Chapter 4

Neutron polarization in
photodisintegration of deuterons

4.1 Introduction

As discussed in Chapter 1, the experimental and theoretical study of the photodisintegration of the deuteron and the radiative capture of neutrons by protons has a long history going back to [47] in the 1930’s. It was known quite early that the capture cross section is dominated by the isovector magnetic dipole emission whose amplitude may be denoted by $M_{1v}$. However in view of the then existing 10% discrepancy between potential model calculations [56] and experiment [51], Breit and Rustgi [59] proposed a polarized target-beam experiment to detect the possibility of an isoscalar magnetic dipole emission whose amplitude may be denoted as $M_{1s}$. The suggestion [59] was more or less ignored in view of the surprising accuracy with which the 10% discrepancy was explained [61] as due to meson exchange currents (MEC). Quoting the theoretical results including MEC, isobar and pair currents of Sato et al. [20] it was pointed out twenty five years late by Nagai et al. [23] that the theory deviated by about 15% from the earlier near threshold $d(\gamma, n)p$ cross section measurements [62],
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which are more relevant to astrophysics, although the theory is in good agreement with the measured cross section for neutrons above 14 MeV. Moreover, the measured [66] angular distribution and neutron polarization at photon energy of 2.75 MeV as well as between 6 to 13 MeV [67] were found to be in disagreement with the theory including MEC. Analyzing power measurements [68] in $p(\vec{n}, \gamma)d$ were found to be consistent with [67] and it was also found [24] earlier that the effect of including MEC is to increase the discrepancy between theory and experiment. Rustgi, Vyas and Chopra [57] drew attention to the unambiguous disagreement between experiment and theory on $d(\gamma, n)p$ at a photon energy of 2.75 MeV, which widens when two body effects are taken into account. There is considerable current experimental interest [13, 14, 15, 16, 17, 18] in studying $d(\vec{\gamma}, n)p$ at energies close to threshold, in view of the highlighted need [11] for precise knowledge of the reaction at astrophysically relevant energies as the Big Bang Nucleosynthesis (BBN) entered the precision era [9], where the primordial deuterium abundance is considered to be the Cosmic Baryometer.

The reaction $d(\vec{\gamma}, n)p$ with linearly polarized photons was discussed using a model independent theoretical formalism in Chapter 2, which revealed that the interference between the $E1_v$ and $M1_s$ amplitudes lead to a term proportional to $\cos \theta$ in the cross section, which is non-zero so long as the three $E1_j$ amplitudes in different total angular momentum channels $j = 0, 1, 2$ differ from each other. Recent experimental evidence [19] shows that these three amplitudes are unequal. It may also be anticipated that the current experimental studies on $d(\vec{\gamma}, n)p$ may be expected to extend to measure neutron polarization in the final state. Therefore, extending our model independent theoretical formalism, we have also envisaged a comprehensive study of the neutron polarization in photodisintegration of deuterons. In the section 4.2, neutron polarization with initially unpolarized deuteron and polarized photon (in general) is discussed, particular
cases of which are presented in the subsequent sections.

4.2 Cross section for Photodisintegration of unpolarized deuterons using polarized photons

The momentum $k$ of the photon in the c.m. frame is chosen to be along the $Z$-axis of a right handed coordinate system while the two independent linearly polarized states of the photon to be $\hat{\epsilon}_x$ and $\hat{\epsilon}_y$ along the $x$ and $y$ axes respectively. The left and right circular polarization states, $\mu = \pm 1$ are defined following [91]. The neutron momentum $p$ in the c.m. frame is chosen to have polar coordinates $(p, \theta, \phi)$ following [13]. The irreducible tensor amplitudes, $F^\lambda(s, \mu)$ of rank $\lambda$ are the same as in Chapter 2.

