CHAPTER-III

GENERATION OF PULSE-POSITION-PULSE-WIDTH-MODULATED (PPPWM) CONTROL SIGNAL THROUGH STATE-EQUIVALENCE TRANSFORMATION OF LINEAR DISCRETE CONTROL (LDC) SIGNAL

3.1 General comments

In this chapter, the details of the formulation of PPPWM control signal from LDC signal will be presented. The formulation will be dealt in four perspectives,

(i) Mathematical definition of PPPWM signals
(ii) Derivation of PPPWM control signal by the 'principle of state equivalence'.
(iii) Formulation of simplified PPPWM signals by approximation of state equivalence for real-time PPPWM implementation in higher order systems, where exact state equivalence formulation is not suitable and
(iv) Derivation of equations for position and width variables for different simplified PPPWM signals.

3.2 Mathematical Definition of PPPWM signal.

State space methods of describing control laws for discrete time systems in which the input amplitude is continuously variable are well known. In PPPWM systems the control variable is not the amplitude of the
input but rather the duration of a sequence of pulses of constant amplitude. The generally used PWM signal waveforms of unipolar and bipolar types have been adopted for PPPWM signal and they are designated as follows,

(i) Unipolar PPPWM

(ii) Bipolar PPPWM

3.2.1. **Unipolar PPPWM signal definition**

Fig. 3.1 and fig. 3.2 show the nature of a Unipolar PPPWM signal for a $n^{th}$ order plant. In Unipolar PPPWM system, PPPWM signal $u(t)$ in the $k^{th}$ sampling interval contains pulses all having the same polarity. The sign of the PPPWM signal is the same as the sign of the linear discrete signal $v(k)$.

The PPPWM signal will have 'q' variables which must at least be equal to the order of the plant, 'n' in order to ensure exact state equivalence between the states of unmodulated and the modulated systems at the sampling instants, $kT$ (Appendix-A).

It should be noted that the total number of variables including position $p_i's$ and widths $w_i's$ of the pulse sequence is 'q'. In case 'q' is odd, the last pulse of the pulse sequence in each interval must be
extended up to \((kT+T)\) and the last position variable will be \(p_{(q+1)/2}\). Since the number of variables for PPPWM signal, 'q' must at least be equal to the order of the plant, 'n', for subsequent mathematical treatment 'q' will be assumed to be equal to 'n'.

The Unipolar PPPWM, as shown in fig.3.1 and fig.3.2, can be defined mathematically as follows,

**Case 1 'n' is even:**

\[
\begin{align*}
\text{Case 1: } & n \text{ is even:} \\
\text{for } kT < t \leq kT + p_1 & \Rightarrow u(t) = 0 \\
& = h.\text{sgn}[v(k)] \text{ for } kT + p_1 < t \leq kT + p_1 + w_1 \\
& = 0 \text{ for } kT + p_1 + w_1 < t \leq kT + p_1 + w_1 + p_2 \\
& \vdots \\
& = 0 \text{ for } kT + p_1 + \ldots + w_{n/2} - 1 < t \leq kT + p_1 + \ldots + p_{n/2} \\
& = h.\text{sgn}[v(k)] \text{ for } kT + p_1 + \ldots + p_{n/2} < t \leq kT + p_1 + \ldots + p_{n/2} + w_{n/2} \\
& = 0 \text{ for } kT + p_1 + \ldots + w_{n/2} < t \leq kT + T
\end{align*}
\]  

--- (3.1)

**Case 2 'n' is odd:**

\[
\begin{align*}
\text{Case 2: } & n \text{ is odd:} \\
\text{for } kT < t \leq kT + p_1 & \Rightarrow u(t) = 0 \\
& = h.\text{sgn}[v(k)] \text{ for } kT + p_1 < t \leq kT + p_1 + w_1 \\
& \vdots \\
& = 0 \text{ for } kT + p_1 + \ldots + w_{n-1} - 1 < t \leq kT + p_1 + w_1 + \ldots + w_{n-1} + p_{n+1} \\
& = h.\text{sgn}[v(k)] \text{ for } kT + p_1 + \ldots + p_{n+1} < t \leq kT + T
\end{align*}
\]  

--- (3.2)
FIG 3.1 UNIPOLAR PPPWM SIGNAL FOR \( n^{th} \) ORDER PLANT

WHEN \( n \) IS EVEN

FIG 3.2 UNIPOLAR PPPWM SIGNAL FOR \( n^{th} \) ORDER PLANT

WHEN \( n \) IS ODD
3.2.2 Bipolar PPPWM signal definition

In Bipolar PPPWM, the signal \( u(t) \) takes both polarities in each sampling interval. The pulse sequence of bipolar PPPWM for an \( n^{th} \) order plant is given in fig.3.3 and fig.3.4. The definition of bipolar PPPWM can be expressed mathematically similar to eqn.(3.1) and eqn.(3.2) as follows.

