APPENDIX - I

Linear Quadratic Gaussian (LQG) Design Procedure

The plant considered for LQG Controller design is of the form

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + v(t) \\
y(t) &= Cx(t) + Du(t) + w(t)
\end{align*}
\] (I.1)

where \( v(t) \) and \( w(t) \) represent noise terms, which are assumed to be white, gaussian and having zero mean. It is observed that all the states are not available for feedback, and there is measurement, as well as process noise. Hence, both state estimation and optimal controller design are to be carried out. By applying the separation theorem, this problem may be split into two sub problems:

a) The state estimation problem – to compute online, a minimum variance estimate \( \hat{x}_e(t) \) of \( x(t) \) using the system inputs \( u(\tau), t_0 < \tau < t \), and the outputs \( y(\tau), t_0 < \tau < t \).

b) The optimal control problem – To compute the optimal control law \( u^*(t) = K'(t)x(t) \), which would be applied if there were no noise and if \( x(t) \) were available.

By replacing \( x(t) \) by its estimate \( \hat{x}_e(t) \) we get the optimal control law for the noisy problem. For the time invariant system given in equation (I.1), the performance index may be defined as

\[
F[x(t_0), u(\cdot), t_0] = \int_{t_0}^{\infty} u^T(t)R u(t) + x^T(t)Q x(t)dt
\] (I.2)

where, \( R \) is positive definite and \( Q \) is a non-negative matrix.

Minimization of this quadratic performance index will yield a linear control law. The minimization problem is the task of finding an optimal control \( u^* \) which minimizes \( F \), and the associated optimum performance index, \( F^* [x(t_0), u^* t_0] \).

The optimal estimator gain \( K_e \) and the optimal controller gain \( K_c \) are obtained by solving matrix Riccati equations.
Fig: I.1  Plant and Controller with State Estimate Feedback
APPENDIX II
Controllers for Exoatmospheric Flight Regime

1. LQG Controller

(a) Attitude path controller:

\[(47.93 \ s^9 + 2384 \ s^8 + 6.147e004 \ s^7 + 1.317e005 \ s^6 + 1.372e006 \ s^5 + 2.004e006 \ s^4 + 8.024e006 \ s^3 + 9.304e006 \ s^2 + 3.393e006 \ s + 2.541e005)\]

\[(s^{10} + 1225 \ s^9 + 1.796e004 \ s^8 + 1.737e005 \ s^7 + 8.03e005 \ s^6 + 3.495e006 \ s^5 + 9.325e006 \ s^4 + 1.929e007 \ s^3 + 2.889e007 \ s^2 + 2.705e006 \ s)\]

(b) Rate path controller:

\[(39.54 \ s^9 + 2030 \ s^8 + 5.369e004 \ s^7 + 1.859e005 \ s^6 + 1.272e006 \ s^5 + 2.86e006 \ s^4 + 7.918e006 \ s^3 + 1.156e007 \ s^2 + 1e006 \ s + 3.021e005)\]

\[(s^{10} + 1225 \ s^9 + 1.796e004 \ s^8 + 1.737e005 \ s^7 + 8.03e005 \ s^6 + 3.495e006 \ s^5 + 9.325e006 \ s^4 + 1.929e007 \ s^3 + 2.889e007 \ s^2 + 2.705e006 \ s)\]

2. H\infty Controller

\[(2.813 \ s^{14} + 8582 \ s^{13} + 8.87e006 \ s^{12} + 3.251e009 \ s^{11} + 1.536e011 \ s^{10} + 3.747e012 \ s^9 + 1.037e013 \ s^8 + 6.858e013 \ s^7 + 1.443e014 \ s^6 + 3.559e014 \ s^5 + 5.771e014 \ s^4 + 3.55e014 \ s^3 + 5.139e013 \ s^2 + 1.017e011 \ s + 5.068e007)\]

\[(s^{15} + 2597 \ s^{14} + 2.226e006 \ s^{13} + 6.637e008 \ s^{12} + 3.496e010 \ s^{11} + 9.598e011 \ s^{10} + 8.03e012 \ s^9 + 2.846e013 \ s^8 + 1.354e014 \ s^7 + 2.786e014 \ s^6 + 6.473e014 \ s^5 + 9.035e014 \ s^4 + 4.664e014 \ s^3 + 4.712e013 \ s^2 + 1.357e012 \ s + 1.31e009)\]
APPENDIX – III

Linear Short Period System model for Descent Vehicle

\[
\begin{bmatrix}
\frac{m U_o}{S_q} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{m U_o}{S_q} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{I_{yy}}{S_q} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{m U_o}{S_q} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{I_{xx}}{S_q b} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{U} \\
\dot{\alpha} \\
\dot{q} \\
\dot{\theta} \\
\dot{\beta} \\
\dot{p} \\
\dot{\phi} \\
\dot{\psi}
\end{bmatrix} =

\begin{bmatrix}
C_{xU} & C_{x\alpha} & C_{xq} & \frac{c}{2 U_o} & C_{xq} & C_{wx} \cos \theta_0 & 0 & 0 & 0 & 0 & 0 \\
C_{zU} & C_{z\alpha} & \left( \frac{m U_o}{S_q} + \frac{c C_{zq}}{2 U_o} \right) & C_{wz} \sin \theta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
C_{mu} & C_{ma} & \frac{c}{2 U_o} & C_{mq} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C_{y\beta} & \left( \frac{b C_{yp} + m W_o}{2 U_o} \right) & C_{y\phi} & \left( \frac{b C_{yr} - m U_o}{2 U_o} \right) & C_{y\psi} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{b}{2 U_o} & C_{lp} & 0 & \frac{b}{2 U_o} & C_{lr} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{b}{2 U_o} & C_{lr} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{n\beta} & \frac{b}{2 U_o} & C_{np} & 0 & \frac{b}{2 U_o} & C_{nr} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
U \\
\alpha \\
q \\
\theta \\
\beta \\
p \\
\phi \\
\psi
\end{bmatrix} +