Choosing $\mu = \pm 1$ as basis states, the $2 \times 2$ density matrix $\rho^\gamma$ for polarized photons may be expressed [83] in the form

$$\rho^\gamma = \frac{1}{2I} [S_0 + \sigma^\gamma \cdot S],$$  \hspace{1cm} (4.1)

where $\sigma^\gamma$ denote the Pauli spin matrices for photon polarization while

$$S_0 = I,$$ \hspace{1cm} (4.2)

denotes the intensity. The three Stokes parameters, expressed in terms of the intensities in the polarized states, are given by

$$I S_1 = Tr(\sigma_\gamma^\gamma \rho^\gamma) = I(\hat{\epsilon}_x) - I(\hat{\epsilon}_y),$$  \hspace{1cm} (4.3)

$$I S_2 = Tr(\sigma_y^\gamma \rho^\gamma) = [I(\hat{\epsilon}_x + \hat{\epsilon}_y) \sqrt{2} - I(\hat{\epsilon}_x - \hat{\epsilon}_y) \sqrt{2}],$$ \hspace{1cm} (4.4)

$$I S_3 = Tr(\sigma_z^\gamma \rho^\gamma) = I(+1) - I(-1).$$  \hspace{1cm} (4.5)
The differential cross section for $d(\gamma, \vec{n})p$ is now given, in general, by

$$\frac{d\sigma}{d\Omega} = \frac{1}{3} \sum_{\mu\mu'} Tr [M(\mu) \rho_{\mu\mu'}^\gamma M(\mu')]$$, \hspace{1cm} (4.6)$$

where $Tr$ denotes the trace or spur with respect to the hadron spin states. Clearly, the above reduces to

$$\frac{d\sigma_0}{d\Omega} = \frac{1}{6} \sum_{s,\mu} (2s + 1) |F_\nu^\lambda(s,\mu)|^2$$, \hspace{1cm} (4.7)$$

with initially unpolarized photons.

### 4.3 Neutron polarization in experiments using 100% linearly polarized photons

In experiments[13, 14, 15, 16, 17, 18] using 100% linearly polarized photons, we have

$$M = M(+1) + M(-1)$$, \hspace{1cm} (4.8)$$

as was already noted in Eq. (2.50) so that

$$\frac{d\sigma}{d\Omega} = \frac{1}{6} Tr MM^\dagger.$$ \hspace{1cm} (4.9)$$

We may express

$$M = \sum_{s=0} \sum_{\lambda} (S^\lambda(s,1) \cdot F^\lambda(s))$$, \hspace{1cm} (4.10)$$

so that

$$\langle \frac{1}{2} m_n \frac{1}{2} m_p | M | 1 m_d \rangle = \sum_{\nu} C(\frac{1}{2} 1; s; m_n m_p m_s)(-1)^{\nu}[\lambda]$$

$$C(1\lambda s; m_d - \nu m_s) F_\nu^\lambda(s).$$ \hspace{1cm} (4.11)$$
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The density matrix, $\rho$ characterizing the neutron polarization in the final state is then defined in terms of its elements

$$
\rho_{m_n m'_n} = \frac{1}{6} \sum_{m_p, s, s'} C\left(\frac{1}{2} \frac{1}{2}; m_n m_p m_s\right) C\left(\frac{1}{2} \frac{1}{2}; m'_n m_p m'_{s}\right) \text{MM}^\dagger. \quad (4.12)
$$

Using Eq. (2.68) for $\text{MM}^\dagger$ we have

$$
\rho_{m_n m'_n} = \frac{1}{6} \sum_{m_p, s, s', \lambda, \lambda', K} C\left(\frac{1}{2} \frac{1}{2}; m_n m_p m_s\right) C\left(\frac{1}{2} \frac{1}{2}; m'_n m_p m'_{s}\right) [s']
$$

$$
(-1)^{s'-1}[\lambda][\lambda'] W\left(s' \lambda' s \lambda; 1K\right) (S^K(s, s') \cdot (F^\lambda(s) \otimes F^{\lambda'}(s'))^K). \quad (4.13)
$$

