X= \{a, b, c, d\}$

$\tau_1 = \{\emptyset, X, \{a, b\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$

$\tau_2 = \{\emptyset, X, \{a, b, c\}, \{b, c, d\}\}$

The $\tau_1$ closed sets are $\{\emptyset, X, \{c, d\}, \{a, b\}, \{b, d\}, \{d\}, \{b\}\}$

The $\tau_2$ closed sets are $\{\emptyset, X, \{d\}, \{a\}\}$

$\{c, d\}$ and $\{a, c\}$ are $\tau_1 \tau_2$ * open sets but $\{c\}$ is not $\tau_1 \tau_2$ * open set.

Remark: 2.1.13

The collection of $\tau_1 \tau_2$ * open sets are formed generalized topological space.

Properties: 2.1.14

Let $(X, \tau_1 \tau_2)$ be a bigeneralized topological space and $A$ is a subset of $X$.

i) If $A$ is $\tau_1$ closed then $A$ is $\tau_1 \tau_2$ * closed

ii) If $A$ is $\tau_2$ closed then $A$ is $\tau_1 \tau_2$ * closed

iii) If $A$ is $\tau_1$ open then $A$ is $\tau_1 \tau_2$ * open

iv) If $A$ is $\tau_2$ open then $A$ is $\tau_1 \tau_2$ * open

Proof:

i) $A$ is $\tau_1$ closed \(\Rightarrow\) $A = \tau_1 \text{cl} \ A$

\[\Rightarrow A \cap \tau_2 \text{cl} \ A = \tau_1 A \cap \tau_2 \text{cl} \ A\]

\[\Rightarrow A = \tau_1 \text{cl} \ A \cap \tau_2 \text{cl} \ A\]

\[\Rightarrow A \text{ is } \tau_1 \tau_2 \text{ * closed}\]
Similarly ii) is true

iii) $A$ is $\tau_1$ open $\Rightarrow A^c$ is $\tau_1$ closed

$\Rightarrow A^c$ is $\tau_1 \tau_2^*$ closed

$\Rightarrow A$ is $\tau_1 \tau_2^*$ open

Similarly iv) is true

**Theorem: 2.1.15**

Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space and $A$ be a subset of $X$. Then

i) If $A$ is $\tau_1 \tau_2$ regular closed then $A$ is $\tau_1 \tau_2^*$ closed.

ii) If $A$ is $\tau_1 \tau_2$ closed then $A$ is $\tau_1 \tau_2^*$ closed.

**Proof:**

i) Since $A$ is $\tau_1 \tau_2$ regular closed, $\tau_1 \text{Cl}(\tau_2 \text{int}(A)) = A$. Therefore $A$ is $\tau_1$ closed.

Hence $A$ is $\tau_1 \tau_2^*$ closed.

ii) Since $A$ is $\tau_1 \tau_2$ closed, $\tau_1 \text{Cl}(\tau_2 \text{Cl}(A)) = A$. Therefore $A$ is $\tau_1$ closed. Hence $A$ is $\tau_1 \tau_2^*$ closed.

**Remark: 2.1.16**

The converse of the theorem is not true.

A bigeneralized topological space $(X, \tau_1, \tau_2)$ has $\tau_1 \tau_2^*$ closed set $A$, but $A$ is neither $\tau_1 \tau_2$ regular closed set nor $\tau_1 \tau_2$ closed set.
For example,
Let $X = \{a, b, c, d\}$ and

$\tau_1 = \{\emptyset, X, \{d\}, \{a, b\}, \{a, b, d\}\}$

$\tau_2 = \{\emptyset, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$

$\tau_1$ closed sets = $\{\emptyset, X, \{a, b, c\}, \{a, c\}, \{c, d\}, \{c\}\}$

$\tau_2$ closed sets = $\{\emptyset, X, \{b, c, d\}, \{a, b\}, \{b\}, \{c\}, \{a\}, \{a, c\}\}$

Now $\{a, b\}$ is a $\tau_1 \tau_2^*$ closed set. But $\{a, b\}$ is neither $\tau_1 \tau_2$ closed set nor $\tau_1 \tau_2$ regular closed set.

**Theorem: 2.1.17**

Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space and $A$ be any subset of $X$. $A$ is $\tau_1 \tau_2^*$ closed set if and only if for any $a \notin A$ if every $\tau_1$ neighbourhood of $a$ intersect $A$ then there exist a $\tau_2$ neighbourhood of $a$ is not intersecting $A$.

