Chapter 4
Electrohydrodynamic Kelvin-Helmholtz Instability

4.1 Introduction

The Kelvin-Helmholtz instability occurs when there is a relative motion between different layers. The fundamental of the instability lies in the fact that the pressure perturbation does work on the interface between the two layers. The instability investigation of the interface between two layers of different velocities is important in several cosmical and laboratory situations, such as generation of Inertial Fusion Energy, meteor entering the Earth’s atmosphere, when the air is blown over mercury, wind blowing over the ocean, in the theory of solar winds, studies of the Earth’s magnetosphere instability, the stream structure of solar corona and the helical wave motion observed in ionized comet tails, etc. The study of the electrohydrodynamic Kelvin-Helmholtz instability for an interface separating two semi-infinite poorly conducting and dielectric streaming fluids influenced by an electric field has been discussed by Lyon[49], El-Shehawey[24] and El-Shehawey and Abd El-Gawad[25]. Similar studies are made by Malik and Singh[120] and Deysikder and Das[116] for highly conducting incompressible fluids in the presence of horizontal external uniform magnetic fields. Nonlinear developments of Kelvin-Helmholtz instability in electrically non-conducting incompressible fluids have been investigated by Drazin[93]
and Nayfeh and Saric[5] for the case where the amplitude of an unstable wave is uniform in space and growing only in time.

This study has later been extended by Weissman[76] for packets of waves in which the amplitude is a function of space as well as time. Elhefnawy[4] has studied the nonlinear electrohydrodynamic Kelvin-Helmholtz instability under the influence of an electric field. In this chapter, we study the linear instability of Kelvin-Helmholtz problem for poorly conducting fluids under the influence of an electric and magnetic fields. The instability of the accelerated interface between two superimposed liquids has been considered by many investigators(see [49], [24], [25] and [120]). An assumption usually adopted is that both liquids are inviscid and irrotational. Despite numerous studies on theoretical and experimental aspects of parallel flow stability, many basic issues remain unresolved. The initiation of instability under quiescent conditions is found to be in general agreement with the results of linear theory. The stability of a liquid layer subjected to a unidirectional force is a problem of long standing. Lamb[38] considers the stability of a horizontal interface between two semi infinite, inviscid incompressible fluids in a gravitational field normal to their interface. He also considers the effect of surface tension which is to stabilize the motion for wavelengths shorter than a cutoff wavelength $l$.

Taylor[126] investigated the instability of a horizontal liquid layer of constant, finite thickness when submerged in a lighter fluid and accelerated in a direction perpendicular to its two bounding surfaces. Additional effects on the instability of superposed fluids, e.g. variable density in fluids, streaming, viscosity, surface tension, rotational motion of the fluids and compressibility, are reported by Bellman and Pennington[11], Chandrasekhar[115], Drazin[20], Gillis and Kaufman[47], Anliker and Pi[72] and Chang and Russell[44]. The effect of a constant shear on the instability of a liquid is studied by
Rajappa and Chang[88]. They studied the Taylor-Helmholtz instability between a liquid of constant shear and a vacuum, and it is found that a more stable flow will be resulted from the shear if the speed of the liquid increases as depth increases.

The recent studies concerning shear flows are investigated by Maslowe and Thompson[118], Shivamoggi[13]. Satyanarayana, Lee and Huba[119] and Akhatov[46] among others for a stratified shear layer, and by Wang and Pritchett[137], Subbiah and Jain[75], and Vyas and Srivastava[19] among others for compressible stratified shear flows. It is believed that the shear induced oscillations in such flows are of significance in a variety of natural phenomena. These include the generation of clear air turbulence in the atmosphere and the propagation of waves in the oceanic thermocline. For an excellent review, see Landau and Lifshitz[17]. On the other hand, electrohydrodynamics is the study of fluid motions driven by external electrostatic fields. Melcher and Taylor[53] summarize early developments in the area while surveys by Melcher[52] provide somewhat more recent perspectives. Melcher’s Continuum electromechanics[99] contains a lucid development of theoretical aspects. Problems of electrohydrodynamic stability have been extensively treated by many authors, e.g. Melcher[99], Woodson and Melcher[40], Taylor and McEwan[35], Kim et al.[60] and Mohamed et al.,[2]. In the linear stability theory, the tangential field has a stabilizing effect[99], while the normal field has a destabilizing influence[99]. Electrohydrodynamics (EHD) is therefore the dual case of magnetohydrodynamics (MHD), where we have conducting liquids with large electrical current, negligible electric fields and strong magnetic fields. In this paper, we consider the conditions for interfacial electrohydrodynamic stability between incompressible poorly conducting viscous fluid bounded above poorly conducting fluid saturated porous layer under the influence of both electric and magnetic fields to the interface between the two layers.
To our knowledge, no previous studies have been made to investigate the combined effects of electric and magnetic fields on the electrohydrodynamic KHI of a poorly conducting fluid. The addition of the electric and magnetic fields to the previous stability problems has many applications in plasma physics, astrophysics, geophysics, chemical engineering and industry. The plan of this work, which is of Kelvin-Helmholtz instability type, is outlined as follows: In section 4.2, we give a description for the problem including the basic equations of fluid mechanics and electrodynamics governing the motion for our model. In section 4.3, we write down the equations of motion suitable to derive the velocity distribution using the appropriate surface and boundary conditions. In section 4.4, we obtain the dispersion relation and in section 4.5, we discuss the stability analysis in the presence of the electric and magnetic field effects.

