Chapter 1

Introduction

A flow cannot occur in Nature even if it is described by an exact solution of Cauchy equation, or Navier Stokes equation in the case of Newtonian fluids, unless it is also stable. Stability analysis consists in determining whether a flow, under given boundary conditions and external fields is resilient to perturbations. In this thesis, I undertook a theoretical study of surface instability of poorly conducting rheological fluids under the influence of electric and magnetic fields.

We can understand the effect of electric and magnetic fields on materials only if we know the stresses induced by them in the materials. This subject has been developed for Newtonian fluids and Hookean solids. Further, the theory is developed only for linear materials. We extend the existing work for materials with simultaneous viscous and elastic behaviour. We also relax the requirements of linear response and extend our treatment to fluids whose polarization (magnetization) is a non-linear function of applied electric (magnetic) field.
The mathematical theory of hydrodynamic theory has been very well developed for Newtonian fluids. We extend the ideas, deriving necessary relations to suit non-Newtonian fluids. We also comment on the various techniques available to study stability and point out the ones that can be used in the case of fluids under the influence of electric and magnetic fields.

Approximating a flow to be irrotational is of immense help even while analysing flows of Newtonian fluids. It is more so when applied to non-Newtonian fluids. We examine the idea of viscous potential flows and remark how we can exploit it for the understanding of non-Newtonian flows. In particular, we point out how several constitutive relations simplify under this approximation and the equations become analytically tractable. We also emphasize that assuming the velocity field to be irrotational is not the same as treating the fluid to be inviscid.

After developing the basic machinery rheological fluids, we begin applying it to the three important classes of instabilities:

1. Rayleigh-Taylor instability arising when a heavy fluid is superposed on a lighter one.

2. Kelvin-Helmholtz instability arising when two fluids are moving relative to each other and

3. Richtmyer-Meshkov instability arising when a fluid is impinged by a shock wave.

To summarize, the thesis solves five problems:

1. Development of the theory of stresses due to static electric and magnetic
fields in materials with simultaneous elastic and viscous behaviour and an arbitrary response to the applied fields.

2. Development of the theory of instability to linear viscoelastic materials and the analytical framework to study their flows.

3. Rayleigh-Taylor instability in presence of electric and magnetic fields.

4. Kelvin-Helmholtz instability in presence of electric and magnetic fields.

5. Richtmyer-Meshkov instability in presence of electric and magnetic fields.

Solution to the problems and exposition of the background material required the thesis to be divided in eleven chapters, including the present one.

• Chapter 2 'Basic ideas of Rheology' begins by mentioning the conservation laws governing the flow and deformation of matter. We then point out the non-intuitive behaviour of rheological fluids and mention various classes of constitutive relations. We develop the integral forms of constitutive relations for benefit of stability analysis. Lastly, we mention the range of responses of materials to electric and magnetic fields.

• Chapter 3 'Stress due to Electric and Magnetic Fields' begins with a brief review of the history of the topic, pointing out why it is a subtle problem. We then develop a theory of stresses using fundamental thermodynamic principles. The expression of stress is developed from the Helmholtz free energy of a material in an electric (magnetic) field. The
generality of the thermodynamic framework allows us to derive results applicable to a wide class of materials. To our knowledge, there are no published results for stress in a viscoelastic material immersed in an electric (or a magnetic) field, allowing the material to have an arbitrary response to the field. We end the chapter by deriving expressions for ponderomotive force for our future analysis.

- Chapter 4 'Instability of Fluids' carries over the main ideas related to hydrodynamic stability developed for Newtonian fluids to non-Newtonian ones. We point out the spurious case of instability due to nature of constitutive relations and briefly describe the variety of techniques used to study instability. We review each of those techniques with the aim of applying it to the problem of rheological fluids in electric or magnetic fields.

- Chapter 5 'Causes of Instability' reviews some of the important types of instability and identifies the physical causes triggering them. While most of it is previously known, we believe that our simple analysis of Kelvin-Helmholtz instability, as manifested by waves on the surface of a still pond, is new.

- Chapter 6 'Viscous Potential Flows' explores approximating solutions of the Cauchy equation assuming the flow to be irrotational. We demonstrate that the approximation simplifies the quasi-linear co-rotational constitutive models into linear ones. However non-linear co-rotational models continue to remain intractable unless we also assume that the fluid is initially at rest. This chapters aim is to explore how to use the
viscous potential flow approximation to the problem at hand.

- Chapter 7 'Some General Results' develops a few general theorems to be used in later chapters.

- Chapter 8 'Rayleigh-Taylor Instability' begins with study of the phenomenon in the simplest circumstance of ideal fluids. Doing so illuminates the key physical ideas most clearly and helps us identify the parameters that can help us arrest or delay the instability. We then proceed to apply the analysis and use the insights to study the interface between Newtonian and non-Newtonian fluids.

- Chapter 9 'Richtmyer-Meshkov Instability' begins by briefly reviewing several techniques to analyse the instability of a surface of a fluid impinged by a shock wave. We then use the viscous potential flow approximation to study the growth of the instability on a non-Newtonian fluid. We not only derive a dispersion relation but also find out how a shock wave propagates inside a material. A shock wave is a region of discontinuity in the material. We mention how to modify the conservation laws of mechanics to accommodate such a discontinuity. However, doing so requires a rather elaborate machinery of continuum mechanics, which we develop in the appendices to the chapter.

- Chapter 10 'Kelvin-Helmholtz Instability' starts with remarks on peculiarity of this type of instability and argue why viscous potential flow approximation is effective in understanding the effect of viscosity and electromagnetic fields. We then proceed to analyze the instability at
the interface of two Newtonian fluids, a Newtonian fluid on top of a non-Newtonian fluid and two non-Newtonian fluids.

- Chapter 11 'Conclusions' summarizes our key findings and chart out future course of investigation.