**Proof:**

Let for any $a \notin A$, every $\tau_1$ neighbourhood of $a$ intersect $A$. Then $a \in \tau_1 \text{cl } A$

Since $A$ is $\tau_1 \tau_2^*$ closed, $\tau_1 \text{cl } A \cap \tau_2 \text{ cl } A = A$. Also $a \notin A$, $a \notin \tau_2 \text{ cl } A$. Therefore, there exist a $\tau_2$ neighbourhood of $a$ not intersecting $A$.

Conversely,

To Prove $\tau_1 \text{cl } A \cap \tau_2 \text{ cl } A = A$. Clearly $A \subset \tau_1 \text{cl } A \cap \tau_2 \text{ cl } A$

Suppose $\tau_1 \text{cl } A \cap \tau_2 \text{ cl } A \not\subset A$. Then there exist $a \in \tau_1 \text{cl } A \cap \tau_2 \text{ cl } A$ and $a \notin A$. That implies every $\tau_1$ neighbourhood of $a$ intersect $A$ and every $\tau_2$ neighbourhood of $a$ intersect $A$ and $a \notin A$. Which is contradiction to our hypothesis. Therefore, $\tau_1 \text{ cl } A \cap \tau_2 \text{ cl } A \subset A$

Hence $A$ is $\tau_1 \tau_2^*$ closed set.
**Theorem: 2.1.18**

Let \((X, \tau_1, \tau_2)\) be a bigeneralized topological space. \(A\) is \(\tau_1 \tau_2^*\) closed if and only if there exist a \(\tau_1\) closed set \(B\) and a \(\tau_2\) closed set \(C\) such that \(A = B \cap C\).

**Proof:**

Let \(A\) be \(\tau_1 \tau_2^*\) closed set. Then \(A = \tau_1 \text{cl} A \cap \tau_2 \text{cl} A\)

Take \(B = \tau_1 \text{cl} A\) and \(C = \tau_2 \text{cl} A\). Then \(A = B \cap C\) where \(B\) is \(\tau_1\) closed set and \(C\) is \(\tau_2\) closed set.

Conversely, let there exist a \(\tau_1\) closed set \(B\) and a \(\tau_2\) closed set \(C\) such that \(A = B \cap C\).

To prove \(A\) is \(\tau_1 \tau_2^*\) closed. Since \(B\) is \(\tau_1\) closed, \(B = \tau_1 \text{cl} B\). Since \(C\) is \(\tau_2\) closed, \(C = \tau_2 \text{cl} C\). Clearly, \(\tau_1 \text{Cl} (B \cap C) \subset \tau_1 \text{Cl} B = B\) and \(\tau_2 \text{Cl} (B \cap C) \subset \tau_2 \text{Cl} C = C\)

Therefore, \(\tau_1 \text{Cl} (B \cap C) \cap \tau_2 \text{Cl} (B \cap C) \subset B \cap C\)

Always, \(B \cap C \subset \tau_1 \text{Cl} (B \cap C) \cap \tau_2 \text{Cl} (B \cap C)\)

Hence \(A = B \cap C\) is \(\tau_1 \tau_2^*\) closed.

**2.2 \(\tau_1 \tau_2^*\) derived set**

**Definition: 2.2.1**

Let \((X, \tau_1, \tau_2)\) be a bigeneralized topological space and \(A \subseteq X\). The point \(x \in X\) is said to be \(\tau_1 \tau_2^*\) limit point of \(A\) if \(U \cap (A - \{x\}) \neq \emptyset\) for every \(\tau_1 \tau_2^*\) open set \(U\) containing \(x\).

i.e.) The point \(x \in X\) is said to be \(\tau_1 \tau_2^*\) limit point of \(A\) if every \(\tau_1 \tau_2^*\) open set \(U\) containing \(x\) intersect \(A\) different from \(x\).
Definition: 2.2.2

The set of all $\tau_1 \tau_2^*$ limit point of $A$ is called $\tau_1 \tau_2^*$ derived set and is denoted by $D^*(A)$.

Theorem: 2.2.3

Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space and $A,B \subseteq X$. Then

i) $A \subset B \Rightarrow D^*(A) \subset D^*(B)$

ii) $D^*(A \cup B) \supset D^*(A) \cup D^*(B)$

iii) $D^*(A) \cap D^*(B) = D^*(A \cap B)$

Proof:

i) Let $A \subset B$

Let $x \in D^*(A)$

Then every $\tau_1 \tau_2^*$ open set $U$ containing $x$ intersect $A$.

Since $A \subset B$, every $\tau_1 \tau_2^*$ open set $U$ containing $x$ intersect $B$. That implies $x \in D^*(A)$. Hence $D^*(A) \subset D^*(B)$.

ii) Clearly $A \subset A \cup B \Rightarrow D^*(A) \subset D^*(A \cup B)$ and $B \subset A \cup B \Rightarrow D^*(B) \subset D^*(A \cup B)$

Hence $D^*(A) \cup D^*(B) \subset D^*(A \cup B)$

iii) Since $A \cap B \subset A$ and $A \cap B \subset B$, $D^*(A \cap B) \subset D^*(A)$ and $D^*(A \cap B) \subset D^*(B)$

That implies, $D^*(A \cap B) \subset D^*(A) \cap D^*(B)$.