142
\[
\begin{bmatrix}
C_{f_{x_{a1}}} & C_{f_{x_{b1}}} & C_{f_{x_{p1}}} & C_{f_{x_{p2}}} \\
C_{f_{y_{a1}}} & C_{f_{y_{b1}}} & C_{f_{y_{p1}}} & C_{f_{y_{p2}}} \\
C_{m_{a1}} & C_{m_{b1}} & C_{m_{p1}} & C_{m_{p2}} \\
0 & 0 & 0 & 0 \\
C_{l_{a1}} & C_{l_{b1}} & C_{l_{p1}} & C_{l_{p2}} \\
0 & 0 & 0 & 0 \\
C_{n_{a1}} & C_{n_{b1}} & C_{n_{p1}} & C_{n_{p2}} \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\delta_{e_{1}} \\
\delta_{e_{2}} \\
\delta_{r_{1}} \\
\delta_{r_{2}} \\
\end{bmatrix}
\]
APPENDIX IV
Aerodynamic and Control Coefficients for Aerodynamically Controlled Launch Vehicle

(a) Ascent Vehicle

Fig. IV.1 Pitch aerodynamic coefficient
Fig. IV.2 Yaw aerodynamic coefficient

Fig. IV.3 Mach no: vs Pitching and Yawing moment coefficients
Fig. IV.4 Mach no: vs normal force coefficient slope and side force coefficient slope

(b) Descent Vehicle

Fig. IV.5 Axial Force Coefficients
Fig. IV.6 Side Force Coefficients

Fig. IV.7 Normal Force Coefficients
Fig. IV.8 Rolling Moment Coefficients

Fig. IV.8 Pitching Moment Coefficients
Fig. IV.9 Yawing Moment Coefficients
APPENDIX V
Application of the Kharitonov Theorem and the Mapping Theorem

Four Kharitonov polynomials are associated with each uncertain polynomial

\[ A(s) = a_0 + a_1 s + \ldots + a_n s^n. \]

If the first Kharitonov polynomial is denoted by

\[ K_1(s) = a_0^u + a_1^u s + a_2 s^2 + a_3 s^3 + a_4^u s^4 + \ldots \]

with the repetitive sequence of uncertain parameter combinations \( uull \ldots \), the other three polynomials have the combinations \( uulu \ldots, lulu \ldots, \) and \( lluu \ldots \) respectively, where subscript \( l \) denotes the lower bound and subscript \( u \) denotes the upper bound of each parameter. Using the Generalised Kharitonov Theorem, each of the uncertain polynomials is replaced by the corresponding extremal polynomial manifolds, constructed by joining appropriate pairs of Kharitonov polynomials. The extremal set of transfer functions is then formed and the image set of the extremal systems at \( s = j\omega \) evaluated. By the Mapping theorem, the complex plane image set of the entire family of systems at \( s = j\omega \) is "concave" or bulges inwards and hence is overbounded by the image set of the extremal systems. The analysis of the image set of the extremal systems gives the guaranteed gain and phase margins of the entire family.
APPENDIX VI
Nominal and Off-Nominal Model Parameter Values

<table>
<thead>
<tr>
<th>Fixed parameters</th>
<th>Variable parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>$\omega_a$</td>
<td>18.9 rad/s</td>
</tr>
<tr>
<td>$\zeta_a$</td>
<td>0.7</td>
</tr>
<tr>
<td>$T_c$</td>
<td>1377180 N</td>
</tr>
<tr>
<td>$l_c$</td>
<td>11.91 m</td>
</tr>
<tr>
<td>$l_{yy}$</td>
<td>23495230 kg m$^2$</td>
</tr>
<tr>
<td>$K_A$</td>
<td>8.16</td>
</tr>
<tr>
<td>$K_R$</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

APPENDIX VII
LQG Controllers for Atmospheric Flight Regime

Attitude path filter

\[
\frac{(1736s^6 + 6.423e04s^5 + 1.439e06s^4 + 1.051e07s^3 + 1.592e08s^2 + 2.29e08s + 7.331e07)}{(s^7 + 659.2s^6 + 1.794e04s^5 + 3.279e05s^4 + 3.046e06s^3 + 2.398e07s^2 + 3.822e07s - 6.753e06)}
\]

Rate path filter

\[
\frac{(820.1s^6 + 3.027e04s^5 + 6.771e05s^4 + 4.903e06s^3 + 7.481e07s^2 + 1.031e08s + 5.099e06)}{(s^7 + 659.2s^6 + 1.794e04s^5 + 3.279e05s^4 + 3.046e06s^3 + 2.398e07s^2 + 3.822e07s - 6.753e06)}
\]