Chapter 6

Effect of oblique electric field on the growth rate of Kelvin-Helmholtz instability.

6.1 Introduction

Kelvin–Helmholtz Instability occurs when we consider the character of the equilibrium of a stratified heterogeneous poorly conducting fluid in which different layers are in relative motion. The most important case is when two superposed fluids flow one over the other with a relative horizontal velocity, the instability of the plane interface between the two fluids when it occurs in this instance, is known as "Kelvin-Helmholtz Instability". The KHI of a Newtonian incompressible poorly conducting fluid in a composite layer bounded by a densely packed poorly conducting fluid saturated porous lining on one side and another side by an impermeable rigid surface in the presence of oblique electric field is investigated in this chapter. This can occur when velocity shear is present at the interface within a continuous fluid or when there is sufficient velocity difference across the interface between two poorly conducting fluids.

The stability analysis can be used to predict the onset of instability and transition from
laminar to turbulent flow in fluids of different densities moving at various speeds. The dynamics of two fluids of different densities when a small disturbance such as a wave is introduced at the boundary connecting the fluids was studied by Helmholtz[130]. If surface tension can be ignored, and for some short enough wavelengths, two fluids are in parallel motion with different velocities and densities yielded an interface that is unstable for all speeds. The existence of surface tension stabilizes the short wavelength instability however, and theory then predicts stability until a velocity threshold is reached. The theory with surface tension included broadly predicts the onset of wave-formation in the important case of wind-over-water. Also the study of this instability becomes applicable to inertial confinement fusion and plasma-Beryllium interface. The KHI in a composite layer differs from the KHI in two fluid layers. The KHI arises when two uniform fluids, separated by a horizontal boundary, are in relative motion. Because of its relevance to astrophysical, geophysical and laboratory situations, this problem has been analyzed by several authors (Shore[121], Chandrasekhar[115]). Without surface tension, this streaming is unstable no matter how small the velocity difference between the layers may be. It was shown by Kelvin[64] that the surface tension will suppress the instability if the difference in velocity is sufficiently small. From an industrial view point, the momentum transfer and the KHI in composite region provides impetus for effective design of porous bearings in lubrication process, particularly in the slider bearings and in the effective design of target in inertial fusion energy (IFE). Because of these importance the KHI is investigated in this chapter using the linear stability analysis following Chandrasekhar[115]. Chandrasekhar[115] discussed the effects of surface tension, variable density, streaming velocity, rotation and applications of magnetic field on the instability behavior. The experimental observation of KHI has been given by Francis[54]. The effect of rotation and a general oblique magnetic field on the KHI has been studied by Sharma and Srivastava[101]. The study of electrohydrodynamic (EHD) KHI of free sur-
face charges, separating two semi-infinite dielectric fluids and influenced by an electric field, has been discussed by Melcher[51].

Elhefnawy[24] studied the non-linear KHI problem under the influence of an electric field by employing the method of multiple scales. He found that the nonlinear effects may be stabilized or destabilized depending on both density and dielectric constant. Mehta and Bhatia[128] investigated KHI of two viscous, superposed, rotating and conducting fluids. Singh and Khare[106] have investigated the stability of two semi-infinite homogenous gravitating streams of infinite conductivity under uniform horizontal magnetic field and uniform rotation. Bhatia and Hazarika[97] discussed this stability problem for superposed viscous gravitating fluids. The importance of the KHI problem has been demonstrated by Benjamin and Bridges[127] who have given an excellent reappraisal of the classic KHI problem in hydrodynamics. They have shown that the problem admits of a canonical Hamiltonian formulation and obtained several new results. More recently Sharma and Kumar[102] have studied the RTI of two superposed conducting Walter’s B electroviscoelastic fluids in hydromagnetics. Allah[89] has investigated the effects of magnetic field and heat and mass transfer on the KHI of superposed fluids. To our knowledge KHI of two poorly conducting fluids in the presence of an oblique electric field has not been given any attention in spite of its importance in IFE.