4.2 Mathematical Formulation of the Problem

We consider a target shell (see Figure(2.2)) in the form of a thin film of an unperturbed thickness, $h$, filled with an incompressible poorly conducting fluid called region-1. It is bounded below by a rigid surface and above by an incompressible poorly conducting fluid saturated in a densely packed pours layer of large extent $H$ called region-2. To generate an applied electric field the lower rigid boundary is embedded with the electrodes of higher potential at $y = 0$ and the electrodes of lower potential are embedded at $y = h$ and the constant magnetic field $H_0$ is applied normal to the flow. The shear produced by the motion of fluids in regions 1 and 2 and also the resistance offered by the densely packed couple stress fluid saturated layer sets up the KHI. The assumptions of the nominal surface as in the experimental work of Beavers and Joseph(BJ)[113] and theoretical work of Rudraiah(BJR)[80] are needed to maintain laminar flows as well as to use the BJ slip condition at the interface $y = h$. The fluid in the shell sets in motion by acceleration nor-
mal to the interface and small disturbances are amplified due to shear in the pours layer of large extent $H$ as well as in the thin film of thickness $h$. This instability at the interface, by definition, is KHI. To investigate this KHI, we consider a two dimensional rectangular channel with the coordinate system $(x, y)$ as shown in Figure 2.2 with $x$-axis parallel to the film, $y$-axis normal to it and $\eta(x, y)$ is the perturbed interface between the fluid in the shell as well as in the porous layer.

The basic equations, for a clear fluid in region-1, are the modified Navier-Stokes equations, modified in the sense of including electric and magnetic field effects (see Stokes[58], Eringen[28] and [84]). Those for porous region-2 are governed by the modified Darcy equation as given below:

Region-1:
Conservation of mass for an incompressible fluid:

$$\nabla \cdot \vec{q} = 0 \quad (4.2.1)$$

Conservation of momentum:

$$\rho_f \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \mu_f \nabla^2 \vec{q} + \rho_e \vec{E} + \vec{J} \times \vec{B} \quad (4.2.2)$$

Region-2:
Modified Darcy equation (modified in the presence of electric force and Lorenz force) in a densely packed porous layer

$$\nabla p = -\frac{\mu}{k} \vec{Q} + \rho_e \vec{E} + \vec{J} \times \vec{B} \quad (4.2.3)$$

These equations have to be supplemented with the conservation of charges for incom-
pressible fluid:

\[
\frac{\partial \rho_e}{\partial t} + (\vec{q} \cdot \nabla) \rho_e + \nabla . \vec{j} = 0
\]  

(4.2.4)

where \( \vec{j} \) is the conduction current density given by

\[
\vec{j} = \sigma \vec{E}
\]  

(4.2.5)

**Maxwell field equations**

Gauss law,

\[
\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0}
\]  

(4.2.6)

where \( \varepsilon_0 \) is the dielectric constant for free space. In a poorly conducting fluid the induced magnetic field is negligible and there is only constant applied magnetic field, hence the Faraday’s law becomes

\[
\nabla \times \vec{E} = 0
\]  

(4.2.7)

That is, the electric field is conservative, so that

\[
\vec{E} = -\nabla \phi
\]  

(4.2.8)

where \( \phi \) is the electric potential. In a poorly conducting fluid since \( \sigma << 1 \), any perturbation on it is negligible and also \( \sigma \) increases with the concentration of the DT particles, \( C_b \) such that

\[
\sigma = \sigma_0 [1 + \alpha_c (C_b - C_0)]
\]  

(4.2.9)

where \( C_0 \) is the initial concentration, \( \sigma_0 \) that of \( \sigma \) at \( C = C_0 \) and \( \alpha_c \) are the volumetric expansion coefficients of \( \sigma \).

