Chapter 3

Target analyzing powers in Photodisintegration of deuterons

3.1 Photodisintegration of polarized deuterons by unpolarized photons

3.1.1 Introduction

We have examined in the previous Chapter, the utility of employing a beam of linearly polarized photons to study the photodisintegration of deuterons. Complementary to this, we present here a model independent theoretical study of the reaction employing polarized targets but with unpolarized photons. This is done in section 3.1. Since experiments [13, 14, 15, 16, 17, 18, 19] are underway on unpolarized deuterons using linearly polarized lasers, it should be feasible to study photodisintegration of polarized deuterons by linearly polarized lasers. Since aligned deuterons are expected to be present when they are embedded in external electric quadrupole fields, we also present under section 3.2, the photodisintegration of aligned deuterons by linearly polarized photons using the same formalism developed in Chapter 2.
3.1.2 Polarized deuterons

The deuteron is a spin-1 nucleus, which gets polarized in different ways in different external environments. For example, an oriented [93, 94, 95] deuteron target can be produced under the influence of an external uniform magnetic field, whose direction determines the axis of orientation. On the other hand, when the deuteron is exposed to external electric quadrupole fields generated by surrounding electrons in crystal lattice sites [96], the spin of the deuteron is aligned. States of polarization of spin-1 nuclei exist, which are more complex and are in fact multiaxial [97, 98, 99, 100, 101]. The state of polarization of a spin-1 nucleus is, in general, specified by the spin density matrix

$$\rho = \frac{\text{Tr} \rho}{3} \left\{ \begin{array}{ccc}
1 + \frac{3}{2}t^0_1 + \frac{1}{\sqrt{2}}t^2_0 & \frac{3}{2}(t^1_{-1} + t^2_{-1}) & \sqrt{3}t^2_{-2} \\
-\frac{3}{2}(t^1_1 + t^2_1) & 1 - \sqrt{3}t^2_0 & \frac{3}{2}(t^1_{-1} + t^2_{-1}) \\
\sqrt{3}t^2_2 & -\frac{3}{2}(t^1_1 + t^2_1) & 1 - \frac{3}{2}t^1_0 + \frac{1}{\sqrt{2}}t^2_0
\end{array} \right\}, \quad (3.1)$$

which is hermitian and is parametrised in terms of the Fano [102] statistical tensors $t^k_q, q = k, k - 1, \ldots, -k$. The $t^k_q$ with $k = 1$ refer to the vector polarization and those with $k = 2$ refer to the tensor polarization of the deuteron. The normalization used for the $t^k_q$ in Eq. (3.1) differs from [102] and follows [97, 98, 99, 100, 101, 103, 104, 105, 106, 107]. The rows and columns of the above matrix are labeled by the states $|1m\rangle$ of the deuteron spin-1 with magnetic quantum numbers $m = +1, 0, -1$ in that order w.r.t a chosen quantization axis which is referred to usually as the $Z$-axis. The target is said to be unpolarized if all the $t^k_q = 0$. Defining vector polarization through

$$t^0_0 = \sqrt{\frac{3}{2}}P_z \quad t^{1\pm1}_1 = \mp \frac{1}{2}\sqrt{3}(P_x + iP_y), \quad (3.2)$$
the target is said to be purely vector polarized if the vector polarization $\vec{P} \neq 0$ but $t_q^2 = 0$. Defining the tensor polarization through a traceless symmetric 2\textsuperscript{nd} rank cartesian tensor $T_{\alpha\beta}; \alpha, \beta = x, y, z$ given by

$$t_0^2 = \frac{1}{\sqrt{2}}(T_{zz}); \quad t_{\pm 1}^2 = \mp \frac{1}{\sqrt{3}}(T_{xx} \pm iT_{yz}) \quad t_{\pm 2}^2 = \frac{1}{2\sqrt{3}}(T_{xx} - T_{yy} \pm 2iT_{xy}),$$

(3.3)

the target is said to be purely tensor polarized or aligned if $\vec{P} = 0$.

### 3.1.3 Differential cross section with a polarized target

We extend our model independent theoretical approach to discuss $d^+ + \gamma \rightarrow n + p$, for which the differential cross-section in c.m. frame may be written, using the same notations as in Chapter 2, in the form

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{\mu} Tr[M(\mu)\rho M(\mu)^\dagger],$$  \hspace{1cm} (3.4)

where the state of polarization of the deuteron in the initial state is specified by the density matrix

$$\rho = \frac{1}{3} \sum_{k=0}^{2} (S^k(1, 1) \cdot t^k),$$  \hspace{1cm} (3.5)

in terms of Fano statistical tensors $t^k$ of rank $k$.