We also note that the flow through porous medium has been of considerable importance in recent years particulary among geophysical fluid dynamicists and petroleum engineers. In this chapter the flow in the porous layer is governed by the Darcy equation and that in a thin fluid film is governed by Navier-Stokes equation. Following Babchin et al.,[10] and Rudraiah et al.,[84], a simple theory based on Stokes and lubrication approximations is used in this study by replacing the effect of the boundary layer with a
Beavers and Joseph[113] slip condition, with the primary objective of using porous layer to suppress the growth rate of KHI. The electrohydrodynamic Kelvin-Helmholtz instability of the interface between two uniform superposed Rivlin-Ericksen viscoelastic dielectric fluid-particle mixtures in porous medium is investigated by El-Sayed[24]. El-Dib and Matoog[136] have studied the Electrorheological Kelvin-Helmholtz instability of a fluid sheet. This work deals with the gravitational stability of an electrified Maxwellian fluid sheet shearing under the influence of a vertical periodic electric field. The field produces surface charges on the interfaces of the fluid sheet. Khalil Elcoot[23] has studied the new analytical approximation forms for non-linear instability of electric porous media. In this work, we have examined the effects of normal and horizontal electric field on the EKHI in the rectangular channel with the two layers, lower one is poorly conducting fluid bounded above by poorly conducting fluid saturated porous layer and below by rigid impermeable surface. The objective of this chapter is to predict the effects of an oblique electric field on electrohydrodynamic KHI with porous layer.

To achieve this objective we plan this chapter as follows. In section 6.2 we derive the basic equation related to relevant to this work. Section 6.3 is devoted to the basic state, in this section we assumed the flow is initially static and derived the initial electric potential, distribution of charge density. The velocity, dispersion relation for two case (normal and aline electric field) are drawn in section 6.4. The important results and conclusions are given the final section 6.5 and we predict that the normal electric field give the stabilizing effect on the interface elevation where as the aline electric field gives the infinitely small scale turbulence.
6.2 Mathematical Formulation

We consider a rectangular channel in the form of a thin film of an unperturbed thickness, $h$, filled with an viscous incompressible poorly conducting fluid called region-1. It is bounded below by a rigid surface and above by an incompressible poorly conducting fluid saturated densely packed pours layer of large extent $H$, called region-2 as shown in Figure2.2. The electric field is applied along and/or normal to flow. The shear produced by the motion of fluids in regions 1 and 2 and also the resistance offered by the densely packed fluid saturated layer sets up Kelvin-Helmholtz instability (KHI). To investigate this KHI, we consider a two dimensional rectangular channel with the coordinate system $(x,y)$ as shown in Figure(2.2) with $x$-axis parallel to the film, $y$-axis normal to it and $\eta(x,y)$ is the perturbed interface between the fluid in the shell as well as in the porous layer.

The basic equations, for a clear fluid in region-1, are the modified Navier-Stokes equations, modified in the sense of including electric field effect (see Stokes[58] and Rudraiah and Kalal[84]). Those for porous region-2 are governed by the modified Darcy equation (see chandrashekara and Rudraiah[31] as given below:

Region-1:

Conservation of mass for an incompressible fluid:

$$\nabla \cdot \vec{q} = 0$$  \hspace{1cm} (6.2.1)

Conservation of momentum:

$$\rho_f \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} \right] = -\nabla p + \mu_f \nabla^2 \vec{q} + \rho_c \vec{E}$$  \hspace{1cm} (6.2.2)
Region-2:

Modified Darcy equation:

\[ \nabla p = -\frac{\mu}{k} \vec{Q} + \rho_e \vec{E} \quad (6.2.3) \]

where \( \vec{Q} \) is the velocity of the fluid in region-1, \( \rho_f \) and \( \mu_f \) are the density and the coefficient of viscosity of the fluid, \( \vec{E} \) the electric field, \( p \) the pressure, \( \vec{Q} \) the Darcy velocity of fluid in region-2, \( k \) the permeability of a porous medium and \( \rho_e \) is the density of charge distribution.

The conservation of energy in both the regions is given by

\[ \left[ \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T \right] = \kappa \nabla^2 T \quad (6.2.4) \]

The conservation of charges for incompressible fluid:

\[ \frac{\partial \rho_e}{\partial t} + (\vec{v} \cdot \nabla) \rho_e + \nabla \cdot \vec{J} = 0 \quad (6.2.5) \]

where \( \vec{J} \) is the conduction current density given by

\[ \vec{J} = \sigma \vec{E} \quad (6.2.6) \]

The **Maxwell Field equations** are

Gauss law is

\[ \nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \quad (6.2.7) \]

where \( \varepsilon_0 \) is the dielectric constant for free space.

