Chapter 3

Two Phase Linear Quadratic Gaussian Controller Design for Launch Vehicle Autopilot and Comparison with Existing Techniques

3.1 Introduction

Before investigating the advanced methods of autopilot design, it is appropriate to do a critical evaluation of the state-of-the-art design techniques for a real life launch vehicle problem. This evaluation is done in the present chapter by applying the state-of-the-art techniques to the autopilot design problem of the final stage of a typical launch vehicle, so as to actively stabilize the slosh dynamics over the entire stage duration, while accounting for the parametric uncertainty at each flight instant. A controller is first designed by the $H_\infty$ design method and compared with the benchmark classical design. A modified LQG design procedure is subsequently adopted to improve the design robustness. Each method is evaluated with respect to tracking performance, robustness, disturbance rejection, noise rejection and controller complexity.

For the phase of flight beyond the atmosphere, the aerodynamic effects are negligibly small. Due to the absence of the lower stages, the vehicle has low length to diameter ratio and high stiffness and hence the bending modes are at high frequencies with respect to the rigid body. Hence flexibility effects are low. The variation of the system parameters as flight progresses is comparatively slow, and hence a time invariant controller design is feasible. However, the oscillations of the liquid fuel and oxidizer in the tanks (slosh dynamics), have very little damping. In the frequency domain, this manifests as very large amplitude modes. Hence stabilization of lightly damped multiple slosh modes is one of the main challenges in this phase. This is achieved in classical control by the addition of passive damping structures, called baffles, in the tanks in the areas where significant slosh excitation is expected to take place. These baffles significantly increase the weight of the tanks. For the terminal stage of a launch vehicle mission, any increase in mass of the structure has a direct bearing on the payload mass, which will reduce proportionally. If active stabilization of the slosh modes is possible, the slosh baffles
could be removed, resulting in substantial weight saving, which translates into payload increase.

The controller based on classical methods does not attempt to optimize in general any of the performance parameters. For the terminal stage of any launch vehicle, any mass saving will directly translate to payload gain (each kg of mass saved gives 1 kg payload gain). Hence optimal design techniques such as the Linear Quadratic Gaussian (LQG) optimal controller are particularly attractive in terminal phase autopilot design. The LQG optimal controller does not ensure robustness to parameter perturbations; the robustness has to be checked after the design is completed, by evaluating the perturbed system behavior with the controller in loop. The $H_\infty$ robust controller design methodology, on the other hand, provides robustness to plant uncertainties which are assumed to be non-parametric in nature. The design trade-off between active stabilization of slosh dynamics through suitable controller design as against passive stabilization through baffles has been evaluated.

The significant contribution of this Chapter is the development of a new Two Phase LQG approach (TP-LQG), which combines the advantages of the time domain techniques with the physical insight of the frequency domain methods to provide a robust LQG design with near perfect phase stabilization of the lightly damped slosh dynamics.

Section 3.2 describes the challenges in slosh stabilization, while Section 3.3 presents the plant model description, and Section 3.4 gives the typical specifications for classical autopilot design. Sections 3.5, 3.6 and 3.7 address the design and analysis with the classical, $H_\infty$ and LQG design methodologies, respectively. The comparison of the performance by all three methods is given in Section 3.8. The major results are summarised in the last section.

3.2 Challenges in Slosh Stabilization

Defined perturbation bounds are associated with each slosh parameter at each flight instant. In the nominal case, the dominant slosh modes of both fuel and oxidizer tanks are at more or less the same frequency over most of the flight regime, leading to a single large mode corresponding to both the tank dynamics. In perturbation cases, the
frequencies shift and the slosh response in the frequency domain can change from a single mode to multiple modes, with large swing in phase. During the transition phase from the lower to the upper stage, when the engines are ignited, the disturbances on the vehicle are to be kept to a minimum to ensure smooth control capture. Sustained slosh oscillations will have a significant influence on the rigid body rates, and impede smooth capture. The baffles are normally provided to take care of such transitions. Once the baffles are uncovered, slosh has only viscous damping; hence once excited, the oscillations will continue for a long time. Slosh baffles provided at the top of the fuel and oxidiser tanks are effective in damping out fuel slosh oscillations for the first ten seconds. Thereafter, slosh has only viscous damping. As flight progresses, the total system mass and inertia progressively come down, while free surface area in the tanks and the sloshing mass progressively increase till the tanks are half full. Hence the influence of the slosh mass on the vehicle response increases as time progresses and reaches a maximum when the ratio of slosh mass to vehicle mass is maximum. The influence of slosh on the vehicle response is largest between 300s and 400s after ignition.

3.3 Plant Model

The launch vehicle in the exo-atmospheric phase has slowly varying plant dynamics. The stage duration is large (typically greater than 400s). Using a time-slice approach, the short period matrices at different time instants during the phase are formulated, with rigid body, actuator dynamics, and the dominant slosh modes for fuel and oxidiser tanks considered. The vehicle dynamics in pitch and yaw planes is very similar since the vehicle is symmetric. Roll dynamics are much simpler, since slosh influence is not significant. Accordingly, the design process is studied for pitch plane only.