Here \( \vec{q} \) is the velocity of the fluid in region-1, \( \rho_f \) and \( \rho_p \) are the density and the coefficient
of viscosity of the fluid, \( p \) the pressure, \( \tilde{Q} \) the Darcy velocity of fluid in region-2, \( k \) the permeability of a porous medium, \( \rho_e \) the distribution of charge density and \( \vec{E} \) is the electric field.

The conservation of concentration of DT particles

\[
\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla)C = D_e \nabla^2 C \quad (4.2.10)
\]

where \( C \) is the concentration of Deuterium–Tritium(DT) particles in the Inertial Fusion Confinement (IFC) and \( D_e \) is the diffusion coefficient. In region 2, the poorly conducting fluid is saturated in a densely packed porous layer so that we assume it as creeping flow and which is in relative motion with the fluid in region 1 and has uniform density distribution. From the concentration equations, assuming the fluid in a porous layer and in clear fluid layer are in advection concentration, we get

\[
\frac{d^2 C_b}{dy^2} = 0 \quad (4.2.11)
\]

The boundaries are assumed to be conducting and isothermal so that we have the boundary conditions:

\[
C_b = \begin{cases} 
C_0 & \text{at } y = 0 \\
C_1 & \text{at } y = 1 
\end{cases} \quad (4.2.12)
\]

Solving, Eq. (4.2.11) using the conditions Eq. (4.2.12) and using Eq. (4.2.9), we get,

\[
\sigma = \sigma_0 \left[ 1 + \frac{\alpha_c \Delta C}{h} y \right] \approx \sigma_0 e^{\alpha y} \quad (4.2.13)
\]

where \( \alpha = \alpha_c \Delta C \) and \( \Delta C = C_1 - C_0 \).

Further, we assume the frequency of charge distribution is smaller than the corresponding
relaxation frequency of the electric field so that the convective current \( \rho_e \vec{v} \) and the time derivative of \( \rho_e \) are negligible compared to \( \nabla \cdot (\sigma E) \), in Eq. (4.2.6). Then we get

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \alpha \frac{\partial \phi}{\partial y} = 0
\]  

(4.2.14)

Satisfying the boundary conditions,

\[
\phi = \begin{cases} 
\frac{V_x}{h} & \text{at } y = 0 \\
\frac{V(x-x_0)}{h} & \text{at } y = 1
\end{cases}
\]  

(4.2.15)

These conditions arise due to the embedded electrodes of different potentials at \( y = 0 \) and \( y = h \) as shown in Figure2.2 and permits a linear variation of \( \phi \) with respect to \( x \).

Solving Eq. (4.2.14), using the conditions Eq. (4.2.15), after making them dimensionless using the scales \( V \) for potential and \( h \) for length, we get

\[
\phi = x - x_0 \left( \frac{1 - e^{-\alpha y}}{1 - e^{-\alpha}} \right)
\]

(4.2.16)

With this, using Eqs. (4.2.6)–(4.2.8), we get,

\[
E_x = -\frac{\partial \phi}{\partial x} = -1
\]

(4.2.17)

\[
E_y = -\frac{\partial \phi}{\partial y} = \frac{x_0 \alpha e^{-\alpha y}}{1 - e^{-\alpha}}
\]

(4.2.18)

and

\[
\rho_e = -\varepsilon_0 \frac{\partial^2 \phi}{\partial y^2} = -\frac{x_0 \varepsilon_0 \alpha^2 e^{-\alpha y}}{1 - e^{-\alpha}}
\]

(4.2.19)
From these equations we obtain,

$$\rho_e E_x = \frac{x_0 \varepsilon_0 \alpha^2 e^{-\alpha y}}{m} \quad (4.2.20)$$

and

$$\rho_e E_y = -\frac{x_0^2 \varepsilon_0 \alpha^3 e^{-2\alpha y}}{m} \quad (4.2.21)$$

where \( m = 1 - e^{-\alpha} \)

