CHAPTER - 7

τ₁τ₂ – REGULAR GENERALIZED STAR STAR CLOSED SETS

In this chapter, we introduce τ₁ τ₂ – regular generalized star star closed sets and we study some properties of τ₁ τ₂ – rg** closed set. Also, we introduce τ₁ τ₂ – rg** open set and we obtain the necessary and sufficient condition for a set to be a τ₁ τ₂ – rg** open set.

7.1 τ₁τ₂ – Regular generalized star star closed sets

Definition: 7.1.1

A subset A of a Bigeneralized topological space (X, τ₁, τ₂) is called

(a) τ₁τ₂ –regular closed if τ₁ –c1 [τ₂-int(A)] = A.

(b) τ₁τ₂ – regular open if τ₁ – int [τ₂-cl(A)] = A.

(c) τ₁τ₂ – regular generalized closed (τ₁ τ₂ – rg closed) in X if τ₂ – cl (A) ⊆ U whenever A ⊆ U and U is τ₁ τ₂ – regular open in X.

(d) τ₁τ₂ –regular generalized open (τ₁τ₂-rg open) in X if F ⊆ τ₂ – int(A) whenever F ⊆ A and F is τ₁τ₂ – regular closed in X.

(e) τ₁ τ₂ – regular generalized star closed (τ₁ τ₂ – rg* closed) in X if

τ₂ – rcl(A) ⊆ U whenever A ⊆ U and U is τ₁ τ₂ – regular open in X.

(f) τ₁ τ₂ – regular generalized star open (τ₁ τ₂ – rg* open) in X if its complement is τ₁ τ₂ – regular generalized star closed (τ₁τ₂ – rg* closed) in X.
Definition: 7.1.2

A subset A of a bigeneralized topological space \((X, \tau_1, \tau_2)\) is called \(\tau_1 \tau_2\) \(-\) regular generalized star star closed \((\tau_1 \tau_2 \text{ rg** closed})\) in \(X\) if \(\tau_2\text{-cl}[\tau_1\text{-int}(A)] \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1 \tau_2\)-regular open in \(X\).

Example: 7.1.3

Let \(X = \{a, b, c, d\}\)
\[\tau_1 = \{\phi, X, \{a,b\}, \{b,d\}, \{a,b,d\}\}\]
\[\tau_2 = \{\phi, X, \{a\}, \{d\}, \{b,d\}, \{a,d\}, \{a,b,d\}\}\].
Clearly \(U = \{b, d\}\) is \(\tau_1 \tau_2\) regular open in \(X\).
\{d\} is \(\tau_1 \tau_2\)-rg** closed set and \{b, d\} is not \(\tau_1 \tau_2\)-rg** closed set.

Result: 7.1.4

The Union of \(\tau_1 \tau_2\)-rg** closed set is not \(\tau_1 \tau_2\)-rg** closed.

For example,
Let \(X = \{a,b,c,d\}\)
\[\tau_1 = \{\phi, X, \{a,b\}, \{b,d\}, \{a,b,d\}\}\]
\[\tau_2 = \{\phi, X, \{a\}, \{d\}, \{b,d\}, \{a,d\}, \{a,b,d\}\}\].
Clearly \(U = \{b, d\}\) is \(\tau_1 \tau_2\) regular open in \(X\).
\{b\} and \{d\} are \(\tau_1 \tau_2\)-rg** closed sets but \{b,d\} is not \(\tau_1 \tau_2\)-rg** closed.

Theorem: 7.1.5

Let A be a subset of a bigeneralized topological space \((X, \tau_1, \tau_2)\). If A is \(\tau_1 \tau_2\)-rg** closed then \(\tau_2\text{-cl}[\tau_1\text{-int}(A)] - A\) does not contain a non empty \(\tau_1 \tau_2\)-regular closed set.
Proof:

Suppose that $A$ is $\tau_1 \tau_2$-rg** closed. Let $F$ be a $\tau_1 \tau_2$-regular closed set such that $F \subseteq \tau_2\text{-}cl[\tau_1\text{-}int(A)] - A$. Then $F \subseteq \tau_2\text{-}cl[\tau_1\text{-}int(A)] \cap A^C$. Since $F \subseteq A^C$, we have $A \subseteq F^C$.

Since $F$ is $\tau_1 \tau_2$ – regular closed set, $F^C$ is $\tau_1 \tau_2$-regular open. Since $A$ is $\tau_1 \tau_2$-rg ** closed, we have $\tau_2\text{-}cl[(\tau_1\text{-}int(A))] \subseteq F^C$. Therefore, $F \subseteq [\tau_2\text{-}cl[\tau_1\text{-}int(A)]]^C$. Also $F \subseteq \tau_2\text{-}cl[\tau_1\text{-}int(A)]$.

Hence $F \subseteq \phi$. Therefore $F = \phi$.