The plant is controlled by gimballing the twin liquid engines. Tail-wags-dog phenomenon can exist because of the gimbaled engine and is considered in the study. The vehicle bending mode frequencies are beyond 16 Hz and their influence on the rigid body and slosh dynamics is low. Hence they are not considered significant for this study. Variation in rigid body dynamics over the entire regime and nominal, lower and upper bound data for slosh parameters (mass, pendulum length, hinge location and frequency) is used to generate the set of uncertain transfer functions.
The short period equations of motion in the pitch plane are

\[
\begin{align*}
\ddot{\theta} &= -\frac{(I_y + m_r l_e)}{I_{yy}} \ddot{\theta}_A - \sum_i m_{pi} l_{pi} \ddot{U}_o \frac{I_{pp}}{I_{yy}} \tau_{ipi} + \left[ \frac{T_c l_e + m_r l_e \dot{U}_o}{I_{yy}} \right] \delta_A \\
\dot{\tau}_{pi} &= \frac{L_{pi}}{L_{pi}} (\ddot{\theta} - \omega_{pi}^2 \tau_{pi} - 2\zeta_{pi} \omega_{pi} \dot{\tau}_{pi}) \\
\ddot{\delta}_A &= -2\zeta_A \omega_A \dot{\delta}_A - \omega_A^2 \delta_A + \omega_A^2 \delta_c
\end{align*}
\]

The state vector in the pitch plane is

\[
X^T = [\theta \ \dot{\theta} \ \tau_{lp} \ \dot{\tau}_{lp} \ \tau_{2p} \ \dot{\tau}_{2p} \ \delta_p \ \dot{\delta}_p]
\]

where, \( \phi \) is pitch attitude angle, \( \dot{\phi} \) pitch body rate, \( \tau_i \) is oxidiser tank slosh angle (pitch), \( \dot{\tau}_i \) is oxidiser tank slosh rate (pitch), \( \tau_{2p} \) is fuel tank slosh angle (pitch), \( \dot{\tau}_{2p} \) is fuel tank slosh rate (pitch), \( \delta_p \) is engine gimbal deflection angle (pitch) and \( \dot{\delta}_p \) is engine gimbal deflection rate (pitch).

\( U = [\delta] \) is the command to the actuator. Only attitude and body rate are sensed. Hence \( Y^T = [\theta \ \dot{\theta}] \) corresponds to the outputs of the attitude and rate sensors.

### 3.4 System Specifications

The prime specification for autopilot design is tracking error, which needs to be maintained below 1deg. The rigid body bandwidth and damping ratio are chosen accordingly.

The standard classical frequency domain specifications for the launch vehicle autopilot design, defined for the nominal plant are

- Aeromargin (low frequency gain margin): 6dB
- Rigid body gain margin: 6dB
- Rigid body phase margin: 30deg
- Slosh minimum phase margin: 35deg

These specifications are ideally to be met for any other controller, for the nominal plant.
3.5 Classical Controller Design

The benchmark classical controller consists of gains which are scheduled as a function of time to account for the variations in the vehicle parameters, integrator and fixed filters in the position and rate paths. The system gains are designed to achieve bandwidth and damping ratio for the dominant rigid body poles so as to satisfy the tracking error requirement (Fig 3.1). The filters are designed to provide adequate phase lead margin to the slosh modes and to provide sufficient attenuation to the high frequency dynamics. Since the disturbances in this flight regime are very low frequency disturbances such as thrust misalignment and centre of gravity (cg) offset, an integrator is designed (with gain $K_i = 0.1$) to provide disturbance rejection of around 18dB at low frequency (0.1 rad/s). The position and rate path filters are given in Fig 3.2 and Fig 3.3.

![Classical gain schedule](image)

Fig 3.1 Classical Gain Schedule
The nominal system response from ignition to burnout is given in Fig 3.4
Slosh has good phase margins at all time instants; however, the rigid body phase margin is significantly affected by the proximity of the slosh dynamics, especially in the 320-360s regime. The phase margin drops to 30 deg in this regime, which just meets the specifications. The placement of the slosh modes is not symmetric with respect to the real axis – the phase lag margin is low while the phase lead margin is large. Since the slosh phase is unfavourable, the classical controller achieves slosh stabilization in the region of high slosh influence by reducing the system bandwidth in this phase, so as to maintain a good separation between rigid body and slosh dynamics. The penalty for this is seen in terms of the reduced aero margin (minimum aeromargin: 9.5dB) and correspondingly high peak overshoots in the transient response (Fig 3.5). It is seen that peak overshoots in this region are around 40% and settling time, more than 10 s. The effect of the comparatively low slosh stability margins is seen in the continuous oscillation of the slosh modes with peak to peak values of 0.1 deg/s and 0.2 deg/s respectively for the two modes. The stability margins are given in Table 3.1
Fig 3.5a - Slosh behaviour with classical design: slosh mode 1

