CHAPTER V
Superdeformation

5.1 Introduction

Shape transition and shape coexistence are phenomena which are well known to exist in various regions of nuclear chart [1]. Over the past decade, the study of superdeformation has occupied one of the foremost venues in the nuclear spectroscopy. Since the discovery of first superdeformed (SD) band in $^{152}$Dy [2], SD states at high spin have been recognized in the so-called $A \approx 240, 190, 150, 130, 110, 80, 60$, and $40$ mass regions. Meticulous theoretical and experimental efforts have been devoted to explore the nature of SD shapes in nuclei [3-7].

Earlier, mean field formalisms were employed for an intuitive understanding of nuclear superdeformation and nuclear superconductivity [8]. Configuration dependent cranked Nilsson–Strutinsky (CNS) calculations are very useful in describing SD bands at high-spin states [9]. Ground state potential energy surface (PES) for SD bands is evaluated by Strutinsky renormalization procedure with the Woods–Saxon potential and total energy surface (TES) [10] obtained with CNS or constrained Hartree–Fock [11] also gives a good prediction of the stability of SD configurations.

All the above-mentioned models have been very successful in unfolding high-spin rotational bands. However, pairing correlations are neglected in those models although there has been no indication that pairing plays a minor role in atomic nuclei.
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Next comes the projected shell model, a shell model truncated in the Nilsson single-particle basis, with pairing correlations incorporated into the basis by a BCS calculation for the Nilsson states [12]. A generator coordinate method with the projected Skyrme HF + BCS states is also used to analyze SD bands [13]. In the Lipkin–Nogami method and particle number projection pairing is properly treated. However, both the models are restricted to axial symmetric shapes and hence the triaxial shapes cannot be discussed. Thus the aim of this work is to provide a suitable approach which can handle the pairing correlation and the evolution of all the nuclear shapes at high-spin states.

Significant features of the microscopic structure of SD bands are described by means of occupation of high-$N$, high-$j$ intruder orbital, which are brought down in energy close to the Fermi level at large deformation and high rotational frequency. Their excitation energies relative to the Fermi level are found to decrease rapidly with increasing deformation and rotational frequency. Theoretical calculations of nuclear level densities as a function of quadrupole deformation specify that SD bands depend on spin and temperature associated with the SD states [14]. The population of SD states depends strongly on the excitation energy of residual nuclei relative to the yrast line. At very high spins and low excitation energies there is a strong tendency for the side feeding of SD bands to increase as the temperature decreases. The impact of temperature on the configuration of SD states has been observed experimentally [15].

A major activity in the study of shape-phase transitions for nuclei in the ground state has been carried out with the IBM [16] at zero temperature. Previously, the Landau theory of phase transitions was used in the analysis of shape transitions in
hot rotating nuclei, i.e., from spherical to deformed shapes or axially deformed to non-axially deformed shapes [17]. Within this approach, simple rules for numerous shape-phase transitions were found as a function of spin and temperature. In our recent article [18], we have studied shape-phase transitions as a function of spin and temperature for hot rotating nuclei and the same formalism when employed to explore SD bands, yields successful results concerning SD shape transition and shape coexistence [19].

In this chapter, an elaborate study of SD states at high-spin states has been carried out using the configuration-dependent cranked Nilsson–Strutinsky model. Energy versus spin curves as well as dynamical moment of inertia ($J^{(2)}$) and rotational frequencies ($\hbar \omega$) are considered and a good understanding of SD bands in several mass regions has been acquired by this formalism. We focus our analysis mainly on rotational energy and dynamical moment of inertia with main emphasis to shape transition and shape coexistence. Spectacular instances of shape coexistence are obtained and have found clear explanation for the existence of different shapes and of the evolution of these minima. Total energy surfaces (TES) are also generated for better understanding of the underlying mechanism of shape evolution and it offers a fair description of the stability of SD configurations.