Noting that $\langle sm_{s'} | S^K_{s}(s, s') | s' m_s \rangle = [K]C(s'sK; m'_s - qm_s)$, we have

$$
\rho_{m_n m'_n} = \frac{1}{6} \sum_{m_p, s, s', \lambda, \lambda', K, q} C\left(\frac{1}{2} \frac{1}{2}; m_n m_p m_s\right) C\left(\frac{1}{2} \frac{1}{2}; m'_n m_p m'_{s}\right)
$$

$$
(-1)^{s'-1}[\lambda][\lambda'] W\left(s' \lambda' s \lambda; 1K\right) (-1)^q[K]
$$

$$
C(s'K s; m'_s - qm_s) (F^\lambda(s) \otimes F^{\lambda'}(s'))^K. \quad (4.14)
$$

Consider the three Clebsch-Gordan coefficients. Using the symmetry properties, they may be written as

$$
\sum_{m_p} C\left(\frac{1}{2} \frac{1}{2}; m_n m_p m_s\right) C\left(\frac{1}{2} \frac{1}{2}; m'_n m_p m'_{s}\right) C(s'K s; m'_s - qm_s)
$$

$$
= (-1)^{1+s+K} [s]^2 [K]^2 \frac{1}{\sqrt{2}[s']} W\left(s \frac{1}{2} \frac{1}{2} K\right) C\left(\frac{1}{2} \frac{1}{2} K \frac{1}{2} m'_n - qm_n\right). \quad (4.15)
$$

Thus

$$
\rho_{m_n m'_n} = \frac{1}{6 \sqrt{2}} \sum_{s, s', \lambda, \lambda', K, q} (-1)^{s' + s + K} (-1)^q[s][\lambda][\lambda'][K]^3 W\left(s' \lambda' s \lambda' ; 1K\right)
$$

$$
W\left(s \frac{1}{2} \frac{1}{2} K\right) C\left(\frac{1}{2} \frac{1}{2} K \frac{1}{2} m'_n - qm_n\right) (F^\lambda(s) \otimes F^{\lambda'}(s'))^K. \quad (4.16)
$$
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which may be written as

$$\rho_{m_n m'_n} = \sum_{K,q} (-1)^{q/2} [K] C\left(\frac{1}{2} K \frac{1}{2}; m'_n - q m_n\right) P^K_q, \quad (4.17)$$

where

$$P^K_q = \frac{1}{3\sqrt{2}} \sum_{s,s',\lambda,\lambda'} (-1)^{s+s'+K} [K] [s][\lambda][\lambda'] W(s'ss\lambda; 1K') \left(\mathcal{F}^\lambda(s) \otimes \mathcal{F}^{1\lambda'}(s')\right)_q^K. \quad (4.18)$$

Expressing $\rho$ in the standard form

$$\rho = \frac{1}{2} [1 + \sigma \cdot P]. \quad (4.19)$$

The neutron polarization $P$ is readily obtained in terms of the spherical components of $P^K_q$ on comparing $\rho$ with the standard form. Parity conservation implies that $P$ should be normal to the reaction plane, i.e., along $k \times p$, which is given by

$$P = \sqrt{2} \pi^2 \sin \theta \{2\sqrt{2} Im(E_{1v}^0 M_{1v}^*) + \frac{1}{\sqrt{3}} Im[(4E_{1v}^0 + 3E_{1v}^1 + 5E_{1v}^2) M_{1s}^*] + \sqrt{2} \cos \theta Im[(E_{1v}^0 + E_{1v}^1) E_{1v}^{2*}]\}, \quad (4.20)$$

where the first and the second terms represent respectively the interference of $M_{1v}$ and $M_{1s}$ with the $E_{1v}$ amplitudes, while the third represents interference between $E_{1v}$ amplitudes. It is interesting to add that polarizing the photon linearly, produces a component of $P$ along $k$, which is proportional to $\sin 2\phi \sin^2 \theta Im[(2E_{1v}^0 + 3E_{1v}^1) E_{1v}^{2*}]$, so that the third term in Eq. (4.20) can be estimated empirically from the measurement of this component of $P$. 
4.4 Spin entanglement in the final state