Fig 3.5b - Slosh behaviour with classical design: slosh mode 2

Fig 3.5c

Fig 3.5d

Fig 3.5 - Transient Response with Classical Controller

Table 3.1 Stability Margins for Nominal System

<table>
<thead>
<tr>
<th>Time from stage ign. (s)</th>
<th>Rigid body</th>
<th>Slosh margins</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aero-</td>
<td>Mode 1</td>
</tr>
<tr>
<td></td>
<td>margin AM</td>
<td>Lead PM (deg)</td>
</tr>
<tr>
<td></td>
<td>(dB)</td>
<td>Lag PM (deg)</td>
</tr>
<tr>
<td></td>
<td>Phase</td>
<td>Peak Magn.</td>
</tr>
<tr>
<td></td>
<td>margin PM</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(deg)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>&gt;20</td>
<td>Gain stabilized</td>
</tr>
<tr>
<td>20</td>
<td>&gt;20</td>
<td>Gain stabilized</td>
</tr>
<tr>
<td>200</td>
<td>18</td>
<td>&gt;90</td>
</tr>
<tr>
<td></td>
<td>31</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>320</td>
<td>9.6</td>
<td>&gt;90</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>45</td>
</tr>
<tr>
<td>360</td>
<td>9.6</td>
<td>&gt;90</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>47</td>
</tr>
<tr>
<td>400</td>
<td>20</td>
<td>&gt;90</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gain stabilized</td>
</tr>
</tbody>
</table>
The magnitude and phase of the two lightly damped slosh modes at almost identical frequency (one corresponding to the fuel and the other to the oxidizer) are sensitive to parameter perturbations. The upper and lower bounds on each parameter over the entire flight regime are considered for generating the perturbation cases corresponding to the worst case response. These worst cases are chosen based on the designer’s experience. Under perturbation of the system parameters, slosh dynamics manifests as a couple of modes in phase opposition, leading to reduced margins, as shown in Fig 3.6. For the off nominal cases also, slosh remains stable with margins >30deg.

Fig 3.6 Perturbation Case with Classical Design

3.6. Design and Evaluation of $H_\infty$ Controller for the Exo Atmospheric Phase

$H_\infty$ controller design involves the synthesis of an augmented plant model with appropriate frequency domain loop shaping weighting functions to obtain desired tracking performance and disturbance rejection while keeping the control effort sufficiently low, and giving good rejection to high frequency noise. The method is a norm based approach, with emphasis on minimizing the norm of appropriate transfer functions to obtain robust performance.
The first step in the design of $H_\infty$ controllers is the choice of weighting functions to meet the system specifications. The plant augmented with the weighting functions is given in Fig 3.7. The bandwidth of the classically designed controller for the launch vehicle is in the range of 1 to 4 rad/s to meet the tracking error requirements while maintaining sufficient separation from higher order dynamics. This has been taken as one of the guidelines in shaping the weighting functions. $W_1^{-1}$ defines the low frequency performance, $W_2^{-1}$ the control energy requirement and $W_3^{-1}$ the high frequency robustness. The design and analysis of $H_\infty$ controllers is done in the MATLAB environment, making use of the Robust Control toolbox for design and evaluation of frequency domain response, and SIMULINK for the simulation of the closed loop system to observe the time domain performance.

3.6.1 Controller Design

The flight instant where the influence of slosh on the system response is a maximum has been taken as the design point, in order to obtain a controller, which will stabilize the nominal as well as the off-nominal plant throughout the stage. The weightage on the error, $W_1$ is chosen so as to give around 60dB sensitivity reduction at very low frequency, while the choice of $W_3$ along with $W_1$ gives a closed loop bandwidth in the desired range of 1-3 rad/s, and gives good high frequency disturbance rejection. $W_2$ is chosen to ensure that the actuator operates in its linear range (the maximum allowable deflection of the actuator is around 4deg).
With these criteria, the following set of weighting functions were chosen and the controller designed:

\[
W_1 = 0.00036 \frac{(s+100)^2}{(s+0.06)^2}
\]

\[
W_2 = \frac{s+4}{0.004s+4}
\]

\[
W_3 = \frac{0.1111s^2+0.66667s+1}{(0.0000015s^3+0.0030015s^2+1.503s+1.5)}
\]

The sensitivity and complementary sensitivity functions along with the inverse weights \(1/W_1\), \(1/W_2\) and \(1/W_3\) are given in Fig. 3.8 - Fig. 3.10.

**Fig. 3.8** Sensitivity Function & \(W^{-1}\)

**Fig. 3.9** \(W^{-2}\)

**Fig. 3.10** Complementary Sensitivity Function & \(W^{-3}\)

It is seen that the achieved sensitivity and complementary sensitivity functions closely follow the desired shapes. The open loop response of the plant with controller is plotted in Fig 3.11. The sharp peak around 3 rad/s corresponds to the slosh mode and the dip at around 15 rad/s corresponds to the imaginary axis zero pair corresponding to the system tail-wags- dog phenomenon. It is seen that the loop gain of the system with controller at low frequencies is of the order of 60dB. The controller response per se, however, has a
low gain at low frequencies (Fig 3.12). This is because the open loop system gain is very high and the controller is simply satisfying the given sensitivity requirements, as seen from Fig 3.13.