**Case 1 'n' is even:**

\[
\begin{align*}
\text{u}(t) &= -h \quad \text{for } kT < t \leq kT + p_1 \\
&= +h \quad \text{for } kT + p_1 < t \leq kT + p_1 + w_1 \\
&\quad \cdots \\
&= -h \quad \text{for } kT + p_1 + \ldots + w_{n/2-1} < t \leq kT + p_1 + \ldots + w_{n/2-1} + p_{n/2} \\
&= +h \quad \text{for } kT + p_1 + \ldots + p_{n/2} < t \leq kT + p_1 + \ldots + p_{n/2} + w_{n/2} \\
&= -h \quad \text{for } kT + p_1 + \ldots + w_{n/2} < t \leq kT + T \\
\end{align*}
\]

(3.3)

**Case 2 'n' is odd:**

\[
\begin{align*}
\text{u}(t) &= -h \quad \text{for } kT < t \leq kT + p_1 \\
&= +h \quad \text{for } kT + p_1 < t \leq kT + p_1 + w_1 \\
&\quad \cdots \\
&= -h \quad \text{for } kT + p_1 + \ldots + w_{n-1} < t \leq kT + p_1 + \ldots + w_{n-1} + p_{n+1} \\
&= +h \quad \text{for } kT + p_1 + \ldots + p_{n+1} < t \leq kT + T \\
\end{align*}
\]

(3.4)
FIG. 3.3 BIPOLAR PPPWM SIGNAL FOR $n^{th}$ ORDER PLANT

WHEN $n$ IS EVEN

FIG. 3.4 BIPOLAR PPPWM SIGNAL FOR $n^{th}$ ORDER PLANT

WHEN $n$ IS ODD
3.3 Description of Plant Dynamics and State-Transition Equation

Let the dynamic equation of the plant in MIMO control system as shown in Fig.3.5, be described by a linear differential equation as given below.

\[ \dot{X} = AX + Bu \]
\[ y = CX \]

where

\[ X \in \mathbb{R}^{n \times 1}, A \in \mathbb{R}^{n \times n}, u \in \mathbb{R}^{m \times 1}, B \in \mathbb{R}^{m \times m}, Y \in \mathbb{R}^{1 \times 1} \& C \in \mathbb{R}^{1 \times n} \]

For a MIMO control system, the plant input \( u(t) \) has 'm' channels. During the control signal modulation by PPPWM, each channel \( v_i(k) \) of the discrete control signal \( v(k) \) is modulated to yield the corresponding channel \( u_i(t) \) of the plant input signal \( u(t) \). Since the plant represented by eqn.(3.5) is linear, the net effect of the multidimensional input signal \( u(t) \) can be studied by means of superposition theorem; i.e. by assuming only one input signal \( u_i(t) \) at a time and then superimposing the effects of the various inputs on each state to get the net effect.

Hence the mathematical treatment in subsequent sections will be for a single input and the results derived for the single input will be
FIG. 3.5  PPPWM FEEDBACK CONTROL SYSTEM.
therefore, directly applicable to the individual channels of the input vector \( u(t) \).

Now the solution of the state equations (eqn. 3.5) can be given as

\[
X(t) = e^{A(t-t_o)}X(t_o) + \int_{t_o}^{t} e^{A(t-\lambda)}u(\lambda)d\lambda B \quad (3.6)
\]

If \( u(\lambda) = u_{t_0} \), a constant for \( t_o < \lambda \leq t \), eq. (3.6) reduces to

\[
X(t) = e^{A(t-t_o)}X(t_o) + \int_{t_o}^{t} e^{A(t-\lambda)}d\lambda B u_{t_0} \quad (3.7)
\]