### 4.3 Velocity Distribution

In this section we find the velocity distribution by solving the momentum Eq. (4.2.2), Darcy Eq. (4.2.3) with suitable boundary conditions. To study the problem of Electrohydrodynamic KHI posed in this paper, we consider a steady, unidirectional and fully developed flow of poorly conducting fluid in region-1 and a poorly conducting fluid saturated porous layer in region-2, with \( \eta(x, t) \) as the elevation of the interface at \( y = h \). The required equations are obtained using the Stoke and lubrication approximation as given chapter 2 section 2.5 and following [80] and [125]. These assumptions of the unidirectional flow in both clear poorly conducting fluid flow in thin film as well as in the fluid saturated porous layer enabled us to use the creeping flow approximations which allow us to neglect certain terms in the perturbation equations to obtain linear equations for the interface elevation. With these assumptions, the basic Eqs. (4.2.2), (4.2.1) and (4.2.3) respectively, become

**Region-1:**

$$0 = -\frac{\partial p}{\partial x} + \mu_j \frac{\partial^2 u}{\partial y^2} + \rho_e E_x - \sigma \mu_0^2 H_0^2 u \quad (4.3.1)$$

$$0 = -\frac{\partial p}{\partial y} + \rho_e E_y \quad (4.3.2)$$
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4.3.3)
\]

**Region-2:**

\[
\frac{\partial p}{\partial x} = -\frac{\mu}{k} Q_x + \rho_e E_x - \sigma \mu_0^2 H_0^2 Q_x \quad (4.3.4)
\]

\[
0 = Q_y + \rho_e E_y \quad (4.3.5)
\]

Making these equations dimensionless using the scales, \( h \) for length, \( \delta h \) for pressure, \( \frac{\delta h^2}{\mu_f} \) for velocity, \( \frac{\mu \gamma}{h^3 \delta^2} \) for time, \( \frac{V}{h} \) for electric field, \( \frac{\varepsilon_0 V}{h} \) for density of the charges and \( V \) for electric potential, take the form

\[
\frac{\partial^2 u}{\partial y^2} - M^2 u = \frac{\partial p}{\partial x} - mw_e e^{-\alpha y} \quad (4.3.6)
\]

and

\[
Q_x = -\frac{1}{M^2 + \sigma_p^2} \left[ \frac{\partial p}{\partial x} - w_e m e^{-\alpha y} \right] \quad (4.3.7)
\]

where \( M^2 = \mu_0 H_0 h \sqrt{\frac{\sigma}{\mu}} \), is the Hartman’s number, \( w_e = \frac{\varepsilon_0 V^2 \alpha^2 x_0}{\delta h^3} \), is the electric number which is the ratio of electric energy to kinetic energy of the system and \( \sigma_p = \frac{h}{\sqrt{k}} \), is the porous parameter.

These equations have to be solved using the following non-dimensional boundary and surface conditions on velocity:

The no-slip boundary conditions at the rigid surface are

\[
u = v = 0 \quad at \quad y = 0 \quad (4.3.8)
\]

The Saffman [21] slip condition at the interface between porous layer and clear fluid layer
is

\[
\frac{\partial u}{\partial y} = -\alpha_1 \sigma_p (u_b - Q_z) \quad \text{at} \quad y = 1
\]  

(4.3.9)

where \( \sigma_p = \frac{h}{\sqrt{k}} \) is the porous parameter and \( \alpha_1 \) is the slip coefficient.

The dynamic surface condition, at the interface, is

\[
p = -\eta[1 - w_e] - \frac{1}{B} \frac{\partial^2 \eta}{\partial x^2} \quad \text{at} \quad y = 1
\]  

(4.3.10)

where \( B = \frac{\delta h^2}{\gamma} \) is the Bond number.