In a poorly conducting fluid the induced magnetic field is negligible and also there is no
applied magnetic field, hence the Faraday’s law becomes

$$\nabla \times \vec{E} = 0$$  \hspace{1cm} (6.2.8)

That is, the electric field is conservative, so that

$$\vec{E} = -\nabla \phi$$  \hspace{1cm} (6.2.9)

where $\phi$ is the electric potential.

In a poorly conducting fluid $\sigma \ll 1$, any perturbation on it is negligible and which increases with the conduction temperature, $T_b$, such that

$$\sigma_b = \sigma_0 \left[1 + \alpha_b(T_b - T_0)\right]$$  \hspace{1cm} (6.2.10)

where $T_0$ is the atmospheric temperature, $\sigma_0$ that of $\sigma_b$ at $T = T_0$ and $\alpha_b$ is the volumetric expansion coefficient of $\sigma_b$.

### 6.3 Basic State

The fluid in this study is assumed to be highly is viscous and electrically poorly conducting, so that inertial terms become negligible compared to diffusion terms. Initially, the fluid is assumed to be quiescent with an applied uniform electric field $\vec{E}_0$ in $xy$ plane inclined at angel $\theta$. We assume the following basic state, initially the fluid is at static state that is

$$\vec{q} = (0, 0)$$  \hspace{1cm} (6.3.1)

and

$$\vec{E} = \vec{E}_0 = (E_{0x}, E_{0y})$$  \hspace{1cm} (6.3.2)
where \( E_{0x} = E_0 \cos \theta \) and \( E_{0y} = E_0 \sin \theta \), \( \theta \) is an angle of inclination of an electric field \( E_0 \), \( \vec{Q} = (0, 0) \), \( \phi = \phi_b, p = p_b, \rho_e = \rho_{eb} \) and \( \sigma = \sigma_b \).

Under these basic state Eqs. (6.2.2), (6.2.5), (6.2.6) and (6.2.7) respectively becomes:

\[
0 = -\frac{\partial p_b}{\partial x} + \rho_{eb} E_{0x} \quad (6.3.3)
\]

\[
0 = -\frac{\partial p_b}{\partial y} + \rho_{eb} E_{0y} \quad (6.3.4)
\]

\[
\nabla \cdot (\sigma_b \vec{E}_0) = 0 \quad (6.3.5)
\]

\[
\nabla \vec{E}_0 = \frac{\rho_{eb}}{\varepsilon_0} \quad (6.3.6)
\]

and

\[
\nabla \times \vec{E}_0 = 0 \quad (6.3.7)
\]

In region 2, the poorly conducting fluid is highly denser and porous layer is densely packed so that we assume that it as almost static and has uniform density and temperature distribution. From the energy equation, by assuming the fluid in a porous layer is static and is in thermal equilibrium, we get

\[
\frac{d^2 T_b}{dy^2} = 0 \quad (6.3.8)
\]

The boundaries are assumed to be conducting and isothermal so that we have the boundary conditions:

\[
T_b = \begin{cases} 
T_0 & \text{at } y = 0 \\
T_1 & \text{at } y = 1 
\end{cases} \quad (6.3.9)
\]
Solving Eq. (6.3.8) using the conditions Eq. (6.3.9) and using Eq. (6.2.9), we get,

$$\sigma_{b} = \sigma_{0}[1 + \alpha_{h}\Delta T y] \approx \sigma_{0} e^{\alpha y} \quad (6.3.10)$$

where $\alpha = \alpha_{h} \Delta T$ and $\Delta T = T_1 - T_0$.

Further, we assume that initially the frequency of charge distribution is smaller than the corresponding relaxation frequency of the electric field so that the convective current $\rho_{eb}\vec{q}$ and the time derivative of $\rho_{eb}$ are negligible compared to $\nabla \cdot (\sigma_{b}\vec{E}_0)$, in Eq. (6.2.5). Then we get

$$\frac{\partial^2 \phi_{b}}{\partial x^2} + \frac{\partial^2 \phi_{b}}{\partial y^2} + \alpha \frac{\partial \phi_{b}}{\partial y} = 0 \quad (6.3.11)$$

Satisfying the boundary conditions,

$$\phi_{b} = \begin{cases} \frac{V_{x}}{h} & \text{at } y = 0 \\ \frac{V(x-x_0)}{h} & \text{at } y = 1 \end{cases} \quad (6.3.12)$$

These conditions arise due to the embedded electrodes of different potentials at $y = 0$ and $y = h$ as shown in Figure(2.2) and permits a linear variation of $\phi_{b}$ with respect to $x$.