Theorem: 7.1.6

If $A$ is $\tau_1 \tau_2$-rg **closed and $B$ is $\tau_1 \tau_2$-rg closed, then $A \cup B$ is $\tau_1 \tau_2$-rg** closed.

Proof:

Let $A \cup B \subseteq U$ and $U$ is $\tau_1 \tau_2$ – regular open in $X$. Since $A \subseteq U$ and $A$ is $\tau_1 \tau_2$-rg** closed, we have $\tau_2\text{-}cl[\tau_1\text{-}int(A)] \subseteq U$. Since $B \subseteq U$ and $B$ is $\tau_1 \tau_2$-rg closed, we have $\tau_2\text{-}cl(B) \subseteq U$.

Now $\tau_2\text{-}cl[\tau_1\text{-}int(A \cup B)] \subseteq \tau_2\text{-}cl[\tau_1\text{-}int[A \cup \tau_2\text{-}cl(B)]] \subseteq \tau_2\text{cl}_{\tau_1}\text{int }U \subseteq U$. Therefore, $A \cup B$ is $\tau_1 \tau_2$-rg** closed.

Theorem: 7.1.7

If a subset $A$ is $\tau_1 \tau_2$-rg closed then $A$ is $\tau_1 \tau_2$-rg** closed.

Proof:

Let $A \subseteq U$ and $U$ is $\tau_1 \tau_2$-regular open. Since $A$ is $\tau_1 \tau_2$-rg closed, we have $\tau_2\text{-}cl(A) \subseteq U$.

Hence $\tau_2\text{-}cl\left[\tau_1\text{-}int(A)\right] \subseteq U$. Therefore $A$ is $\tau_1 \tau_2$-rg** closed.

The converse of the above theorem is not true as seen from the following example.
Example: 7.1.8

Let $X = \{a,b,c,d\}$ and $\tau_1 = \{\phi, X, \{a,b\}, \{b,c\}, \{a,b,c\}\}$.

$\tau_2 = \{\phi, X, \{c,d\}, \{a\}, \{a,c,d\}\}$. Clearly $\{b, c\}$ is $\tau_1 \tau_2$ regular open in $X$. Then $\{c\}$ is $\tau_1 \tau_2$ -rg** closed set but not $\tau_1 \tau_2$ –rg closed.

Theorem: 7.1.9

Let $A$ and $B$ be subsets of $X$ such that $A \subseteq B \subseteq \tau_2$-$\text{cl}[^{\tau_1}\text{-int}(A)]$. If $A$ is $\tau_1 \tau_2$-rg** closed, then $B$ is $\tau_1 \tau_2$ –rg** closed.

Proof:

Let $B \subseteq U$ and $U$ is $\tau_1 \tau_2$-regular open in $X$. Since $A \subseteq B$, we have $A \subseteq U$. Since $A$ is $\tau_1 \tau_2$-rg** closed, we have $\tau_2$-$\text{cl}[^{\tau_1}\text{-int}(A)] \subseteq U$. Since $B \subseteq \tau_2$-$\text{cl}[^{\tau_1}\text{-int}(A)]$, we have $\tau_2$-$\text{cl}[^{\tau_1}\text{-int}(B)] \subseteq \tau_2$-$\text{cl}(B) \subseteq \tau_2$-$\text{cl}[^{\tau_1}\text{-int}(A)] \subseteq U$. Therefore, $B$ is $\tau_1 \tau_2$-rg** closed.

Theorem: 7.1.10

Suppose that $\tau_1 \tau_2$-$\text{R.O}(X, \tau_1, \tau_2) \subseteq \tau_2$-$\text{C}(X, \tau_1, \tau_2)$. Then every subset of $X$ is $\tau_1 \tau_2$-rg**closed.

Proof:

Let $A$ be a subset of $X$. Let $A \subseteq U$ and $U$ is $\tau_1 \tau_2$-regular open in $X$. Since $\tau_1 \tau_2$ –R.O $(X, \tau_1, \tau_2) \subseteq \tau_2$-$\text{C}(X, \tau_1, \tau_2)$ we have $U$ is $\tau_2$-closed in $X$. Since $A \subseteq U$, we have $\tau_2$-$\text{cl}(A) \subseteq \tau_2$-$\text{cl}(U) = U$. Therefore, $\tau_2$-$\text{cl}[^{\tau_1}\text{-int}(A)] \subseteq \tau_2$-$\text{cl}[A] \subseteq U$. Hence $A$ is $\tau_1 \tau_2$ –rg** closed.
7.2 \(\tau_1\tau_2\)-REGULAR GENERALIZED STAR STAR OPEN SETS

Definition: 7.2.1

A subset \(A\) of a bigeneralized topological space \((X,\tau_1,\tau_2)\) is called \(\tau_1\tau_2\)-regular generalized star star open (\(\tau_1\tau_2\text{-rg}^{**}\) open) in \(X\) if its complement is \(\tau_1\tau_2\)-regular generalized star star closed (\(\tau_1\tau_2\text{-rg}^{**}\) closed) in \(X\).