The kinematic surface condition, at the interface, is

\[
v = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \quad \text{at} \quad y = 1
\]  

(4.3.11)

For the linear case the above Eq. (4.3.11) reduces to the form

\[
v = \frac{\partial \eta}{\partial t} \quad \text{at} \quad y = 1
\]  

(4.3.12)

Solution of Eq. (4.3.6), satisfying the conditions Eq. (4.3.8) and (4.3.9), is

\[
u = C_1 \cosh My + C_2 \sinh My - \frac{1}{M^2} \frac{\partial p}{\partial x} - w_e m^* e^{-\alpha y}
\]  

(4.3.13)

where

\[
C_1 = \frac{1}{M^2} \left[ \frac{\partial p}{\partial x} + w_e m^* M^2 \right],
\]

\[
C_2 = -\frac{\tanh M}{M^2} \left[ \frac{\partial p}{\partial x} - w_e m^* M^2 \right] - \frac{w_e m^* \alpha e^{-\alpha}}{\cosh M}
\]

and \( m^* = \frac{m}{\alpha^2 - M^2} \).
Using the condition \( u = u_b \) at \( y = 1 \) in Eq. (4.3.13) and using Eq. (4.3.7), we get

\[
u_b = \frac{1}{M - \alpha \sigma \tanh M} \left( k_1 \frac{\partial p}{\partial x} + k_2 w_e m^* M^2 - \frac{w_e m^* e^{-\alpha} \sinh M}{M} \right)
\]

(4.3.14)

where

\[
k_1 = \frac{\cosh M - 1}{M} + \tanh M \left( \frac{\sinh M}{M} + \frac{\alpha \sigma}{M^2 + \sigma_p^2} \right)
\]

\[
k_2 = \frac{\cosh M}{M} + \tanh M \left( \frac{\sinh M}{M} + \frac{\alpha e^{-\alpha}}{M^2} \right).
\]

From Eqs. (4.3.7) and (4.3.14), we get

\[
u_b - Q_x = \frac{1}{a_1} \left( k_1 \frac{\partial p}{\partial x} + k_2 w_e m^* M^2 - k_3 \right) - \frac{1}{a_2} \left[ \frac{\partial p}{\partial x} - w_e m^* e^{-\alpha y} \right]
\]

(4.3.15)

where \( k_3 = \frac{w_e m^* e^{-\alpha} \sinh M}{M} \), \( a_1 = M - \alpha \sigma \tanh M \) and \( a_2 = M^2 + \sigma_p^2 \).

### 4.4 Dispersion Relation

Integrating Eq. (4.3.3) from 0 to 1 and using the conditions Eq. (4.3.8), we get

\[
v(1) = \int_0^1 \frac{\partial u}{\partial x} \, dy
\]

(4.4.1)

This, using Eq. (4.3.13) and integrating, we get

\[
v(1) = \left\{ \frac{1}{3} - \left[ \frac{k_3}{3M} + \frac{\alpha^2 \sigma_p^2 (\cosh M - 1)}{M \alpha \sigma_p \cosh M} k_4 + \frac{\alpha^2 \sigma_p^2 (\cosh M - 1)}{M \cosh^2 M} k_5 \right] \right\} \frac{\partial^2 p}{\partial x^2}
\]

(4.4.2)

where

\[
k_3 = \frac{M^2 - 1 + 3(2 \sinh M - \tanh M)}{M^2},
\]

\[
k_4 = \frac{(1 + \alpha \sigma_p) \tanh M - M}{(M^2 + \sigma_p^2)(M - \alpha \sigma_p \tanh M)}
\]

and
\[ k_5 = \frac{M \cosh M (\cosh M - 1) + \sinh M^2}{M - \alpha \sigma_p \tanh M} \]

From Eq. (4.3.12), using the normal mode solution of the form

\[ \eta = \eta_0 e^{ix + nt} \]  \hspace{1cm} (4.4.3)

and using Eqs. (4.3.10) and (4.4.2), we get the dispersion relation in the form

\[ n = \left\{ \frac{1}{3} - \left[ \frac{k_3}{3M} + \frac{k_6}{M} \left( \frac{k_4}{\alpha \sigma_p} + \frac{k_5}{\cosh M} \right) \right] \right\} l^2 \left( 1 - w_e - \frac{l^2}{B} \right) \]  \hspace{1cm} (4.4.4)

where \( n \) is the growth rate in the presence of porous media, electric and magnetic fields and \( k_6 = \frac{\alpha^2 \sigma_p^2 (\cosh M - 1)}{\cosh M} \).