This is one of the deficiencies of the traditional $H_\infty$ synthesis model structure, in systems where the plant resembles a double integrator at very low frequencies, as in the exoatmospheric phase launch vehicle. The plant inversion compensator in this case includes two very low-frequency zeros, which makes the low frequency compensator gain to be very small ($<< 1$). This effect has been reported earlier, while applying the $H_\infty$ design technique to a plant of similar nature. Since the low frequency gain of the compensator, which represents the “stiffness” of the compensators to low frequency disturbances is approximately the inverse of the low frequency gain of the plant ($K_{plant}$) times $W_1(s)$, the only way to overcome this problem is to shape $W_1$ such that the low frequency gain is improved. This is achieved by adding an integrator term in $W_1$. 
The new weight on W1 is

\[
\frac{0.00033(s+100)^2(s+0.2)}{(s+0.6)^2(s+0.001)}
\]

The shaped W1 with 100dB sensitivity reduction at low frequencies is given in Fig 3.14, along with the sensitivity function of the system with the corresponding controller. W3 and the complementary sensitivity function are given in Fig. 3.15.

Fig 3.14 Sensitivity Function with Modified Weights

Fig 3.15 Complementary Sensitivity Function with Modified Weights

Fig 3.16- Controller Response with Modified Gains
It is seen that with modified weights, the low frequency response has been significantly improved (Fig 3.16). However, beyond a point, further low frequency sensitivity reduction to increase the controller gain is not feasible, since it starts affecting the response of the rigid body and higher dynamics.

The sharp kink seen in the controller response corresponds to the slosh dynamics. It is seen that the design gives good high frequency attenuation (more than −35dB at 300 rad/s), and correspondingly good noise rejection.

3.6.2 Controller Robustness Evaluation

The design is evaluated with respect to the same criteria as for the other design methods. Frequency domain analysis is carried out for the nominal plant at the different flight instants (Fig. 3.17) as well as the earlier defined perturbation cases.

![Nyquist plot](image)

**Fig. 3.17 – Nominal System Response throughout the Flight Regime**

It is seen that good stability margins exist at all flight instants, for the nominal plant, and hence the design is robust to variations in the flight parameters over the duration, as in the case of the classical design. The minimum margins are:

- Gain margin: 13dB
- Phase margin: 38deg
- Slosh lead phase margin: >120deg
- Slosh lag phase margin: >40deg

It is seen that even though the specifications on the nominal plant are satisfied, the slosh modes are not perfectly phase stabilized, but have significantly lower lag phase margin. Assessment of the perturbation cases shows that while most of the cases are stable, a few are not, as seen in Fig. 3.18, where the slosh modes are in phase opposition.
Fig. 3.18- Response of Perturbation Cases with $H_\infty$ Design

Around 30deg rigid body phase margin is maintained for all cases. However, robustness to slosh parameter perturbation is less than with the other designs. Step response analysis with the $H_\infty$ controller gives very small rise time (less than 1.5s) and almost no peak overshoot.

Representative step response at T+360s is given in Fig. 3.19. It is seen that the rigid body transient response characteristics are much better than that of the classical controller. However, since the slosh modes have large magnitudes but are not perfectly phase stabilized, they are clearly seen in the response, with contribution around 0.12 deg/s peak to peak in the body rate. It is also seen that slosh exhibits a beating due the proximity of two modes at nearly the same frequency. This is clearly seen in the slosh pendulum angle responses.

Fig 3.19(a) Attitude Angle  Fig 3.19(b) Body Rate
3.6.3 Noise Rejection with $H_\infty$ Design

The noise rejection of the system is quite good, as seen from the responses with noise injected at plant input and output. Noise effect on body rate is around 0.01 deg/s, as in LQG design, with noise injected at plant input (Fig. 3.20).
Clean response is obtained with noise injected at the plant output (Fig 3.21).

3.6.4 Disturbance Rejection with $H_{\infty}$ Design

Since the low frequency gain of the $H_{\infty}$ controller for systems with double integrator at the origin cannot be boosted beyond a limit without affecting the system response, it has limited low frequency disturbance rejection for such systems, compared to the other design methods. It is seen that introduction of integral action does not automatically follow when the weighting functions are shaped; it depends strongly on the inherent nature of the plant.

It is seen that for this system with double pole at the origin, and with lightly damped, multiple modes at nearly the same frequency, the norm based approach is not giving good
robustness results. Hence an alternate approach based on the Linear Quadratic Gaussian (LQG) optimal control design, is explored in the next section.

3.7 Two Phase Linear Quadratic Gaussian Controller Design

Linear Quadratic Gaussian (LQG) optimal control design, which minimizes quadratic performance indices subject to linear state space dynamics driven by Gaussian white noise, has been used in the Ariane series of vehicles as well as in the Apollo and Space Shuttle. In this section, the standard LQG procedure is modified to incorporate robustness to parameter perturbations, and a controller is designed for the plant described in Section 3.3. The standard LQG design procedure is given in Appendix 1.