In the absence of modulation, the plant is driven directly by \( v(k) \) which is limited by the maximum value of the input voltage. From practical considerations the limiting voltage can be assumed to be \( \pm h \) and hence the state transition equation can be written as,

\[
X(k+1) = e^{AT}X(k) + \left[ e^{AT} - I \right] A^{-1}B h \text{sat}[v(k)/h] \quad (3.8)
\]

where,

\[
\text{sat}(x) = \begin{cases} 
1 & , x \geq 1 \\
 x & , 0 \leq |x| < 1 \\
-1 & , x \leq -1 
\end{cases}
\]
3.4 Transformation of LDC signal to equivalent PPPWM signal

The linear discrete control (LDC) signal during a particular sampling interval is transformed to PPPWM control signal by using the 'principle of state equivalence' for the plant described in eqn.(3.5) for the single input condition.

3.4.1 Condition of state equivalence for Unipolar PPPWM and solution of Unipolar PPPWM signal variables

The state transition equation of the system under the influence of the unipolar PPPWM can be derived as follows.

Assume that n is even. Then \( u(t) \) is given by eqn.(3.1). From eqn.(3.1) and eqn.(3.7) we obtain,

\[
X(kT+T) = e^{AT}X(kT) + \sum_{j=1}^{n/2-1} e^{AT-\sum (p_i+w_i)} \left[ e^{j-1}A \cdot Bh \cdot sgn[v(k)] \right] Aw^{-1}
\]

\[
+ e^{AT-\sum (p_i+w_i)} \left[ e^{Aw^{n/2} - I}A \cdot Bh \cdot sgn[v(k)] \right] Aw^{-1}
\]

--- (3.9)
For the state equivalence of modulated and unmodulated system at sampling instants, the R.H.S of eqn.(3.8) and eqn.(3.9) should be equal. This will lead to the following condition,

\[
\sum_{j=1}^{n/2} \left[ e^{A\sum_{i=1}^{j} (p_i + w_i)} - e^{A\sum_{i=1}^{j} (p_i + w_i)} \right]^{-1} A^2 B + 
\]

\[
\left[ e^{A\sum_{i=1}^{n/2} (p_i + w_i)} - e^{A\sum_{i=1}^{n/2} (p_i + w_i)} \right]^{-1} A^2 B
\]

\[
= [I - e^{-AT}] A^{-1} B \text{ sat}(\gamma) \quad (3.10)
\]

Where \( \gamma = \left| \frac{v(k)}{h} \right| \) \quad (3.11)

Eqn.(3.10) gives the relation between the discrete control signal 'γ' and PPPWM signal variables 'p_i's and 'w_i's.
The exponents in eqn. (3.10) can be expanded using Sylvester expansion [30] yielding the following equation of state equivalence.

\[
\sum_{j=1}^{n/2-1} \left[ \sum_{r=1}^{n} e_{r} \left[ W_{j} - \sum_{i=1}^{\frac{n}{2}} (p_{i} + w_{i}) \right] \right] \Omega_{r} - \sum_{r=1}^{n} e_{r} \sum_{i=1}^{\frac{n}{2}} (p_{i} + w_{i}) \Omega_{r} \right]^{-1} A B +
\]

\[
\left[ \sum_{r=1}^{n} e_{r} \left[ W_{\frac{n}{2}} - \sum_{i=1}^{\frac{n}{2}} (p_{i} + w_{i}) \right] \right] \Omega_{r} - \sum_{r=1}^{n} e_{r} \sum_{i=1}^{\frac{n}{2}} (p_{i} + w_{i}) \Omega_{r} \right]^{-1} A B
\]

\[
= \left[ I - e^{-AT} \right]^{-1} A B \text{sat}(\gamma) \tag{3.12}
\]

where \( \Omega_{r} = \prod_{j=1}^{n} \left[ \frac{A - \lambda_{j} I}{\lambda_{r} - \lambda_{j}} \right] \), \( \lambda_{j} \) being the eigen value of \( A \).

The matrix-vector equation (3.12) is equivalent to a set of \( n \) nonlinear equation in the \( n \) variables \( p_{1}, w_{1}, \ldots, p_{n/2}, w_{n/2} \) and describes PFPWM signal corresponding to the discrete control signal for exact state-equivalence.

However, the real time solution of \( p_{i} 's \) and \( w_{i} 's \) in terms of \( \gamma \) is a difficult task and hence
the result is not suitable for on-line implementation, when the order of the system is higher than 2 due to the increase in real-time computation time.