Solving Eq.6.3.11, using the conditions Eq. (6.3.12), after making them dimensionless using the scales $V$ for potential and $h$ for length, we get

$$\phi_{b} = x - x_0 \left( \frac{1 - e^{-\alpha y}}{1 - e^{-\alpha}} \right) \quad (6.3.13)$$

With this, using Eqs. (6.2.5)–(6.2.7), we get,

$$E_{0x} = -\frac{\partial \phi}{\partial x} = -1 \quad (6.3.14)$$
\[ E_{0y} = -\frac{\partial \phi}{\partial y} = \frac{x_0 \alpha e^{-\alpha y}}{1 - e^{-\alpha}} \]  
(6.3.15)

and

\[ \rho_{eb} = -\varepsilon_0 \frac{\partial^2 \phi_b}{\partial y^2} = -\frac{x_0 \varepsilon_0 \alpha^2 e^{-\alpha y}}{1 - e^{-\alpha}} \]  
(6.3.16)

### 6.4 Perturbed State

In this section, we super impose an infinitely small perturbation on the basic state given in section 6.3 as follows:

\[ \bar{q} = (u, v) \]  
(6.4.1)

and

\[ \bar{E} = \bar{E}_0 + \bar{E}' \]  
(6.4.2)

where \( E_x = E_{0x} + E'_x \) and \( E_y = E_{0y} + E'_y \) and \( \bar{Q} = (Q_x, Q_y) \), \( \phi = \phi_b + \phi' \), \( p = p_b + p' \), \( \rho_c = \rho_{eb} + \rho'_e \).

Using these perturbed quantities in the Eqs. (6.2.1), (6.2.2), (6.2.3) and (6.2.5), and linearizing these equations (neglecting the primes) using the basic state discussed in section 6.3, we get,

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  
(6.4.3)

\[ \frac{\partial p}{\partial x} = \mu_f \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho_c E_{0x} + \rho_{eb} E_x \]  
(6.4.4)

\[ \frac{\partial p}{\partial y} = \mu_f \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho_c E_{0y} + \rho_{eb} E_y \]  
(6.4.5)

\[ \frac{\partial p}{\partial x} = -\frac{\mu}{k} Q_x + \rho_c E_{0x} + \rho_{eb} E_x \]  
(6.4.6)

\[ \frac{\partial p}{\partial y} = -\frac{\mu}{k} Q_y + \rho_c E_{0y} + \rho_{eb} E_y \]  
(6.4.7)
and

\[
\frac{\partial \rho_e}{\partial t} + v \frac{\partial \rho_{eb}}{\partial y} + \sigma_b \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right) = 0 \tag{6.4.8}
\]

Making Eqs. (6.4.3) to (6.4.8) dimensionless using the scales, \( \delta h \) for length, \( \delta h \) for pressure, \( \frac{\delta h^2}{\mu_f} \) for velocity, \( \frac{\mu_f}{h^3 \delta^2} \) for time, \( \frac{V}{h} \) for electric field, \( \frac{\varepsilon_0 V}{h} \) for density of the charges and \( V \) for electric potential, take the form

\[
\frac{\partial p}{\partial x} = \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + w_1 (\rho_e E_{0x} + \rho_{eb} E_x) \tag{6.4.9}
\]

\[
\frac{\partial p}{\partial y} = \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + w_1 (\rho_e E_{0y} + \rho_{eb} E_y) \tag{6.4.10}
\]

\[
\frac{\partial p}{\partial x} = -\frac{1}{\sigma_p^2} Q_x + w_1 (\rho_e E_{0x} + \rho_{eb} E_x) \tag{6.4.11}
\]

\[
\frac{\partial p}{\partial y} = -\frac{1}{\sigma_p^2} Q_y + w_1 (\rho_e E_{0y} + \rho_{eb} E_y) \tag{6.4.12}
\]

and

\[
\frac{\partial \rho_e}{\partial t} + v \frac{\partial \rho_{eb}}{\partial y} + \sigma_b \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right) = 0 \tag{6.4.13}
\]

where \( w_1 \) is an electric number and \( \sigma_p \) is the porous parameter.