Example: 7.2.2

Let \(X = \{a,b,c,d\}\), \(\tau_1 = \{\phi, X, \{a,c\}, \{c,d\}, \{a,c,d\}\}\) \(\tau_2 = \{\phi, X, \{a\}, \{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}\)

\(\{a,b,c\}\) and \(\{a,b,d\}\) are \(\tau_1\tau_2\text{rg}^{**}\) open sets and \(\{a,b\}\) is not \(\tau_1\tau_2\text{rg}^{**}\) open.

A necessary and sufficient condition for a set \(A\) to be a \(\tau_1\tau_2\text{-rg}^{**}\) open set is obtained in the next theorem.

Theorem: 7.2.3

A subset \(A\) of a bigeneralized topological space \((X, \tau_1, \tau_2)\) is \(\tau_1\tau_2\text{-rg}^{**}\) open if and only if

\(F \subseteq \tau_2\text{-int}[\tau_1\text{-cl}(A)]\) whenever \(F \subseteq A\) and \(F\) is \(\tau_1\tau_2\)-regular closed in \(X\).

Proof:

Necessity: Let \(F \subseteq A\) and \(F\) is \(\tau_1\tau_2\)-regular closed in \(X\). Then \(A^C \subseteq F^C\) and \(F^C\) is \(\tau_1\tau_2\)-regular open in \(X\). Since \(A\) is \(\tau_1\tau_2\text{-rg}^{**}\) open, we have \(A^C\) is \(\tau_1\tau_2\text{-rg}^{**}\) closed. Hence, \(\tau_2\text{-cl}[\tau_1\text{-int}(A^C)] \subseteq F^C\). Consequently, \([\tau_2\text{-int}[\tau_1\text{-cl}(A)]]^C \subseteq F^C\). Therefore, \(F \subseteq \tau_2\text{-int}[\tau_1\text{-cl}(A)]\).

Sufficiency: Let \(A^C \subseteq U\) and \(U\) is \(\tau_1\tau_2\)-regular open in \(X\). Then \(U^C \subseteq A\) and \(U^C\) is \(\tau_1\tau_2\)-regular closed in \(X\). By our assumption, we have \(U^C \subseteq \tau_2\text{-int}[\tau_1\text{-cl}(A)]\). Hence \([\tau_2\text{-int}[\tau_1\text{-cl}(A)]]^C \subseteq U\). Therefore, \(\tau_2\text{-cl}[\tau_1\text{-int}(A^C)] \subseteq U\). Consequently \(A^C\) is \(\tau_1\tau_2\text{-rg}^{**}\) closed. Hence \(A\) is \(\tau_1\tau_2\text{-rg}^{**}\) open.
Theorem: 7.2.4

Let $A$ and $B$ be subsets of $X$ such that $\tau_2\text{-int}[\tau_1\text{-cl}(A)] \subseteq B \subseteq A$. If $A$ is $\tau_1\tau_2$-rg** open, then $B$ is $\tau_1\tau_2$-rg** open.

Proof:

Let $F \subseteq B$ and $F$ is $\tau_1\tau_2$-regular closed in $X$. Since $B \subseteq A$, we have $F \subseteq A$. Since $A$ is $\tau_1\tau_2$-rg** open, we have, $F \subseteq \tau_2\text{-int}[\tau_1\text{-cl}(A)]$ by theorem 7.2.3. Since $\tau_2\text{-int}[\tau_1\text{-cl}(A)] \subseteq B$, we have $\tau_2\text{-int}[\tau_2\text{-int}[\tau_1\text{-cl}(A)]] \subseteq \tau_2\text{-int}(B) \subseteq \tau_2\text{-int}[\tau_1\text{-cl}(B)]$. Hence $F \subseteq \tau_2\text{-int}[\tau_1\text{-cl}(A)] \subseteq \tau_2\text{-int}[\tau_1\text{-cl}(B)]$. Therefore, $B$ is $\tau_1\tau_2$-rg** open.

Theorem: 7.2.5

If a subset $A$ is $\tau_1\tau_2$-rg** closed, then $\tau_2\text{-cl}[\tau_1\text{-int}(A)] - A$ is $\tau_1\tau_2$-rg** open.

Proof:

Let $F \subseteq \tau_2\text{-cl}[\tau_1\text{-int}(A)] - A$ and $F$ is $\tau_1\tau_2$-regular closed. Since $A$ is $\tau_1\tau_2$-rg** closed, we have $\tau_2\text{-cl}[\tau_1\text{-int}(A)] - A$ does not contain nonempty $\tau_1\tau_2$-regular closed by theorem 7.1.5. Therefore, $F = \phi$, hence $\tau_2\text{-cl}[\tau_1\text{-int}(A)] - A$ is $\tau_1\tau_2$-rg** open.