So for higher order systems, there is a need to adopt an approximate solution of eqn. (3.10). One direct straightforward approach for approximation would be as follows.

(i) To expand the matrix exponents on both sides of eq. (3.10) by the infinite series expansion.

(ii) To neglect the terms containing higher powers of $A$ depending on the order of approximation.

(iii) Finally, to equate the coefficients of equal powers of $A$ on both sides.

This will result in simple algebraic equations in terms of $p_t$'s, $w_t$'s, $T$ & $r$. Solving them we can obtain $p_t$'s and $w_t$'s. Larger the number of terms used, better will be the approximation. Consequently, larger the number of pulses ($p_t$'s and $w_t$'s) used for PPPWM, better will be the accuracy of state equivalence. Ideally the number of variables $q$ must tend to $\infty$ to achieve exact state equivalence by this method.
\[
\lim_{q \to \infty} w_q = 0 \quad \text{and} \\
\lim_{q \to \infty} e^{Aw_{q/2}} = 1
\]

So the term \( e^{Aw_{q/2}} \) in eqn. (3.10) will be equal to 1. Subsequently eqn. (3.10) will reduce to the following equation of state equivalence condition.

\[
\sum_{j=1}^{\infty} A^{j-1} \left[ e^{A\left( \sum_{i=1}^{j} (p_i + w_i) \right)} - e^{A\sum_{i=1}^{j} (p_i + w_i)} \right] \left[ I - e^{-AT} \right]^{-1} B = \left[ I - e^{-AT} \right] A B \operatorname{sat}(r)
\]

--- (3.13)

### 3.4.2 Formulation of simplified Unipolar PPPWM signal

Eqn. (3.13) can be expanded using infinite series expansion and then approximated as indicated in the previous section. This will result in different types of Unipolar PPPWM signals depending on the levels of approximation considered in the state equivalence. These signals will be termed as **First order**, **Second order**, **Third order** etc. according to the levels of approximation.
3.4.2.1 First order Unipolar PPPWM

The condition that $p_i$'s and $w_i$'s should satisfy to achieve exact state equivalence is given by eqn. (3.10). Expanding the exponents on both sides of eqn. (3.13) and approximating them by the first two terms, we get the following relations.

\[
\sum_{j=1}^{\infty} \left\{ A \left[ \sum_{i=1}^{j} (p_i \cdot w_i) \right] + A \left[ \sum_{i=1}^{j} (p_i \cdot w_i) \right] \right\}^{-1} A B = B.T.\text{sat}(\gamma) \quad \text{--- (3.14)}
\]

\[ B \sum_{j=1}^{\infty} w_j = B.T.\text{sat}(\gamma) \quad \text{--- (3.15)} \]

In order to satisfy eqn. (3.15) for all $B$, the following equation should hold:

\[ \sum_{j=1}^{\infty} w_j = T.\text{sat}(\gamma) \quad \text{--- (3.16)} \]

Thus using first degree approximation of exponent, eqn. (3.13) is reduced to the single equation given by eqn. (3.16). Choosing $p_i = 0$ for $i=1,2,\ldots,\infty$ and $w_i = 0$ for $i=2,3,\ldots,\infty$ we obtain,
Thus with first order approximation, the Unipolar PPPWM is reduced to the conventional unipolar PWM which is shown in fig.3.6.

3.4.2.2 Second order Unipolar PPPWM

Expanding the exponents on both sides of eqn. (3.13) and approximating the series with the first three terms, we obtain

\[ B \sum_{j=1}^{\infty} w_j + \frac{A \cdot B}{2} \sum_{j=1}^{\infty} \left\{ w_j - \sum_{i=1}^{j} (p_i + w_i) \right\}^2 - \left[ \sum_{i=1}^{j} (p_i + w_i) \right]^2 \]

\[ = \left[ BT - \frac{A \cdot B}{2} T^2 \right] \text{sat}(\gamma) \quad --- (3.18) \]

In order to satisfy eqn. (3.18) for all values of A and B, the following two equations should hold:

\[ \sum_{j=1}^{\infty} w_j = T \cdot \text{sat}(\gamma) \quad --- (3.19) \]
Thus by second degree approximation
of the matrix exponents of eqn.(3.13) is reduced to
eqn.(3.19) and eqn.(3.20).