In addition these equation Eqs. (6.2.7) and (6.2.8) will reduce to the form:

\[
\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = \frac{\rho_e}{\varepsilon_0} \tag{6.4.14}
\]

\[
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \tag{6.4.15}
\]

### 6.5 Dispersion Relation

In this section we derive the dispersion relation of the interface elevation of KHI. Our interest here is to find the growth rate of a small periodic perturbations of the interface.
For this, we assume all the perturbed quantities to vary in the form

\[
f(x, y, t) = f(y)e^{ix+nt}
\]

where \(l\) denotes the wave number and \(n\) is the frequency or growth rate. Using Eq. (6.5.1) in the Eqs. (6.4.3) and (6.4.9) to (6.4.15) takes the form

\[
Dv + ilu = 0 \quad (6.5.2)
\]

\[
ilp = (D^2 - l^2)u + w_1(\rho_c E_{0x} + \rho_{eb} E_x) \quad (6.5.3)
\]

\[
Dp = (D^2 - l^2)v + w_1(\rho_c E_{0y} + \rho_{eb} E_y) \quad (6.5.4)
\]

\[
ilp = -\frac{1}{\sigma_p^2}Q_x + w_1(\rho_c E_{0x} + \rho_{eb} E_x) \quad (6.5.5)
\]

\[
Dp = -\frac{1}{\sigma_p^2}Q_y + w_1(\rho_c E_{0y} + \rho_{eb} E_y) \quad (6.5.6)
\]

\[
n\rho_c + vD\rho_{eb} + \sigma_0 (ilE_x + DE_y) = 0 \quad (6.5.7)
\]

\[
ilE_x + DE_y = \frac{\rho_c}{\epsilon_0} \quad (6.5.8)
\]

\[
ilE_y - DE_x = 0 \quad (6.5.9)
\]

Eliminating the pressure term between Eqs. (6.5.3) and (6.5.4) and using Eq. (6.5.2), we obtain

\[
(D^2 - l^2)^2v - ilw_1 \cos \theta (D\rho_c E_{0x} + D\rho_{eb} E_x) = 0 \quad (6.5.10)
\]

Similarly eliminating pressure terms in Eqs. (6.5.5) and (6.5.6) and using Eq. (6.5.2), we get

\[
(D^2 - l^2)Q_y - ilw_1\sigma_p^2 \cos \theta (D\rho_c E_{0x} + D\rho_{eb} E_x) = 0 \quad (6.5.11)
\]
Further using $\vec{E} = -\nabla \phi$ Eqs. (6.5.7), (6.5.10) and (6.5.11) takes the form

$$n\rho_e + vD\rho_{eb} - \sigma_b(D^2 - t^2)\phi = 0 \quad (6.5.12)$$

### 6.5.1 Effect of Transverse electric field

In this section we derive the dispersion relation under the influence of applied electric field normal to the flow. In this case the oblique angle $\theta = \frac{\pi}{2}$, hence the Eqs. (6.5.10) and (6.5.11) respectively becomes;

$$\begin{align*}
(D^2 - t^2)\psi &= 0 \quad (6.5.13) \\
(D^2 - t^2)Q_y &= 0 \quad (6.5.14)
\end{align*}$$

These equations have to be solved using the proper boundary condition. The crux of the problem is to specify the proper boundary and surface condition on the velocity.

**Velocity Boundary Conditions:**

The no-slip boundary condition at the rigid surface:

$$u = v = 0 \quad \text{at} \quad y = 0 \quad (6.5.15)$$

this, using the continuity equation, may be written as

$$v = Dv = 0 \quad \text{at} \quad y = 0 \quad (6.5.16)$$

The interface equation is

$$y = h + \eta \quad (6.5.17)$$
where \( \eta(x,t) \) is the departure of the interface from the unperturbed state at \( y = h \) as shown in Figure(2.2).

\[
v = v_0 \quad \text{at} \quad y = 1 \quad \text{(6.5.18)}
\]

The BJ-slip condition:

\[
\frac{\partial v}{\partial y} = -\alpha \sigma (v_0 - Q_y) \quad \text{at} \quad y = 1 \quad \text{(6.5.19)}
\]

and

\[
Q_y = v_0 \quad \text{at} \quad y = 1 \quad \text{(6.5.20)}
\]

where \( \alpha \) is the slip parameter and \( v_0 \) is the slip velocity at the interface \( y = 1 \).