Now, let $x \in D^*(A) \cap D^*(B)$.

Then every $\tau_1 \tau_2^*$ open set $U$ containing $x$ intersect $A$ and every $\tau_1 \tau_2^*$ open set $U$ containing $x$ intersect $B$. That implies, every $\tau_1 \tau_2^*$ open set $U$ containing $x$ intersect $A \cap B$.

Therefore, $x \in D^*(A \cap B)$

Hence $D^*(A \cap B) = D^*(A) \cap D^*(B)$. 

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**Theorem: 2.2.4**

Let \((X, \tau_1, \tau_2)\) be a bigeneralized topological space and \(A \subseteq X\).

Then \(\tau_1 \text{Cl} \ A \cap \tau_2 \text{Cl} \ A = A \cup D^*(A)\).

**Proof:**

Let \(x \in \tau_1 \text{Cl} \ A \cap \tau_2 \text{Cl} \ A\)

To prove \(x \in A \cup D^*(A)\)

If \(x \in A\) then \(x \in A \cup D^*(A)\).

Suppose \(x \not\in A\), we have to prove \(x \in D^*(A)\).

Suppose \(x \not\in D^*(A)\), then there exist an \(\tau_1 \tau_2^*\) open set \(U\) containing \(x\) such that

\[ U \cap (A - \{x\}) = U \cap A = \emptyset. \]

This implies, \(A \subseteq U^c\) and \(U^c\) is \(\tau_1 \tau_2^*\) closed

This implies, \(\tau_1 \text{Cl} \ A \cap \tau_2 \text{Cl} \ A \subseteq \tau_1 \text{Cl} U^c \cap \tau_2 \text{Cl} U^c\). This implies, \(\tau_1 \text{Cl} \ A \cap \tau_2 \text{Cl} A \subseteq U^c\).

But \(x \in \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A\) and \(x \not\in U^c\).

Which is contradiction. Hence \(x \in D^*(A)\).

Therefore, \(x \in A \cup D^*(A)\). Hence, \(\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A \subseteq A \cup D^*(A)\)

Conversely, let \(x \in A \cup D^*(A)\)

If \(x \in A\) then \(x \in \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A\)

If \(x \in D^*(A)\), then every \(\tau_1 \tau_2^*\) open set containing \(x\) intersect \(A\).

Let \(U\) be the \(\tau_1\) open set containing \(x\). Then \(U\) be \(\tau_1 \tau_2^*\) open set containing \(x\). By hypothesis, \(U\) intersect \(A\). Therefore, \(x \in \tau_1 \text{Cl} A\). Similarly \(x \in \tau_2 \text{Cl} A\).

Hence \(x \in \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A\). This implies \(A \cup D^*(A) \subseteq \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A\)

Hence \(A \cup D^*(A) = \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A\)
**Theorem: 2.2.5**

Let \((X, \tau_1, \tau_2)\) be a bigeneralized topological space and \(A \subseteq X\). If \(A\) is \(\tau_1\ tau_2^*\) closed then \(D^*(A) \subseteq A\).

**Proof:**

We have
\[
A \cup D^*(A) = \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A
\]
Since \(A\) is \(\tau_1\ tau_2^*\) closed, \(D^*(A) \cup A = A\).
Hence \(D^*(A) \subseteq A\).

2.3. Star Continuous map

**Definition: 2.3.1**

Let \((X, \tau_1, \tau_2)\) and \((Y, \mu_1, \mu_2)\) be bigeneralized topological space. A function \(f: (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)\) is said to be star continuous if inverse image of \(\mu_1\ mu_2^*\) closed set is \(\tau_1\ tau_2^*\) closed.

**Theorem: 2.3.2**

Let \((X, \tau_1, \tau_2)\) and \((Y, \mu_1, \mu_2)\) be two bigeneralized topological space. \(f: X \to Y\) is star continuous if and only if inverse image of \(\mu_1\ mu_2^*\) open set is \(\tau_1\ tau_2^*\) open.

**Proof:**

Let \(f: (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)\) be star continuous and \(A \subset Y\) be \(\mu_1\mu_2^*\) open set. That implies, \(A^c\) be \(\mu_1\mu_2^*\) closed set. Since \(f\) is star continuous, \((f^1(A^c))^c\) is \(\tau_1\ tau_2^*\) closed.