For Linear Quadratic Gaussian Optimal Controller (LQG) design, the designer needs to specify a matrix of weighting factors in the performance index in order to achieve the desired performance. The overall influence of different weightages in the optimized performance index on the control performance is assessed in terms of transient response as well as robustness to parameter perturbation, and the choice of weights that gives the best relative performance is used for the final controller design. Since thrust misalignments and thrust offset (offset of the thrust line with respect to the center of gravity) can cause very low frequency disturbance, the disturbance internal model is included in the plant in the form of an integrator, whose time constant is chosen to be the same as that of the baseline classical controller. The same design instant as for the H∞ controller is chosen for design. The performance of the controller at different flight instants is evaluated through frequency response and transient response analysis.

3.7.1 Tuning of LQG Controller Weights to Achieve Performance and Robustness

For Linear Quadratic Gaussian Optimal Controller (LQG) design, the designer needs to specify a matrix of weighting factors in the performance index in order to achieve the desired performance, basically in terms of transient response and the control effort. The choice of weights is essentially a trial and error process, and the final design is only as good as the chosen weights. There is no direct way to incorporate robustness to parameter perturbations into LQG design. This is one of the reasons for the limited application of this method in launch vehicle autopilot design. The other reason is the lack of physical
insight in the design procedure, which is the main reason why practicing engineers prefer
the frequency domain tools of the classical design method.

Different approaches to incorporate robustness in LQG design have been reported^{57-60}. This thesis uses a different approach to achieve performance as well as robustness to parameter perturbation, through appropriate tuning of the weighting matrix Q, by making use of a set of worst case plants, which are selected by the designer based on his knowledge of the system behaviour. In standard LQG methods, close control on the system states is possible by manipulating the weighting matrices. This attractive feature is combined with the physical insight offered by the frequency domain techniques to evolve a Two Phase LQG (TP_LQG) design procedure, where the scope of the state weighting matrix is expanded from tuning for performance to tuning for robustness.

The performance of the system is basically dictated by the rigid body response i.e. the tracking error and body rates (which are dictated by the achieved bandwidth) and the control effort define the performance. Hence the states corresponding to the rigid body response (attitude and body rates) and those of the slosh dynamics have been treated separately. The tuning of the weights is done in two steps: in the first step, weights assigned to the rigid body states are tuned to achieve acceptable rigid body response based on transient response results. Once these are fixed, the weights on the slosh states have been tuned with a unique approach, based on the frequency domain analysis over the worst case set of plants. Thus the main emphasis for the choice of the slosh weights has shifted from optimization to robustness. The overall influence of different weightages in the optimized performance index on the control performance is assessed in terms of transient response as well as robustness to parameter perturbation. The choice of weights that gives the best relative performance is used for the final controller design. The LQG controller tuning procedure is given in Fig. 3.22.

With similar relative weights for the rigid body states and control, the peak overshoot is more than 20%. Large relative weight on the control is seen to give a sluggish response with large peak overshoot and settling times. However, the peak rates and control effort are quite small for this case. The conflicting requirements of control effort minimization and tracking error reduction can be met by putting large weights on the attitude( for
reducing tracking error, which is the highest priority), somewhat lower weights on the body rate (which directly reduces the peak control effort) and unity weight on the control (which gives good transient response). Attitude weightage of 750, weightage of 10 for the rate and unity for the control are seen to give the best results, with rise time of around 2s, peak overshoot of around 7% and settling time around 5.5s. These rigid body weights have been chosen for further studies. It may be noted that since the rigid body dynamics are very slowly varying, these results hold good over the entire flight regime.

Formulate worst case plant set

Assign
\[ Q = [I] \]
\[ R = [I] \]

Design LQG Controller

Transient Response Satisfactory?

Yes

Design LQG controller

Tune diagonal slosh mode state weights

No

Frequency Response satisfactory?

Yes

Freeze design

No

Tune Rigid body weights

\[ Q(1,1) \]
\[ Q(2,2) \]

**Fig 3.22 LQG Design Tuning Procedure**

With zero slosh weights, the controller performance is seen to be sensitive to slosh parameter perturbations, with significant slosh activity observed in the body rate and control signals, reaching 20% at the end of 10s for off-nominal cases. Hence weights on the slosh states were tuned based on robustness evaluation over the flight regime.
Weights of 10 or more on the slosh states are seen to stabilize the slosh modes at all instants; however the rigid body response starts deteriorating, with increased rise time and settling time. Hence the weightage on slosh has been fixed at the minimum possible value, which ensures stable response at all flight instants for nominal and perturbation cases; i.e. the appropriate diagonal Q matrix has weights

\[ q_{11} = 750, q_{22} = 10, q_{33..q_{66}} = 10, q_{77} = q_{88} = 1, \text{ and } R = [1]. \]

### 3.7.2 Integrator for Disturbance Rejection

Since the disturbances in this flight regime are very low frequency disturbances such as those due to thrust misalignment and centre of gravity offset, these can be taken care of in LQG design by adding the integrator as a part of the plant. An integrator with gain \( K_i = 0.1 \) (same as for the classical design) was included in the plant model to provide adequate disturbance rejection. The plant model is augmented as shown in Fig. 3.23.