Our observations are listed below:

i) The majority of the SD shapes are predominantly axially symmetric in nature. Evidences are also there for triaxial SD bands like $^{154}$Er based on the development of single-particle triaxial shell gaps in non-axially deformed nuclei, but in several cases triaxiality is found to be suppressed due to pairing correlations.
ii) The highly deformed and SD bands in $A \approx 130$ region are similar to the one in $A \approx 60$ region where the proton $g_{9/2}$ holes play an important role in stabilization of superdeformation and the contribution to the maximum spin of the $f_{7/2}$ holes is comparable with the contribution from $g_{9/2}$ particles.

iii) The dynamical moment of inertia $I^{(2)}$ in $A \approx 150$ region shows a variety of behavior as a function of $\hbar \omega$ such as a turnover of $I^{(2)}$. This result suggests that there may exist a competition between pairing and antipairing effects in $A \approx 150$ region at least for the one where $I^{(2)}$ exhibits a turnover with increasing $\hbar \omega$. Experimentally determined properties of SD states reasonably agree with our theoretical predictions.

iv) In the $A \approx 80$ SD nuclei, the population of $h_{11/2}$ intruder orbitals is responsible for SD shapes whereas $g_{9/2}$ intruder orbitals are responsible for $A \approx 60$ region. The coexistence of prolate–oblate shape dominates the structure of low-spin states in $A \approx 80$ region. The characteristic features of TES’s are the $\gamma$ softness for $A \approx 60$ region ($^{62}$Zn) and oblate prolate coexistence for $A \approx 80$ region ($^{72}$Kr).

In our investigation, we have noticed that the effect of superdeformation vanishes above $T = 0.5$ MeV. Hence the most probable values of all the parameters are obtained after minimizing the free energy and the values of temperatures are chosen below $T = 0.5$ MeV. Also we have calculated rotational energy $E_{\text{rot}}$, moments of inertia $I^{(1)}$ and $I^{(2)}$ of the systems using Eqs. (4.32 - 4.34) respectively.
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Since every region of superdeformation possesses its own characteristics, systematic investigations on similarities and differences among the SD bands in different mass regions are needed for a deeper understanding of SD structure. In this section, we present detailed information on rotational energies, spins, moment of inertia of the SD rotational bands taking into account of one critical nucleus in each region. The nuclei selected are $^{147}$Gd (neutron deficient even–odd), $^{136}$Pm (neutron rich odd–odd), $^{60}$Zn (doubly-magic $N = Z$), and $^{40}$Ca (doubly magic $N = Z$) in the mass regions $A \approx 150$, $A \approx 130$, $A \approx 60$, and $A \approx 40$, respectively.

5.2.1. SD in $A \approx 150$ Region

In recent years, neutron-deficient rare earth nuclei have been the subject of many experimental studies. The $A \approx 150$ region ranges from $^{142}$Eu to $^{155}$Dy with the Gd isotopes forming the longest chain of SD nuclei. A unique feature of the SD bands in $A \approx 150$ region is that their underlying shell structure, i.e. their high-$N$ intruder content, is clearly reflected in their moment of inertia. Nuclei very close to $^{146}$Gd are not expected to show deformed rotational bands at low spin because of the nature of the orbital closer to the Fermi surface. However, as M increases, intruder orbitals become closer to the Fermi surface and deformed bands are expected to occur.

Experimentally, the behavior of $\gamma^{(2)}$ is known to pursue well the predicted intruder occupation of the particle configurations. This also leads to a sudden increase of nuclear moment of inertia in the yrast band at a certain angular momentum or rotational frequency and the reason being the rotational alignment of angular
momenta of a nucleon pair occupying a high-\( j \) intruder orbital near the Fermi surface leading to back bending.