**Interface conditions:**

**The dynamic surface condition:** The normal stress at the interface is given by the combination of the stress gradient, surface tension and viscosity terms.

\[
p = -\delta \eta - \gamma \frac{\partial^2 \eta}{\partial x^2} + \mu_f \frac{\partial v}{\partial y} \quad \text{at} \quad y = 1 \quad \text{(6.5.21)}
\]

where \( \delta \) is the stress gradient and \( \gamma \) is the surface tension.

**The kinematic surface condition:**

\[
v = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \quad \text{at} \quad y = 1 \quad \text{(6.5.22)}
\]

The solution of Eqs. (6.5.13) and (6.5.14) satisfying the boundary conditions Eqs. (6.5.15) to (6.5.18) is of the form

\[
v = (A_1 + A_2 y) \cosh ly + (A_3 + A_4 y) \sinh ly \quad \text{(6.5.23)}
\]

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and

\[ Q_y = B_1 \cosh ly + B_2 \sinh ly \]  \hspace{1cm} (6.5.24)

where \( l \) is wave number and the constants \( A_i, i = 1, 2, 3, 4 \) and \( B_j, j = 1, 2 \) are interrelated in view of the boundary conditions Eqs. (6.5.15) to (6.5.18) and are given by

\[
\begin{align*}
    A_1 &= 0, \\
    A_2 &= \frac{IV_0 \cosh l + IV_0 \sinh l - K \alpha \sigma_p \sinh l + V_0 \alpha \sigma_p \sinh l}{l(\cosh^2 l - 2 \sinh^2 l)}, \\
    A_3 &= -\frac{IV_0 \cosh l + IV_0 \sinh l - K \alpha \sigma_p \sinh l + V_0 \alpha \sigma_p \sinh l}{l(\cosh^2 l - 2 \sinh^2 l)}, \\
    A_4 &= -\frac{IV_0 \sinh l - \alpha \sigma_p (V_0 - K)(\sinh l - \cosh l)}{V_0 \sinh 2l}, \\
    B_1 &= -\frac{V_0 \cosh 2l}{\cosh 2l \sinh l - \sinh 2l}, \\
    B_2 &= \frac{V_0 \cosh 2l}{\cosh 2l \sinh l - \sinh 2l}.
\end{align*}
\]

and

Using Eqs. (6.5.7) and (6.5.8)

\[ (D^2 - l^2)\phi = -\frac{vD\rho_e b}{\epsilon_0 n + \sigma_b} \]  \hspace{1cm} (6.5.25)

Equation (6.5.25) can be solved for \( n \), which is the frequency or growth rate of the interface and it is called the dispersion relation, as follows;

\[ n = -\frac{vD\rho_e b + \sigma_b (D^2 - l^2)\phi}{\epsilon_0 (D^2 - l^2)\phi} \]  \hspace{1cm} (6.5.26)

The dispersion relation given by Eq. (6.5.26) pertains to linear KHI of an incompressible viscous poorly conducting fluid in a thin film bounded below by a rigid surface and above by a densely packed poorly conducting fluid saturated porous layer. This dispersion relation is similar to the one given by Brown[14].
6.5.2 Effect of Horizontal electric field

In this section we derive the dispersion relation under the influence of applied electric field 
alone or parallel to the flow. In this case the oblique angle \( \theta = 0 \). This aligned electric 
field (i.e. electric field parallel to \( x \) axis) offer a rigidity to the fluid and hence produce a 
tension. The effect of this tension is to stretch the lines force resulting in oscillations. In 
this case Eqs. (6.5.10) and (6.5.11) becomes;

\[
(D^2 - l^2)^2 v - il w_1 (D \rho_e E_{0x} + D \rho_{cb} E_x) = 0 \tag{6.5.27}
\]

\[
(D^2 - l^2) Q_y - il w_1 \sigma_p^2 (D \rho_e E_{0x} + D \rho_{cb} E_x) = 0 \tag{6.5.28}
\]