\[
\begin{align*}
\begin{bmatrix}
\dot{X} \\
\delta_i
\end{bmatrix} &= \begin{bmatrix}
A & BK_i \\
0 & 0
\end{bmatrix} \begin{bmatrix}
X \\
\delta_i
\end{bmatrix} + \begin{bmatrix}
B \\
1
\end{bmatrix} \delta_{DAP} \\
\begin{bmatrix}
Y
\end{bmatrix} &= \begin{bmatrix}
C & 0
\end{bmatrix} \begin{bmatrix}
X \\
\delta_i
\end{bmatrix} + \begin{bmatrix}
0
\end{bmatrix} \delta_{DAP}
\end{align*}
\]

Fig 3.23 Plant Augmented with Integral Controller

An LQG controller is designed for the augmented system with unity weight on the corresponding additional state in the Q matrix, and the rest of the weights maintained same as in the previous case. The controller frequency response is given in Fig 3.24. It is seen that the controller has a high gain of 20dB in the very low frequency region corresponding to the integrator, and a considerable attenuation of >28 dB in the high frequency regime, beyond 50 Hz.
Even though LQG design cannot explicitly handle low frequency disturbances, by including the integrator in the plant dynamics for LQG controller design, we have been able to get the classical desired open loop controller shape with good low frequency gain (≈ 20dB at 0.1 rad/s), thus giving good low frequency disturbance rejection. The controller transfer function is given in Appendix 2.

3.7.3 Robustness Assessment through Frequency Domain Analysis

Frequency domain analysis has been carried out with the designed filter at different instants from ignition to burnout and the response assessed. The response at the different flight instants are given in Fig 3.25.
It is seen that for the nominal system throughout the entire duration, very good stability margins are obtained for the rigid body as well as the slosh dynamics, which are almost perfectly phase stabilized. Due to the very low damping on slosh, the slosh mode magnitude in the Nyquist plot is very large. However, due to the near perfect phase stabilization, this large mode magnitude is an added advantage, as it leads to fast quenching of slosh oscillations, whenever excited. The nominal response results are given in Table 3.2.

The gain margin is more than 12dB at all time instants. In the initial phase of flight, the slosh modes are quite small. This is due to the effectiveness of baffles. However, as time progresses, we have two distinct modes, one large and one small, at very close frequencies. From 320 s onwards, the two slosh modes merge into one large mode since the frequencies are same for both tanks. Towards burnout, due to the smaller slosh mass, the mode again shrinks. It is seen that very good stability margins are obtained for both the rigid body and the slosh modes. The system being slowly time varying, the aeromargin (low frequency gain margin) point is seen to slowly shift; however, more than 12dB margins are available at all flight instants. Thus the LQG controller is robust to the parameter variations from ignition to burnout.

Table 3.2 Stability Margins for the Nominal Compensated System-LQG Design

<table>
<thead>
<tr>
<th>Time from stage ign. (s)</th>
<th>Rigid body</th>
<th>Slosh margins</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aeromargin - AM (dB)</td>
<td>Phase margin - PM (deg)</td>
</tr>
<tr>
<td></td>
<td>Lead PM (deg)</td>
<td>Lag PM (deg)</td>
</tr>
<tr>
<td>3</td>
<td>12.8</td>
<td>&gt;40</td>
</tr>
<tr>
<td>20</td>
<td>13</td>
<td>&gt;40</td>
</tr>
<tr>
<td>200</td>
<td>15.4</td>
<td>&gt;40</td>
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<tr>
<td>320</td>
<td>15.56</td>
<td>&gt;40</td>
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<td>&gt;40</td>
</tr>
<tr>
<td>400</td>
<td>16.6</td>
<td>&gt;40</td>
</tr>
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</table>

Under parameter perturbation, similar behaviour as in the classical case is exhibited (Fig. 3.26).
The minimum slosh phase margin over all the perturbation cases throughout the flight regime is 60deg which is much higher than the nominal with classical design. Even at the region of maximum slosh influence on the body rates i.e. between 300 and 400s of flight, slosh has very good stability margins. Thus it is seen that even though the LQG controller is not designed for robustness to parameter perturbations, it is in fact giving very good robustness.

The rigid body stability margins for the perturbed cases are more or less same as for the nominal case. This is because of the very slowly time varying nature of the plant. Since the design is carried out in the middle of the flight regime, the effective bandwidth is slightly higher in the earlier flight regime i.e. before the design point, and slightly lower in the later flight regime i.e. beyond 360s. The variation is manifested as small changes in the phase crossover frequency in frequency domain and as corresponding changes in the transient response characteristics in the time domain.

Step response analysis with the LQG controller gives rise time less than 2s, peak overshoot less than 10% and settling time of less than 5s at all flight instants. Representative step response at the design instant is given in Fig. 3.27. It is seen that because of the good aeromargin, the transient response characteristics are much better than that of the classical controller.
It is seen that because of the perfect phase stabilization offered by LQG, slosh oscillations are damped fast and the contribution of slosh in the body rates is very small. Thus active damping is provided for slosh.