We might point out that Eq. (4.12) is sufficiently general that it can be used to study also the spin entanglement between the neutron and proton in the final state. For this purpose, we may note that $S_q^K(s, s')$ are expressible in terms of $\sigma_n$ and $\sigma_p$ as

$$S^0_0(00) = \frac{1}{4} (1 - \sigma_n \cdot \sigma_p)$$ (4.21)

$$S^0_0(11) = \frac{1}{4} (3 + \sigma_n \cdot \sigma_p)$$ (4.22)

$$S^1_\nu(01) = \frac{1}{2}\sqrt{\frac{2}{2}} (\sigma_n \otimes \sigma_p)^1_\nu - \frac{1}{4} (\sigma_n - \sigma_p)^1_\nu$$ (4.23)

$$S^1_\nu(10) = \frac{\sqrt{3}}{2\sqrt{2}} (\sigma_n \otimes \sigma_p)^1_\nu + \frac{\sqrt{3}}{4} (\sigma_n - \sigma_p)^1_\nu$$ (4.24)

$$S^1_\nu(11) = \frac{\sqrt{3}}{2\sqrt{2}} (\sigma_n + \sigma_p)^1_\nu$$ (4.25)

We may express the full density matrix using (2.56) and (2.63) as

$$\rho_{m_n,m_p;m_n',m_p'}^f = \frac{1}{6} \sum_{s,s'} C\left(\frac{\Lambda}{2}s; m_n m_p m_s\right) C\left(\frac{\Lambda}{2}s'; m_n' m_p' m_{s'}\right)$$

$$\sum_{\lambda,\lambda'} (S^\lambda(s, 1) \cdot F^{\lambda'}(s)) (-1)^{s'-1} \frac{[s']}{\sqrt{3}} (S^{\lambda'}(1, s') \cdot F^{\lambda'}(s'))$$

$$= \frac{1}{6} \sum_{s,s',\lambda,\lambda',\Lambda} (-1)^{s'-1} \frac{[s']}{\sqrt{3}} (-1)^{\lambda + \lambda' - \Lambda} C\left(\frac{\Lambda}{2}s; m_n m_p m_s\right) C\left(\frac{\Lambda}{2}s'; m_n' m_p' m_{s'}\right)$$

$$(S^\lambda(s, 1) \otimes S^{\lambda'}(1, s'))^\lambda \cdot (F^\lambda(s) \otimes F^{\lambda'}(s'))^\Lambda, \quad (4.26)$$

Using the known properties of the irreducible tensor operators, we have

$$\rho_{m_n,m_p;m_n',m_p'}^f = \frac{1}{6} \sum_{s,s',\lambda,\lambda',\Lambda} (-1)^{s'-1} [s'][\lambda][\lambda'] C\left(\frac{1}{2}s; m_n m_p m_s\right)$$

$$C\left(\frac{1}{2}s'; m_n' m_p' m_{s'}\right) W(s'\lambda s\lambda; 1\Lambda)$$

$$(S^\lambda(s, s') \cdot (F^\lambda(s) \otimes F^{\lambda'}(s'))^\Lambda) \cdot (4.27)$$
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Using \(< s m s | S_{-q}^\Lambda (s, s') | s' m_{s'} > = [\Lambda] C(s' \Lambda s; m_{s'} - q m_s)\), we have

\[
\rho^f_{m_n m_p; m'_{s'} m_p} = \frac{1}{6} \sum_{s, s', \lambda, \lambda', q} (-1)^q (-1)^{s'-1} \langle s' | [\lambda] | \lambda' \rangle C(\frac{1}{2} \frac{1}{2} s; m_n m_p m_s) \\
C(\frac{1}{2} \frac{1}{2} s'; m'_n m'_p m'_s) [\Lambda] C(s' \Lambda s; m_{s'} - q m_s) W(s' \lambda' s \lambda; 1 \Lambda) \\
(\mathcal{F}^\lambda(s) \otimes \mathcal{F}^{1\lambda'}(s'))^\Lambda.
\] (4.28)