The solution of Eqs. (6.5.27) and (6.5.28) satisfying the boundary conditions Eqs. (6.5.15) 
to (6.5.18) is of the form

\[
v = (A_1 + A_2 y) \cosh ly + (A_3 + A_4 y) \sinh ly + f(y) \tag{6.5.29}
\]

and

\[
Q_y = B_1 \cosh ly + B_2 \sinh ly + g(y) \tag{6.5.30}
\]

where \( l \) is wave number, \( A_i, i = 1, 2, 3, 4 \) and \( B_j, j = 1, 2 \) are constants interrelated in 
view of the boundary conditions Eqs. (6.5.15) to (6.5.18),

\[
f(y) = \frac{il w_1 e^{-\alpha y}}{(\alpha^2 - l^2)^2} \tag{6.5.31}
\]

and

\[
f(y) = \frac{il w_1 e^{-\alpha y}}{(\alpha^2 - l^2)} \tag{6.5.32}
\]
From Eq. (6.5.26) we have the frequency of growth rate $n$:

$$n = \frac{D\phi D\sigma_b - vD\rho_{eb} + \sigma_b(D^2 - l^2)\phi}{\epsilon(D^2 - l^2)\phi} \quad (6.5.33)$$

The dispersion relation given by Eq. (6.5.33) pertains to linear KHI of an incompressible viscous poorly conducting fluid in a thin film bounded below by a rigid surface and above by a densely packed poorly conducting fluid saturated porous layer. The direct effect of an electric field is clearly seen in Eq. (6.5.33), through the term in the denominator besides there is an indirect effect from the modification velocity and Darcy velocity at $y = h$ given by Eqs. (6.5.29) and (6.5.30).

### 6.6 Results and Conclusions

The growth rate of KH instability in an electrically poorly conducting fluid in the presence of an electric field inclined at an angle $\theta$ to the $x$-axis is investigated using the general dispersion relation Eq. (6.5.26) with object of controlling the undesirable effect of shell breakup. When we set both $E_{0x}$ and $E_{0y}$ are zero this dispersion relation will reduces to the form given by Brown[14] for the hydrodynamic case as follows:

$$n = \frac{(\delta - \gamma \lambda^2)V\lambda^2}{\mu_f(2\lambda^2DV - D^2V)} \quad (6.6.1)$$

If we set $E_{0y} = 0$, we obtain the dispersion relation given by Eq.(6.5.16) for the aligned electric field. In this case the expression for the velocity is given by Eqs. (6.5.29) and (6.5.30) and it is a function of hyperbolic and trigonometric functions, exhibits oscillatory nature depending on the strength of an electric field $E_{0x}$. In this case $n$ is evaluated for different values of $w_e$ and $l$ and the results are depicted graphically in Figure(6.1). We found that the growth rate increases for short period and then becomes stable for all values
Figure 6.1: Graph of $n$ versus $l$ for different values of $w_e$

of $\lambda$ and $w_e=2,20,200$ and 2000.

If we set $E_{0x} = 0$, we obtain the dispersion relation given by Eq. (6.5.33) for the transverse electric field. In this case the expression for the velocity is given by Eqs.(6.5.29) and (6.5.30) and it is a function of hyperbolic analogous to hydrodynamic case. The dispersion relation $n$ is evaluated for different values of $w_e$ and $l$ and the results are depicted graphically in Figure(6.2). We found that the interface becomes stable for all values of $\alpha$ and $w_e > 1$ in contrast to the instability that exists in the hydrodynamic case.
Figure 6.2: Graph of $n$ versus $l$ for different values of $w_e$. 
In general, we found that the effect of the transverse or aligned electric field is to suppress the KH instability incase of a thin film \((h < 1)\) even for small strength electric fields. The increase in the strength of the electric field causes the growth rate to exhibit both negative and positive values when the layer thickness is small. Physically this behavior may be attributed to the fact that the high electric field and shear produced by the fluid saturated porous layer may amplify the disturbance in a manner that produces an inflectional profile which is the characteristic feature of the instability phenomena. It is interesting to note that Electric force balances the viscous force \((We = 1)\) the interface is stable in case of transverse electric field for all layer thickness but same also true for aligned electric field only for \(h < 10\). We observe that the peaks in the dispersion in Figures(6.1) and (6.2) are attributed to the resonance effect due to the balance of shear terms with applied electric field. Finally, we conclude that the surface instability may be controlled by a suitable strength of electric field and porous layer thickness for a given surface tension and stress gradient. We note that, in case of porous layer large thickness without without performing a non-linear analysis it is rather difficult to point out exactly how an electric field controls the KH instability.