In the absence of electric field, couple stress, porous layer and laser radiation that is as \( w_e \to 0, M \to 1 \) and \( \sigma_p \to 0 \), the dispersion relation Eq. (4.4.4) reduces to

\[ n_b = \frac{l^2}{3} \left( 1 - \frac{l^2}{B} \right) \]  \hspace{1cm} (4.4.5)

which coincides with the expression given by Babchin et.al.,[10] and we call it as the classical growth rate. Then Eq. (4.4.4) can be written as

\[ n = n_b - \left\{ \frac{M^2 - 1}{3M^3} - \left[ \frac{k_3}{3M} + \frac{k_6}{M} \left( \frac{k_4}{\alpha \sigma_p} + \frac{k_5}{\cosh M} \right) \right] \right\} l^2 \left( 1 - w_e - \frac{l^2}{B} \right) \]  \hspace{1cm} (4.4.6)

### 4.5 Results and Conclusions

The linear KHI of an incompressible viscous poorly conducting fluid in a thin film which is in relative motion with poorly conducting fluid saturated in the densely packed porous layer in the presence of an electric and magnetic fields, is studied using linear stability
analysis combined with the normal mode solution. The dispersion relation given by Eq. (4.4.4), following Takabe[125] and Rudraiah and Kalal [84] can be expressed in the form:

\[ n = n_b - \beta l v_a \]  \hspace{1cm} (4.5.1)

where

\[ v_a = \left\{ \frac{M^2 - 1}{3M^3} - \left[ \frac{k_3}{3M} + \frac{k_6}{M} \frac{k_4}{\alpha \sigma_p} + \frac{k_5}{\cosh M} \right] \right\} l \left( 1 - w_e - \frac{l^2}{B} \right) \]  \hspace{1cm} (4.5.2)

is the transverse velocity at the interface, \( \beta = \frac{1}{M^2} \), \( n_b \) is given by Eq. (4.4.5).

Setting \( n = 0 \) in the dispersion relation Eq. (4.4.4), we get the critical wave number, \( l_{ct} \), in the form

\[ l_{ct} = \sqrt{(1 - w_e)B} \]  \hspace{1cm} (4.5.3)

The maximum wave number, \( l_m \), obtained from Eq. (4.4.4), by setting, \( \frac{\partial n}{\partial l} = 0 \), is

\[ l_m = \frac{l_{ct}}{\sqrt{2}} = \sqrt{\frac{(1 - w_e)B}{2}} \]  \hspace{1cm} (4.5.4)

The relations Eqs. (4.5.3) and (4.5.4) are true even in the absence of couple stress, electric field and laser radiation effects and for convenience we call them as classical results denoted by suffix b. Substituting Eq. (4.5.3) into Eq. (4.4.4), we get the maximum growth rate in the form

\[ n_m = B(1 - w_e)^2 \left\{ \frac{1}{3} - \left[ \frac{k_3}{3M} + \frac{k_6}{M} \left( \frac{k_4}{\alpha \sigma_p} + \frac{k_5}{\cosh M} \right) \right] \right\} \]  \hspace{1cm} (4.5.5)
Similarly, using Eq. (4.4.5) and (4.5.3), we get the classical maximum growth rate as

\[ n_{bm} = \frac{B}{12} \]  

(4.5.6)

The ratio of the growth rates, \( G_m = \frac{n_m}{n_{bm}} \), is obtained from Eqs. (4.5.5) and (4.5.6) as

\[ G_m = (1 - w_e)^2 \left\{ 1 - 3 \left[ \frac{k_3}{3M} + \frac{k_6}{M} \left( \frac{k_4}{\alpha_{sp}} + \frac{k_5}{\cosh M} \right) \right] \right\} \]  

(4.5.7)

We note that in the limit \( M \to 0 \) and \( w_e \to 0 \) this ratio of growth rates given by Eq. (4.5.7) reduces to

\[ G_m = \frac{4 + \alpha_{sp}}{4(1 + \alpha_{sp})} \]  

(4.5.8)

this coincides with the one given by Rudraiah[82] in the absence of couple stress and electric field effects, called NR-formula.

This, \( G_m \), given by Eq. (4.5.7), is computed for different values of the Hartman number, \( M \), electric number, \( w_e \) and porous parameter and the results are tabulated in tables I and II. From these tables, we note that the reduction of growth rate is 99% in the presence of laser radiation; couple stress and electric field, compared to 45% reduction predicted by Takabe[125] and 79% by Rudraiah[82] in the absence of these quantities.