The \( J^{(2)} \) for most of the SD bands in this mass region are smoothly varying and the magnitude and variation with frequency are characteristics of different high-\( N \) orbital configuration in \( A \approx 150 \) region. However, in this mass region, it turned out that there are considerable variations in the behavior of \( J^{(2)} \) with rotational frequency. Calculation shows that there are larger differences in the behavior of \( J^{(2)} \) as a function of \( \omega \) for nuclei in \( A \approx 150 \). For \(^{150}\text{Gd}\) and \(^{151}\text{Tb}\), the \( J^{(2)} \) values are found to be very high at low frequencies and decrease rapidly with increasing frequencies, while for \(^{149}\text{Gd}\) they decrease less rapidly and for \(^{151,152}\text{Dy}\) they are almost constant. Hence the calculated \( J^{(2)} \) values reproduce the general trend of experimental data for all the nuclei studied although the absolute magnitudes of \( J^{(2)} \) for \(^{149,150}\text{Gd}\) and \(^{151}\text{Tb}\) are slightly low.

These differences arise due to the individual behavior of occupied intruder orbitals originating from high-\( N \) oscillator shells which approach the Fermi surface at very large deformation. In the \( A \approx 150 \) mass region, these orbitals are the \( N = 6 \) proton states and the \( N = 7 \) neutron states. Thus the differences in \( J^{(2)} \) are seen to arise from the way in which various high-\( N \) shell valence nucleon contribute to total moment of inertia. Figure 5.1 (a) shows the agreement between the calculated and observed rotational energy spectrum of \(^{147}\text{Gd}\). The plot of \( \mathcal{M} \) versus \( \hbar \omega \) for \(^{147}\text{Gd}\) shown in Fig. 5.1(b) denotes that the band backbends at \( \hbar \omega = 0.3 \) MeV due to the simultaneous alignment of a pair of \( i_{13/2} \) neutrons and a pair of \( h_{11/2} \) protons. Figure
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5.1(c) also implies that there exist a turnover of $g^{(2)}$ with increasing $\hbar \omega$ due to the competition between the pairing and antipairing effects in the SD states in $^{147}$Gd. The backbend in $g^{(2)}$ may be due to decrease in nuclear pairing as $\hbar \omega$ increases, which is associated with shape transition and this band can be explained in terms of quasi particle effects at low spin and single-particle effects at higher spin. It is suggested that the observed gain in alignment is due to the paired band crossing and evidence for this crossing is observed as a rapid change in the magnitude of $g^{(2)}$. After the crossing $g^{(2)}$ remains constant until the upturn seen in $\hbar \omega$ around 0.6 MeV. This gives a strong indication that there is a correlation between the lowest observed rotational frequency in a SD band and the predicted crossing frequency of the high-$j$ intruder routhians. This indicates that SD bands depopulate in the region of band crossing. Thus the observed gain in alignment is due to the paired band crossing.

Turning to shape coexistence in this region, Lagergren et al. [20] have suggested the first observation of coexisting SD shapes in $^{154}$Er at the prolate and triaxial shapes by identifying a new SD band and this agrees very well with our theoretical prediction of the triaxiality seen in $^{154}$Er [19]. For $^{147}$Gd, in Fig. 5.1(d), the triaxiality is feeble and TES predicts competing prolate and oblate shape for this configuration.
Fig. 5.1: (a) Rotational energy as a function of spin for $^{147}$Gd. (b) Rotational frequency as a function of spin. (c) Dynamical moment of inertia as a function of rotational frequency. (d) Contour plot of energy in the $\varepsilon, \gamma$ plane.
5.2.2. SD in $A \approx 130$ Region

Transitional nuclei with $Z > 50$ and $A \approx 130$ have been a subject of much interest as they reveal considerable variations of shapes and deformations with the configuration of valence quasiparticles. Theoretical calculations for this mass region have shown SD shapes stabilized by multiquasiparticle configurations, including one or more deformation-driving $i_{3/2}$ orbitals. The agreement between the calculated and observed rotational energy values is shown in Fig. 5.2(a). The behavior of generated angular momentum for the corresponding rotational frequency is shown in Fig. 5.2(b).