Choosing $p_i = w_i = 0$ for $i=2,3,\ldots,\infty$, we obtain

\[ \sum_{j=1}^{\infty} \left\{ \left[ w_j - \sum_{i \neq j} (p_i + w_i) \right]^2 - \left[ \sum_{i \neq j} (p_i + w_i) \right]^2 \right\} = T \cdot \text{sat}(\gamma) \quad -- (3.20) \]

Eqn.(3.21) and eqn.(3.22) show that
the pulse is occurring at the middle of the sampling
interval. Thus the second degree approximation of the
exponential terms in eqn.(3.13) reduces the exact PPPWM
to second order PPPWM having a single centralised pulse in each
sampling interval as shown in fig.3.7.
FIG 3.6 FIRST ORDER UNIPOLAR PPPWM SIGNAL

FIG 3.7 SECOND ORDER UNIPOLAR PPPWM SIGNAL
3.4.2.3 Third order Unipolar PPPWM

Expanding the exponents on both sides of eqn.(3.13) and approximating them with the first four terms in their expansion, we get the following relations.

\[
B \sum_{j=1}^{\infty} w_j + \frac{A \cdot B}{2} \sum_{j=1}^{\infty} \left\{ \left[ w_j - \sum_{i=1}^{j} (p_i + w_i) \right]^2 - \left[ \sum_{i=1}^{j} (p_i + w_i) \right]^2 \right\} +
\]

\[
\frac{A \cdot B}{2} \sum_{j=1}^{\infty} \left[ \left( w_j - \sum_{i=1}^{j} (p_i + w_i) \right)^3 + \left( \sum_{i=1}^{j} (p_i + w_i) \right)^3 \right] = \left[ BT - \frac{A \cdot B}{2} T + \frac{A \cdot B}{2} T \right] \text{sat}(\gamma)
\]

--- (3.23)

In order to satisfy eqn.(3.23) for all \(A\) & \(B\), the following three equations should hold.

\[
\sum_{j=1}^{\infty} w_j = T \cdot \text{sat}(\gamma)
\]

--- (3.24)

\[
\sum_{j=1}^{\infty} \left\{ \left[ w_j - \sum_{i=1}^{j} (p_i + w_i) \right]^2 - \left[ \sum_{i=1}^{j} (p_i + w_i) \right]^2 \right\} = -2T \cdot \text{sat}(\gamma)
\]

--- (3.25)

\[
\sum_{j=1}^{\infty} \left\{ \left[ w_j - \sum_{i=1}^{j} (p_i + w_i) \right]^3 + \left[ \sum_{i=1}^{j} (p_i + w_i) \right]^3 \right\} = 9T \cdot \text{sat}(\gamma)
\]

--- (3.26)
Thus by third degree approximation, eqn.(3.13) is reduced to the three equations, eqn.(3.24)-eqn.(3.26).

Let us now assume that,
\[ p_i = w_i = 0 \text{ for } i = 3, 4, \ldots \infty \text{ and} \]
\[ w_2 = T - (p_1 + w_1 + p_2) \tag{3.27} \]

By substituting eqn.(3.27) in eqn. (3.24)-eqn.(3.26), we obtain the following equations from which the PPPWM variables can be solved.

\[ 4p_1^4 - 16rT p_1^3 + (18r^2 T^2 + 6r T^2)p_1^2 \]
\[ - (12r^3 T^3 + 4r T^3)p_1 + (3r^4 T^4 + r^2 T^4) = 0 \tag{3.28} \]
\[ p_2 = T - p_1 \tag{3.29} \]
\[ w_1 = \frac{r T^2 - r^2 T^2}{2(rT - p_1)} \tag{3.30} \]

where \( r = 1 - \text{sat}(\gamma) \) \tag{3.31}

The third order Unipolar PPPWM defined by eqn.(3.27)-eqn.(3.31) is shown in fig.3.8. We can see that the solution of eqn.(3.28) is much involved and is not suitable for practical implementation. But a look up table based on eqn.(3.28) can be used to read the value of \( p_i \) corresponding to any value of \( r(0 \leq r \leq 1) \). In a similar way, we can derive the formulae for higher order PPPWM's.
3.5 Transformation of LDC signal to equivalent Bipolar PPPWM signal

The LDC signal can be transformed to Bipolar PPPWM signal during a particular sampling interval using, as in unipolar case, the principle of state equivalence.

