CHAPTER-II

OVERVIEW OF PWM DESIGN METHODS

2.1 General comments

The PWM analysis and design techniques available can be categorised as,

(i) Modelling

(ii) Stability analysis

(iii) Controller synthesis technique

2.2 PWM modelling and analysis technique

In view of the nonlinear nature of PWM, the exact modelling of PWM is not available. However, attempts have been made to develop approximate models. The exact difference equations for the PWM control system relating the controlled variable to the system error at the sampling instants were considered by Tsypkin (1956) [17] and Nease (1957) [23]. Tsypkin provided a general method for the exact expression of the pulse-width modulated systems by direct superposition of approximately weighted step function responses. In 1960, Andeen developed an approximate linear model of PWM [6, 11] using the principle
of equivalent area for its use in practical design. For sufficiently small inputs, the PWM output was approximated with a sequence of impulse functions of equal area. With this approximation, the PWM control system is equivalent to the conventional linear sampled-data system and could be studied by the use of conventional sampled-data theory. Delfed and Murphy (1961) developed a method [7] for the determination of the response of a closed loop PWM control system to arbitrary input using describing function, where the linear and nonlinear terms were separated by use of difference equation. But all such models assumed either that the inputs are very small or that the pulse duration sampler works into a combination of system elements which includes a low pass element having largest time constant at least twice the sampling period for adequate filtering. So where the sampling frequency is comparable to the system dynamic frequency, application of these models faced serious limitations.

2.3 PWM stability analysis

In 60's the stability problem in feedback systems including pulse modulation (of a specific type) has been studied using variety of techniques including linearisation, Lyapunov's method and application of Popov's criterion [19]. Yet, the results are far from satisfactory often yielding conservative stability
boundaries and / or requiring a difficult transformation of the feedback system as a pre-requisite to the application of the criterion (characteristic of the Lyapunov and Popov development). In particular, the results only apply to restricted class of PWM systems and in general prove only a limited kind of stability (asymptotic stability in the large of the null solution). In regulator systems when the input is known precisely such definitions of stability is suitable. However, when the input is arbitrary or restricted to a certain class of functions, such results are not applicable as a consequence of the inherent nonlinearity of the pulse modulator. Delfeld and Murphy (1961) suggested a technique for stability analysis based on a modified describing function [7]. But, it suffers from the limitations that are always associated with the use of a describing function. A graphical procedure suggested by Polok (1961) is applicable only to the first order system [21]. And a sufficient condition for asymptotic stability in the large suggested by Kodata and Bourne (1961) [4] is, in general, applicable to those PWM control systems in which the linear process actuated by the modulator can be characterised by a transfer function that has neither multiple poles nor complex poles. Again a sufficient condition for asymptotic stability in the large suggested by Murphy and Wu (1964) [8] is applicable in general to the lumped-parameter, single loop PWM feedback control systems that are stationary and linear except for the modulator. Skoog and Blankenship (1969) [16]
developed a sufficient condition for asymptotic stability in the large for PWM feedback systems using Lipschitz continuous operator. Although all these stability studies did help in analysing the oscillations in the PWM feedback systems they hardly could be used as design tools.

2.4 Controller synthesis technique in PWM control system

Though design of PWM systems was attempted by Nelson (1961) through Lyapunov's second method [20, 22] using the model suggested by Andeen, his approach was limited by the assumption that pulse width is so small in every sampling interval that the actual pulses could be replaced by impulse functions. This approach could not meet the design requirement as it failed for large inputs. Design of PWM systems through optimisation was attempted by many (Kirk-1967 [14], and Bekey-1968 [29]) using Pontryagin maximum principle which required the solution of nonlinear two point boundary value problems. The problem of optimisation of PWM control system became difficult due to the nonlinearity inherent in the pulse width modulation of the control signal. Although Kulher and Yeh (1976) [15] considered the problem by optimisation of a class of PWM control systems via an extended maximum principle, the derivation became very complicated because of the inherent nonlinearity of PWM signals. In 80's a set of block-pulse
function has been used by many authors to analyse and identify linear systems (e.g. Chen, 1981 [31], Palinisamy and Bhattacharya, 1981 [32]). This set of functions was applied (e.g. Chen and Wu, 1987, [13]) first to analyse a certain class of PWM systems and then to design the optimal control gain under minimisation of a given quadratic performance index. The solution, however, appears to be an openloop one and would be valid mostly for a regulator problem where the input is defined precisely. However, Bernard Friedland (1976) modelled a linear system [5] for PWM control where he developed a simple, but approximate, linear relationship between the pulse width and a suitably defined state vector which could be used for the purpose of designing a control system. This technique has been approximated to a linear control law through small signal approximation and hence linear law does not ensure performance for large excursions of signal input. The average value of PWM signal over the sampling interval does not ensure the validity of linear model solution at sampling instants. Hence, the inter-sample ripple may lead to instability of the system specially where the plant is not open loop stable.
2.5 General discussion

So going by the existing literature, it is observed that most of PWM control systems were studied by transform methods with various modifications that were found necessary and were concerned primarily with the analysis of the existing closed loop systems rather than design problems. The state space methods were introduced in mid-sixties and it was followed by the use of these methods for pulse width modulated control system investigations mostly related to stability studies and optimisation studies using Pontryagin maximum principle. However, all these techniques are felt not very adequate to handle the problems related to PWM control system design. Hence further research is warranted to find simple practical approach to handle PWM class of control system.