Configuration assignment to the bands is based on the moment of inertia behavior and it led to the assignment of $\pi h_{11/2} \otimes v_{i_{3/2}}$ for SD band in this mass region with the coupling between the lowest $i_{3/2}$ neutron orbital and the favored signature of $h_{11/2}$proton orbital. Larger $J^{(2)}$ are observed in the odd–odd nuclei in the 130 region and especially in $^{136}$Pm, the $J^{(2)}$ behavior in Fig. 5.2(c) is somewhat irregular due to the orbital interactions and the lower $J^{(2)}$ moment of inertia here is justified as a blocking effect for the alignment of the first pair of $h_{11/2}$ protons which is known to occur near $h \omega \approx 0.45$ MeV.

These observations are in sharp contrast to other recent measurements on neighbouring light odd-Z Pr and Pm nuclei where highly deformed second minimum structures involving the $g_{9/2}$ orbitals are found. In odd-Z $^{133,135}$Pm nuclei, bands involving $h_{11/2}$, $d_{5/2}$, $g_{7/2}$, and $g_{9/2}$ proton orbitals are established with the $h_{11/2}$ being yrast and $g_{9/2}$ lying highest in excitation energy. The behavior of these orbitals as a
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function of $\hbar \omega$ and spin are very different and they have $J^{(2)}$ that closely track each other.

Turning to shape coexistence, the ground state yrast band in $^{136}$Pm arise from the $\pi g_{7/2}$ and $\pi d_{5/2}$ orbitals and in addition to the yrast band in this nuclei having oblate deformation, it is also evident that another prolate shape coupled with the same single proton orbital is strongly competing due to the $i_{13/2}$ neutron. Thus the coexistence of both prolate and oblate shapes are evidenced in $^{136}$Pm, as in several other odd-$A$ nuclei in this mass region and is shown in Fig. 5.2(d).

(a)  
(b)  
(c)  
(d)

Fig. 5.2: Same as Fig. 5.1 but for $^{136}$Pm.
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5.2.3. SD in $A \approx 60$ Region

The most fascinating region is the one with $A = 60$ where a large variety of rotational structures like smooth terminating, highly deformed and superdeformed bands are expected to be observed at very high $\hbar \omega$ in the same nucleus [21]. Figure 5.3 (a) shows the comparison of calculated and observed values of rotational energy of $^{60}$Zn. The generated angular momentum versus corresponding rotational frequency is given in Fig. 5.3(b). Of particular interest are the SD bands in this mass region where $\hbar \omega$ extends about 1.8 MeV. Especially in $^{60}$Zn, we note that the band extends up to high $\hbar \omega$ when compared to all other SD bands (Fig. 5.3(b)).

The spins of the yrast band are plotted as a function of $\hbar \omega$ and the observed peak in $^{60}$Zn is unlikely to be the result of a simple single particle like evolution of the structure such as the unpaired crossing that has been observed in $^{147}$Gd. It is found in Fig. 5.3(c) that much lower value of $J^{(2)}$ than $J^{(1)}$ is a general feature in this region, reflecting the limited angular momentum content in these configurations. The $J^{(2)}$ exhibits a large rise at low $\hbar \omega$ and this may be due to the simultaneous alignment of two pairs of $g_{9/2}$ protons and neutrons. It is noted that in the $N = Z$ nucleus, neutron–proton pairing plays a certain role at high spins. On the other hand, the role of $T = 0$ and $T = 1$ pair correlations also play an important role in this $N = Z$ region [22].

In a recent article [23], the properties of SD bands in Zn are explained by a dinuclear model where the states of SD band are described as Be-cluster configuration. In this case, both clusters are nuclei with isospin $T = 0$ in the ground state and the symmetry energy is minimized. In all the favored SD bands in this mass
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region, the neutron and proton orbitals up to the $N, Z = 30$ SD shell gap corresponding to the $f_{7/2}$ hole and $g_{9/2}$ particle configuration are filled. In $^{60}$Zn, the SD configuration and the ground band configuration differ only by the movement of a pair of neutrons from the $f_{7/2}$ to the $g_{9/2}$ orbital. The mixing of these configurations in the presence of pairing correlations at low spin is thus expected to be more significant than in heavier nuclei, where the SD and normal deformed configurations differ by the rearrangement of a substantial number of nucleon pairs. The coexistence of both prolate and oblate shapes for $^{60}$Zn is shown in Fig. 5.3(d).