3.5.1 Condition of state equivalence for Bipolar PPPWM

Bipolar PPPWM which achieves exact state equivalence is defined in eqn.(3.3) & eqn.(3.4). The condition to be satisfied by the positions and the widths of the bipolar PPPWM's to achieve the state equivalence is developed in this section. For the sake of simplicity, the order of the plant is assumed to be even. Following the similar steps used in the derivation of eqn.(3.10), we obtain the state transition of the modulated plant as,

\[
X(kT+p_t) = e^{A_{p_t}} X(kT) - e^{A_{p_t}} [A_{p_t} - I]^{-1} A B h
\]  \hspace{1cm} (3.32)

\[
X(kT+p_t+w_1) = e^{A_{p_t+w_1}} X(kT) - e^{A_{p_t+w_1}} [e^{A_{p_t}} - I]^{-1} A B h
\]  \hspace{1cm} (3.33)

\[
+ e^{A_{w_1}} [A_{w_1} + I]^{-1} A B h
\]
Similarly,

\[
X(kT + p_1 + w_1 + \ldots + p_{n/2} + w_{n/2}) = \left[ e^{-\sum_{i=1}^{n/2} (p_i + w_i)} \right] X(kT)
\]

\[
- \left[ e^{\sum_{i=2}^{n/2} (p_i + w_i)} \right] \left[ e^{-I} A \right]^{-1} A \left[ e^{\sum_{i=2}^{n/2} (p_i + w_i)} \right] \left[ e^{-I} A \right]^{-1} A B h
\]

\[
- \left[ e^{\sum_{i=2}^{n/2} (p_i + w_i)} \right] \left[ e^{-I} A \right]^{-1} A B h + \ldots + \left[ e^{\sum_{i=2}^{n/2} (p_i + w_i)} \right] \left[ e^{-I} A \right]^{-1} A B h
\]

Hence,

\[
X(kT + p_1 + w_1 + \ldots + p_{n/2} + w_{n/2}) = \left[ e^{-\sum_{i=1}^{n/2} (p_i + w_i)} \right] X(kT)
\]

\[
\begin{align*}
&+ \sum_{j=1}^{n/2-1} \left[ e^{\sum_{i=j+1}^{n/2} (p_i + w_i)} \right] \left[ e^{-I} A \right]^{-1} \left[ e^{\sum_{i=j+1}^{n/2} (p_i + w_i)} \right] \left[ e^{-I} A \right]^{-1} A B h \\
&+ \left\{ e^{\sum_{i=2}^{n/2} (p_i + w_i)} - e^{\sum_{i=2}^{n/2} (p_i + w_i)} \left[ e^{-I} A \right]^{-1} A B h \right\}
\end{align*}
\]

\[ (3.35) \]
Consequently,

$$X(kT+T) = e^{AT}X(kT) +$$

$$e \sum_{j=1}^{n/2-1} \left\{ -A \sum_{i=1}^{j} (p_i + w_i) \left[ e^{A_{j}} - I \right] - A [W_j - \sum_{i=1}^{j} (p_i + w_i)] \left[ e^{A_{j}} - I \right] \right\}^{-1} A \cdot Bh +$$

$$A^T \left\{ \sum_{i=1}^{n/2} (p_i + w_i) \right\} \left\{ e^{A_{n/2}} - 2I - e^{-A_{n/2}} \right\}^{-1} A \cdot Bh + A \cdot Bh$$

--- (3.36)

For the state equivalence of the modulated and unmodulated system at each sampling instants, the R.H.S of eqn.(3.36) and eqn.(3.8) should be equal. The resulting matrix-vector equation can be expanded into a set of 'n' nonlinear equations in 'n' variables using Sylvester expansion formula [30] and they can be uniquely solved to get Bipolar PPPWM variables. But this is a tedious task and not feasible for higher order system. Hence we use the infinite series expansion of the matrix exponents to develop Bipolar PPPWM's of various levels of approximation as in the previous chapter. Assuming infinite number of pulses in each sampling interval, the state equivalence condition to be satisfied by the Bipolar PPPWM can be written as,
3.5.2 Formulation of Simplified Bipolar PPPWM signals

For practical implementation of higher order Bipolar PPPWM control systems, eqn.(3.37) and eqn.(3.38) are expanded in infinite series and approximated to get different types of Bipolar PPPWM signals.