3.7.4 Noise Rejection with LQG Design:
Based on flight sensor measurements, as well as hardware in loop simulation tests, rate gyro measurement noise statistics are mean zero and standard deviation $1\sigma = 0.01\text{deg/s}$.
There is no random noise associated with the plant— the disturbances are thrust misalignment and differential thrust which are of very low frequency. The high frequency unmodelled dynamics, delays and non-linearities, however, may be represented as noise at the plant input. In order to assess the noise rejection of the system, simulations are carried out using MATLAB/SIMULINK with step command as well as random noise injected either at the plant input or output, at the design point of 360 s. For both cases, the loop shows a good noise rejection. With the noise injected at the plant input, the effect on the body rate is of the order of 0.01 deg/s (Fig 3.28). With noise injected at the output, the control command excursions are less than 0.02 deg, and the attitude and body responses are clean (Fig 3.29).

Fig 3.28 - Response with Noise Injected at Plant Input

Fig 3.29 Response with Noise Injected at Plant Output

3.7.5 Disturbance Rejection Assessment

The disturbances in this phase are of low frequency, typically due to thrust misalignment or differential thrust. The disturbance rejection is dictated by the low frequency gain of
the controller. With integrator included, the LQG controller has as high a low frequency gain as the classical controller, leading to comparable disturbance rejection.

### 3.8 Comparison Between Performance with Classical, $\mathcal{H}_\infty$ and TP-LQG Designs

There are close similarities between the state space versions of $\mathcal{H}_\infty$ controllers and TP-LQG controllers. Both $\mathcal{H}_\infty$ and LQG controllers are optimal controllers based on minimization of a cost index. The dynamic cost weights have a similar effect in both types of cost function. Closed loop stability is guaranteed for both methods even if the plant is non-min. phase or unstable.

However, the basic conceptual idea behind $\mathcal{H}_\infty$ design involves minimization of transfer function magnitudes, which is quite different to the LQG requirement of minimizing a complex domain integral representing error and control power spectra. $\mathcal{H}_\infty$ methods attempt frequency response shaping of desired transfer functions, similar to the classical method. Standard classical and state space approaches have the advantage of simplicity over the $\mathcal{H}_\infty$ methodology. All three methods involve some amount of trial and error.

It is seen that TP-LQG design is more robust to slosh parameter perturbations than the other two design methods since it gives near-perfect phase stabilization as against classical and $\mathcal{H}_\infty$ design, where the modes have more phase lag for nominal system. Of the three methods, $\mathcal{H}_\infty$ is seen to be least robust to variations in the lightly damped slosh parameters.

The low frequency controller gain (in the frequency range below 0.1 rad/s) is comparable for TP-LQG design and classical design (since the integrator was explicitly built into the plant with LQG design). However, in spite of repeated iterations, the $\mathcal{H}_\infty$ design was seen to give a lower gain at low frequencies. In the frequency range from 0.05 rad/s to 10 rad/s, the $\mathcal{H}_\infty$ controller has highest gain of around $-4$dB, followed closely by LQG controller ($-6$dB). Hence the system bandwidth is higher and the transient response is good for both these design methods, with fast response, low overshoot and settling time. However, in the phase of flight with high slosh activity, the bandwidth of classical
design is low (deliberately lowered to improve slosh robustness) and hence the transient response is affected.

The comparison of the slosh responses for the classical design throughout the regime with the corresponding plots for the TP-LQG and $H_\infty$ design methods shows that the slosh mode magnitudes are similar to that with LQG design and about half of that with the $H_\infty$ design. While the LQG design is giving near perfect phase stabilization for the slosh modes, the placement of the slosh modes is not symmetric with respect to the real axis in either $H_\infty$ or classical designs—additional lag of more than 40 deg is there with respect to LQG, leading to reduced lag margins.

The noise rejection of the loop is better with TP-LQG design up to 100 rad/s, but the response flattens thereafter. Hence high frequency noise beyond 100 rad/s [16 Hz] gets constant attenuation of 34dB. Both classical and $H_\infty$ design give more attenuation at higher frequencies. The EB deck plate where gyros are mounted usually has natural vibration frequency around 50Hz where the three designs give almost equal attenuation. For noise frequency beyond this, classical design gives better rejection. The three designs are compared in Table 3.3.

The comparison table shows the advantage of TP-LQG methods over the other methods for the terminal phase launch vehicle control. The main reason for this is the peculiar nature of the problem, where the most important characteristics are the lightly damped, highly variable slosh dynamics, and the presence of very low frequency disturbances arising out of differential thrust between the engines, and due to the engine misalignment. TP-LQG handles this problem most satisfactorily and the near perfect phase stabilization property for slosh can be further exploited to remove the baffles provided at the top of the tank.