This implies, \((f^1(A))^c\) is \(\tau_1\ tau_2^*\) closed

Hence, \(f^1(A)\) is \(\tau_1\ tau_2^*\) open.
Conversely, let $A \subset Y$ be $\mu_1 \mu_2^*$ closed set.

Then $A^c$ is $\mu_1 \mu_2^*$ open.

By hypothesis, $f^{-1}(A^c)$ is $\tau_1 \tau_2^*$ open.

This implies, $(f^{-1}(A))^c$ is $\tau_1 \tau_2^*$ open.

Hence, $f^{-1}(A)$ is $\tau_1 \tau_2^*$ closed.

By definition of star continuous, $f$ is star continuous.

**Theorem: 2.3.3**

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is star continuous if and only if

$f(\tau_1 \tau_2^* \text{Cl}(A)) \subset \mu_1 \mu_2^* \text{Cl}f(A)$ where $A \subset X$.

**Proof:**

Let $A \subset X$. Then $f(A) \subset Y$. Since $\mu_1 \mu_2^* \text{Cl}f(A)$ is $\mu_1 \mu_2^*$ closed set in $Y$. Then $Y - \mu_1 \mu_2^* \text{Cl}f(A)$ is $\tau_1 \tau_2^*$ open set in $Y$. Since $f$ is star continuous, $f^{-1}(Y - \mu_1 \mu_2^* \text{Cl}f(A))$ is $\tau_1 \tau_2^*$ open in $X$. This implies, $f^{-1}(\mu_1 \mu_2^* \text{Cl}f(A))$ is $\tau_1 \tau_2^*$ closed set in $X$ containing $A$.

Clearly $\tau_1 \tau_2^* \text{Cl}(A) \subset f^{-1}(\mu_1 \mu_2^* \text{Cl}f(A))$. This implies, $f(\tau_1 \tau_2^* \text{Cl}(A)) \subset \mu_1 \mu_2^* \text{Cl}f(A)$.

Conversely, let $B$ be $\mu_1 \mu_2^*$ closed set of $Y$. Then $f^{-1}(B) \subset X$. By hypothesis, $f(\tau_1 \tau_2^* \text{Cl}f^{-1}(B)) \subset (\mu_1 \mu_2^* \text{Cl}f(f^{-1}(B)))$. Hence, $f(\tau_1 \tau_2^* \text{Cl}f^{-1}(B)) \subset \mu_1 \mu_2^* \text{Cl}(B)$. Since $B$ is $\mu_1 \mu_2^*$ closed, $f(\tau_1 \tau_2^* \text{Cl}f^{-1}(B)) \subset B$. This implies, $\tau_1 \tau_2^* \text{Cl}(f^{-1}(B)) \subset f^{-1}(B)$. Also $f^{-1}(B) \subset \tau_1 \tau_2^* \text{Cl}(f^{-1}(B))$. Therefore, $f^{-1}(B)$ is $\tau_1 \tau_2^*$ closed set. Hence $f$ is star continuous.
Definition: 2.3.4

Let \((X, \tau_1, \tau_2)\) and \((Y, \mu_1, \mu_2)\) be two bigeneralized topological space. A function \(f: X \to Y\) is star open map if image of \(\tau_1 \tau_2^*\) open set is \(\mu_1 \mu_2^*\) open.

Theorem: 2.3.5

If \(f: (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)\) is star open map then
\[
f^{-1}(\mu_1 \mu_2^* \text{Cl}(B)) \subseteq \tau_1 \tau_2^* \text{Cl}(f^{-1}(B))\]
for every \(B \subset Y\).

Proof:

Let \(B \subset Y\). Then \(f^{-1}(B) \subset X\). Since \(\tau_1 \tau_2^* \text{Cl}(f^{-1}(B))\) is \(\tau_1 \tau_2^*\) closed set in \(X\). This implies, \(X - \tau_1 \tau_2^* \text{Cl}(f^{-1}(B))\) is \(\tau_1 \tau_2^*\) open set in \(X\). Since \(f\) is star open map, \(f(X - \tau_1 \tau_2^* \text{Cl}(f^{-1}(B)))\) is \(\mu_1 \mu_2^*\) open set in \(Y\). This implies, \(f(\tau_1 \tau_2^* \text{Cl} f^{-1}(B))\) is \(\mu_1 \mu_2^*\) closed set in \(Y\) containing \(B\). Clearly \(\mu_1 \mu_2^* \text{Cl}(B) \subset f(\tau_1 \tau_2^* \text{Cl} f^{-1}(B))\). This implies, \(f^{-1}(\mu_1 \mu_2^* \text{Cl}(B)) \subset \tau_1 \tau_2^* \text{Cl} f^{-1}(B)\).