Also the growth rate \( n \) given by Eq. (4.4.4) is always real. This \( n \) is computed for different values of \( M \), \( w_e \), and \( B \) and the values are depicted in the Figures (4.1) to (4.3). These figures 2 to 5, represent the dispersion relation \( n \) versus cutoff wave number \( l \). Figure 4.1 represents the graph of the growth rate \( n \) versus wave number \( l \) for different values of \( \beta \) and with fixed values of \( w_e = 1, \delta(1) = 10, B = 0.02 \) and \( \sigma = 0.001 \). This figure shows decrease in \( \beta \) that is increase in \( \lambda \) a material property responsible for couple stress the
negative growth rate $n$ decreases. Particularly for $\beta = 0.75$ the growth rate $n$ has almost approaches zero and hence stabilizes the interface elevation. Figure(4.2) represents the graph of the growth rate $n$ versus wave number $l$ for different values of $w_0$ and with fixed values of $\beta = 1$, $\delta(1) = 10$, $B = 0.02$ and $\sigma = 0.001$. This graph shows that for an increase in $w_0$ the growth rate $n$ decreases. Particularly for $w_0$ the growth rate $n$ is very small and hence stabilizes the interface elevation. Figure 4.4 represents the graph of the growth rate $n$ versus wave number $l$ for different values of $\delta(1)$ and with the fixed values of $\beta = 1$, $w_0 = 1$, $B = 0.02$ and $\sigma = 0.001$. This graph shows that for an increase in $\delta(1)$ the growth rate $n$ decreases. Particularly for $\delta(1) = 1$ the growth rate $n$ is very small and hence stabilizes the interface elevation. Figure(4.3) represents the graph of the growth rate $n$ versus wave number $l$ for different values of Bond number $B$ with fixed values of $\beta = 1$, $w_0 = 1$, $\delta(1) = 10$ and $\sigma = 0.001$. This figure shows that decrease in $B$ (increase in surface tension) increases the negative growth rate. Particularly for $B = 0.04$ the growth rate $n$ is very small and hence stabilizes the interface elevation.

Finally, from these graphs, we found that the reduction of maximum growth rate in presence of an electric field, laser radiation and couple stress. Hence, we conclude that the electric field, laser radiation and couple stress are more effective in reducing the asymmetry of IFE caused by fusing DT by laser radiation compared to classical growth rate in the absence of electric field and couple stress parameter. This conclusion on the reduction of growth rate of the EKHI mode is more effective in the design of a suitable IFT for increasing the efficiency of extraction of IFE by reducing the asymmetry caused by laser radiation. These conclusions are also useful in the effective design of artificial organs in biomedical engineering to prevent the side effects like haemolysis.
Table I: Reduction in growth rate for different values Couple stress parameter $\beta$

(For varying electric number $w_e$ and slip parameter $\alpha_1$)

<table>
<thead>
<tr>
<th>$M$</th>
<th>$w_e$</th>
<th>$\alpha_1$</th>
<th>$\sigma_p$</th>
<th>$G_m$</th>
<th>$R_m$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td>0.677</td>
<td>32.21</td>
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<tr>
<td>0.1</td>
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<td></td>
<td></td>
<td>0.068</td>
<td>93.11</td>
</tr>
<tr>
<td>0.15</td>
<td>0.99</td>
<td>0.0001</td>
<td>0.001</td>
<td>0.015</td>
<td>98.447</td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td>0.004</td>
<td>99.96</td>
</tr>
<tr>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td>0.0012</td>
<td>99.98</td>
</tr>
</tbody>
</table>

Table II: Reduction in growth rate for different values Couple stress parameter $\beta$

(For varying electric number $w_e$ and slip parameter $\alpha_1$)

<table>
<thead>
<tr>
<th>$M$</th>
<th>$w_e$</th>
<th>$\alpha_1$</th>
<th>$\sigma_p$</th>
<th>$G_m$</th>
<th>$R_m$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
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<td>0.599</td>
<td>34.11</td>
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<tr>
<td>0.1</td>
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<td></td>
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<td>93.70</td>
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<td>0.005</td>
<td>0.014</td>
<td>98.86</td>
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<tr>
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<tr>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td>0.001</td>
<td>99.99</td>
</tr>
</tbody>
</table>
Figure 4.1: Graph of $n$ versus $l$ for different values of $M$
Figure 4.2: Graph of $n$ versus $l$ for different values of $w_e$
Figure 4.3: Graph of $n$ versus $l$ for different values of $\sigma_p$. 

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