(a)

(b)

(c)

(d)

Fig. 5.3: Same as Fig. 5.1 but for $^{60}$Zn.
5.2.4. SD in \( A \approx 40 \) Region

After nearly 15 years of systematic search, new islands of superdeformation have been found in the nuclear chart around \( A \approx 40 \) mass region (e.g., \(^{36}\text{Ar}\) and \(^{40}\text{Ca}\)). Nuclear deformation plays a vital role in the \( s-d \) shell nuclei, and searching for exotic deformations such as superdeformation in this region is one of the hot subjects. It is well known that a variety of structures appears in excited as well as the ground states. Another important aspect in the light-mass region is cluster structure. In light nuclei, largely deformed states often involve clustering at least in \( Z = N \) nuclei in this mass region such as the \(^{16}\text{O} \) \( \alpha \)-cluster state in \(^{20}\text{Ne}\) [24].

The low-spin behavior of these bands is distinctive as compared to SD bands in other mass regions. Another important observation is that these SD bands are linked by intense, discrete transitions to spherical or normal deformed states in the respective nuclei. In each of these nuclei, the SD structures are attributed to multiparticle–multihole \((np-nh)\) excitations across the \( N, Z = 20 \) spherical shell gaps into the \( f-p \) shell. Super deformed rotational band have now been observed in \(^{36}\text{Ar}\) and \(^{40}\text{Ca}\) [25]. Motivated by this observation, microscopic studies for deformed structures in \( s-d \) shell region have progressed recently. We have investigated the coexistence of spherical, deformed and superdeformed states at low spin in \(^{40}\text{Ca}\) and have found that the SD states of \(^{40}\text{Ca}\) are nearly axially symmetric. An agreement between the calculated and observed values of rotational energy of the system \(^{40}\text{Ca}\) is shown in Fig. 5.4 (a). The generated angular momentum as a function of rotational frequency is given in Fig. 5.4(b).
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The dynamical moment of inertia \( J^{(2)} \) behavior suggests that several states among the SD bands around this region may exhibit the identical band phenomenon and backbending is seen around 30h (Fig. 5.4(c)). In addition to the ground state of \(^{40}\text{Ca}\) having a spherical shape, a SD state is also observed with respect to the spherical ground state. The first excited state being a prolate one has been interpreted as \( 4p - 4h \) excitation across the \( N, Z = 20 \) shell gap and the SD state as \( 8p - 8h \) excitation. The SD band head configuration corresponds to the promotion of four particles from the \( s - d \) to the \( p - f \) shell. The \( g_{9/2} \) orbital is also found to change the structure of \(^{40}\text{Ca}\) which comes down to the \( s - d \) shell as an intruder orbital at superdeformation (\( \beta \approx 0.6 \)).

In our results, the triaxiality in \(^{40}\text{Ca}\) is found to be suppressed and hence the SD states are nearly axially symmetric. The general features of SD bands in all the mass regions are outlined. Calculations performed using a microscopic approach is in good agreement with the experimental predictions [14, 26]. Different SD bands are formed depending on the orbitals occupied by the valence protons and neutrons and this behavior is discussed in light of proposed theoretical explanations. A discussion of possible mechanisms for reproducing the smooth increase in \( J^{(2)} \) observed in all SD bands is provided and it is concluded that differences in the behavior of \( J^{(2)} \) for all the nuclei depend on pairing, deformation and Fermi surface changes. Despite the success for the qualitative explanations, a problem is recognized in relation to the pairing collapse at high spin. Actually, recent evidences [27] show that the pair correlations which are quenched at zero temperature and high rotational frequency reappear at high temperature. Figure 5.4(d) shows the coexistence behaviour of \(^{40}\text{Ca}\).
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(a)

(b)

(c)

(d)

Fig. 5.4: Same as Fig. 5.1 but for $^{40}$Ca.
References