In all three designs, performance assessment is based on the nominal performance, along with that for worst-case combinations of the perturbed parameters, which are heuristically decided by the designer.
Table 3.3 Comparison of Performance with Classical, $H_\infty$ and TP-LQG Designs

<table>
<thead>
<tr>
<th></th>
<th>Classical</th>
<th>$H_\infty$</th>
<th>LQG</th>
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<tbody>
<tr>
<td>Stability</td>
<td>Requires controller parameter variation to</td>
<td>Requires controller parameter variation to</td>
<td>Stabilizes both nominal and perturbed system</td>
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<tr>
<td></td>
<td>maintain stability</td>
<td>maintain stability</td>
<td>with single controller</td>
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<tr>
<td>Robustness</td>
<td>Slosh modes have higher phase lag for nominal</td>
<td>Slosh modes have higher phase lag for nominal</td>
<td>Perfect phase stabilization for slosh in nominal</td>
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<tr>
<td></td>
<td>system. Hence less robust to parameter</td>
<td>system. Hence less robust to parameter</td>
<td>case hence robust to slosh parameter</td>
</tr>
<tr>
<td></td>
<td>perturbations</td>
<td>perturbations</td>
<td>perturbations</td>
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<tr>
<td>Disturbance rejection</td>
<td>High low frequency disturbance rejection,</td>
<td>Low stiffness to low frequency disturbances</td>
<td>Stiffness to low frequency disturbances</td>
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<td></td>
<td>because of integrator in forward path</td>
<td></td>
<td></td>
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<tr>
<td>Transient response</td>
<td>Lower rigid body bandwidth for slosh</td>
<td>Higher rigid body bandwidth. Hence good</td>
<td>Higher rigid body bandwidth. Hence good</td>
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<tr>
<td></td>
<td>stabilization, in the region of high slosh</td>
<td>transient response - fast response, with no</td>
<td>transient response - fast response, with low</td>
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<td>influence. Hence poor transient response in</td>
<td>overshoot and small settling time</td>
<td>overshoot and settling time.</td>
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<td>this region.</td>
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<tr>
<td>Noise attenuation</td>
<td>Less attenuation up to 16Hz</td>
<td>Less attenuation up to 16Hz</td>
<td>Constant attenuation of 34dB beyond 16Hz</td>
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<td></td>
<td>Same rejection as other schemes at EB deck</td>
<td>Same rejection as other schemes at EB deck</td>
<td>Same rejection as other schemes at EB deck</td>
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<td></td>
<td>plate natural vibration frequency (50Hz)</td>
<td>plate natural vibration frequency (50Hz)</td>
<td>plate natural vibration frequency (50Hz)</td>
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<td>Better rejection for noise frequency beyond</td>
<td>Better rejection for noise frequency beyond</td>
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<td></td>
<td>50Hz</td>
<td>50Hz</td>
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<tr>
<td>Optimization criteria</td>
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<td>Optimized with respect to tracking error, slosh</td>
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<td>mode response and control demand with relative</td>
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<td>weighting.</td>
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<tr>
<td>Controller complexity</td>
<td>Fourth order controller in forward path,</td>
<td>14th order equivalent controller</td>
<td>9th order controller in</td>
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<td>second order in rate and first order in</td>
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3.9 Summary

The autopilot design of the terminal phase of a typical launch vehicle has been carried out through both optimal and robust controller design methods and the response compared with that of the benchmark classical controller with respect to the different aspects of controller performance. For the TP-LQG design, appropriate weighting function has been arrived at, which ensures rigid body and slosh mode stability, with good gain and phase margins for the nominal and perturbed plant, throughout the flight regime. Satisfactory performance is obtained throughout with a single controller, with near perfect phase stabilization for the slosh modes. A unique two step approach for incorporation of robustness into TP-LQG design through tuning of weights based on transient response for the rigid body, followed by frequency response for the flexible dynamics, has also been demonstrated.

With the designed $H_\infty$ controller, the nominal system response is satisfactory throughout the regime; however, the robustness to slosh parameter perturbations is less than in the LQG method. It is seen that the LQG design methodology offers an alternate controller design, which actively stabilizes the slosh dynamics of the system without depending on the passive damping, provided by slosh baffles. There is basically a trade-off between extra hardware (slosh baffles) and software complexity (more number of filters and higher order filter) between the different designs. Performance assessment of all three designs is based on the performance for worst-case combinations of the perturbed parameters, which are heuristically decided by the designer. Hence there is always a possibility that some other combination of parameter perturbations can lead to lower margins, and degraded performance. For the exoatmospheric plant considered, due to the comparatively simple plant dynamics and small range of parameter variation, the intuitively identified worst cases may be sufficient for design evaluation. However, for more complex plants such as the aerodynamically unstable, atmospheric phase launch vehicle, the evaluation becomes complicated with this approach. This problem is addressed in subsequent chapters.
The significant contributions in the present chapter are:

(i) The development of a new Two Phase LQG approach (TP-LQG), in which the weights in the state weighting matrix are tuned in two phases: the first corresponding to the transient response and the second, the frequency response over the entire regime. The method combines the advantages of the time domain techniques with the physical insight of the frequency domain methods to provide a robust LQG design with near perfect phase stabilization of the lightly damped slosh dynamics.

(ii) A comprehensive comparison of the autopilot designed with several of the state-of-the-art techniques with respect to the different aspects of control system performance.