Consider the three Clebsch-Gordan coefficients. Using the symmetry properties, they may be written as

\[
C(\frac{1}{2} \frac{1}{2} s; m_n m_p m_s) C(\frac{1}{2} \frac{1}{2} s'; m'_n m'_p m'_s) C(s' \Lambda s; m_{s'} - q m_s) \\
= (-1)^{1-m_n-s' - \Lambda} \langle s' | [\lambda] | \lambda' \rangle C(\frac{1}{2} \frac{1}{2} t_n; m'_n m_n - q_n) C(\frac{1}{2} \frac{1}{2} t_p; -m'_p m_p - q_p) \\
C(t_n t_p \Lambda; -q_n - q_p q) \left\{ \begin{array}{c} \frac{1}{2} \frac{1}{2} s' \\ \frac{1}{2} \frac{1}{2} s \\ t_n \ t_p \ \Lambda \end{array} \right\} \\
(\mathcal{F}^\lambda(s) \otimes \mathcal{F}^{1\lambda'}(s'))
\] (4.29)

\[
\rho^f_{m_n m_p; m'_n m'_p} = \sum_{q_t, t_p, \Lambda} (-1)^{m_p + m_n - q_t} \\
C(\frac{1}{2} \frac{1}{2} t_n; m'_n m_n - q_n) C(\frac{1}{2} \frac{1}{2} t_p; -m'_p m_p - q_p) \\
C(t_n t_p \Lambda; -q_n - q_p q) \left\{ \begin{array}{c} \frac{1}{2} \frac{1}{2} s' \\ \frac{1}{2} \frac{1}{2} s \\ t_n \ t_p \ \Lambda \end{array} \right\} \left\{ \begin{array}{c} t_n \ t_p \ \Lambda \end{array} \right\} \\
(\mathcal{F}^\lambda(s) \otimes \mathcal{F}^{1\lambda'}(s'))^\Lambda
\] (4.30)

where

\[
P^\Lambda_q = \sum_{s, s', \lambda, \lambda'} (-1)^{1-s-s' - \Lambda} \langle s' | [\lambda] | \lambda' \rangle C(\frac{1}{2} \frac{1}{2} s; m_n m_p m_s) \\
C(\frac{1}{2} \frac{1}{2} s'; m'_n m'_p m'_s) [\Lambda] C(s' \lambda s \lambda; 1 \Lambda) \\
W(s' \lambda' s \lambda; 1 \Lambda) \\
(\mathcal{F}^\lambda(s) \otimes \mathcal{F}^{1\lambda'}(s'))^\Lambda [\lambda] [\lambda']
\] (4.31)
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It may be pointed out that the above equation can be used to study the $n - p$ spin entanglement.

## 4.5 Conclusions

We have studied the neutron polarization in the final state, with both initially unpolarized and linearly polarized photons. Expression for the neutron polarization in the final state is derived. We found that the neutron spin is sensitive to the initial polarization of the photon. On deriving the full density matrix for the $n - p$ system in the final state, we also noted that the equation can be used to study the spin entanglement.

It is interesting to note that Eq. (4.20) contains the interference between $M_{1s}$ and $E_{1v}$ amplitudes. It may also be pointed out that the term $c \cos \theta$ exists not only in Eq. (2.79) but also in the unpolarized differential cross section $\frac{d \sigma}{d \Omega}$ itself and $M_{1s}$ produces observable effects in $\tilde{d} (\gamma, n)p$ as well. It is therefore, important to extend the recent measurements [16, 17] to lower energies and study neutron polarization at the energies of astrophysical interest.