3.5.2.1 First order Bipolar PPPWM

Expanding the matrix exponents on both sides of eqn.(3.37) by the infinite series expansion and approximating them by the first two terms, we obtain
\[ B \sum_{j=1}^{n/2-1} \left\{ 2\left[w_j - \sum_{i=1}^{j} (p_i + w_i)\right] + \sum_{i=1}^{j} (p_i + w_i) \right\} + \]

\[ B \left\{ 2\left[w_{n/2} - \sum_{i=1}^{n/2} (p_i + w_i)\right] + \sum_{i=1}^{n/2} (p_i + w_i) - T \right\} \]

\[ = BT \text{ sat}(\phi) \quad (3.39) \]

Eqn. (3.39) can now be reduced to,

\[ B \sum_{j=1}^{\infty} \left[ 2w_j - T \right] = BT \cdot \text{sat}(\phi) \quad (3.40) \]

In order to satisfy eqn. (3.40) for all values of B, the following relation should hold:

\[ \sum_{j=1}^{\infty} 2w_j = [1+\text{sat}(\phi)]T \quad (3.41) \]

Putting \( p_i = w_i = 0 \) for \( i = 2, 3, \ldots, \infty \) and \( p_1 = T - w_1 \), we get

\[
\begin{align*}
w_1 &= [1+\text{sat}(\phi)]T/2 \quad (3.42) \\
p_1 &= [1-\text{sat}(\phi)]T/2 \quad (3.43)
\end{align*}
\]

The first order Bipolar PPPWM represented by eqn. (3.42) & eqn. (3.43) is shown in fig.3.9. This coincides with the conventional bipolar PWM.
3.5.2.2 Second order Bipolar PPPWM

Expanding the exponents on both sides of eqn. (3.37) and approximating them by the first three terms, we obtain the following condition for state equivalence.

\[
B \sum_{j=1}^{\infty} \left[ 2w_j - \frac{A}{2} \sum_{j=1}^{\infty} \right] \left\{ 2 \left[ W_j - \sum_{i=1}^{j} (P_i + W_i) \right] - \left[ \sum_{i=1}^{j} (P_i + W_i) \right] - \left[ \sum_{i=1}^{j-1} (P_i + W_i) \right] \right\}
\]

\[
+ \frac{AB}{2} \left\{ T - \left[ \sum_{i=1}^{\infty} (P_i + W_i) \right] \right\} = \left[ BT - \frac{AB}{2} T \right] \cdot \text{sat}(\phi) \quad --- (3.44)
\]

Assuming \( p_i = w_i = 0 \) for \( i = 2, 3, \ldots, \infty \) in eqn. (3.44) we obtain the following expressions for the second order Bipolar PPPWM.

\[
w_i = \left[ 1 + \text{sat}(\phi) \right] \cdot \frac{T}{2} \quad --- (3.45)
\]

\[
p_i = \left[ 1 - \text{sat}(\phi) \right] \cdot \frac{T}{4} \quad --- (3.46)
\]

The resulting PPPWM signal is shown in fig. 3.10. It can be seen from the figure that the pulse is centralized in each sampling interval.
FIG 3.9 FIRST ORDER BIPOLAR PPPWM

FIG 3.10 SECOND ORDER BIPOLAR PPPWM
3.6 Discussion on PPPWM signal Transformation

This chapter gives the equations of transformation from LDC signal to PPPWM signal and forms the basis for PPPWM control system design. Some of the noteworthy features in this transformation are highlighted below,

(i) Exact state equivalence transformation does not affect the linear discrete design and is suitable for plants of order not more than 2.

However, the exact state equivalence transformation involves plant dynamics and hence is subject to deviation in performance when the plant dynamics vary with time.

(ii) Simplified PPPWM signal equations are independent of plant dynamics and hence solutions are not affected by plant dynamics.

But, the approximation involved in getting simplified PPPWM signals results in state error and, sometimes, in stability problems. These will be addressed in subsequent chapters.

(iii) PPPWM signals, derived either by exact or by approximate state equivalence, will result in inter-sample ripple which has to be kept within specified limit. This will be discussed in later chapter.
(iv) With first order approximation, \text{PPPWM} signal reduces to the conventional \text{PWM} signal. Thus \text{PWM} can be viewed as a specific form of \text{PPPWM}.

(v) Considering the fact that \text{PPPWM} possesses higher degree of freedom, if the number of variables is kept more than the order of the system, equivalence of the states can also be achieved at intersample points in addition to exact state equivalence of states at sampling instants.