Using Eq. (2.53), we have

$$M(\mu)\rho M(\mu)^\dagger = \frac{1}{3} \sum_{s,s',\lambda,\lambda',k} (S^\lambda(s, 1) \cdot F^\lambda(s, \mu))(S^{k}(1, 1) \cdot t^k)(-1)^{s'-1}\frac{[s']}{\sqrt{3}} \quad (S^{\lambda'}(1, s') \cdot F^{\lambda'}(s', \mu)).$$  \hspace{1cm} (3.6)
using the known properties of irreducible tensor operators. We next express

\[ (S^\lambda(s, 1) \cdot F^\lambda(s, \mu))(S^k(1, 1) \cdot t^k) = \sum_{k'} ((S^\lambda(s, 1) \otimes S^k(1, 1))^{k'} \cdot (F^\lambda(s, \mu) \otimes t^k)^{k'}) (-1)^{\lambda+k-k'} . \]  

(3.7)

Noting that

\[ (S^\lambda(s, 1) \otimes S^k(1, 1))^{k'}_{q'} = (-1)^{\lambda+k-k'}[1][\lambda][k]W(1k s\lambda; 1k')(S^k'(s, 1))^{k'}_{q'} , \]  

(3.8)

we have

\[ M(\mu) \rho M^\dagger(\mu) = \frac{1}{3} \sum_{s, s', \lambda, \lambda', k, k'} (-1)^{s'-1[s'][\lambda][k]}W(1k s\lambda; 1k') \]

\[ (S^k'(s, 1) \cdot (F^\lambda(s, \mu) \otimes t^k)^{k'})(S^\lambda'(1, s') \cdot F^{\dagger\lambda'}(s', \mu)) , \]  

(3.9)

Using once again the properties of irreducible tensor operators (refer to Appendix A), we write

\[ (S^k'(s, 1) \cdot (F^\lambda(s, \mu) \otimes t^k)^{k'})(S^\lambda'(1, s') \cdot F^{\dagger\lambda'}(s', \mu)) = \sum_{\Lambda} (-1)^{k'+\lambda'-\Lambda} \]

\[ (S^k'(s, 1) \otimes S^{\lambda'}(1, s'))^\Lambda \cdot ((F^\lambda(s, \mu) \otimes t^k)^{k'} \otimes F^{\dagger\lambda'}(s', \mu))^\Lambda . \]  

(3.10)

Now by interchanging the positions of \( t^k \) and \( F^\lambda(s\mu) \) and by using the property,

\[ (S^k'(s, 1) \otimes S^{\lambda'}(1, s'))^\Lambda_{Q} = (-1)^{k'+\lambda'-\Lambda}[1][\lambda'][k']W(s'\lambda'sk'; 1\Lambda)S^\Lambda_{Q}(s, s') , \]  

(3.11)

we have

\[ (S^k'(s, 1) \cdot (F^\lambda(s, \mu) \otimes t^k)^{k'})(S^\lambda'(1, s') \cdot F^{\dagger\lambda'}(s', \mu)) = \sum_{\Lambda} (-1)^{k+\lambda-k'}[1][\lambda'][k'] \]

\[ W(s'\lambda'sk'; 1\Lambda)(S^\lambda(s, s') \cdot ((t^k \otimes F^\lambda(s, \mu))^{k'} \otimes F^{\dagger\lambda'}(s', \mu))^\Lambda) . \]  

(3.12)
Thus, we have
\[ M(\mu) \rho M^\dagger(\mu) = \sum_{s,s',\lambda,\lambda',k,k',\Lambda} (-1)^{s'-1} [s'][\lambda][k] W(1ks\lambda; 1k') (-1)^{k+\lambda-k'} 
\left( S^\Lambda(s,s') \cdot \left( (t^k \otimes F^\lambda(s,\mu))^{k'} \otimes F^{1\lambda'}(s',\mu) \right) \right)^\Lambda. \] (3.13)

Now consider the coupling of 3 angular momenta,
\[ \left( (t^k \otimes F^\lambda(s,\mu))^{k'} \otimes F^{1\lambda'}(s',\mu) \right)^\Lambda = \sum_{\lambda''} W(k\lambda\lambda'; k'\lambda'') [k'][\lambda''] \nonumber 
\left( t^k \otimes (F^\lambda(s,\mu) \otimes F^{1\lambda'}(s',\mu))^{\lambda''} \right)^\Lambda. \] (3.14)

Thus
\[ M(\mu) \rho M^\dagger(\mu) = \frac{1}{\sqrt{3}} \sum_{s,s',\lambda,\lambda',k,k',\Lambda} (-1)^{s'-1} (-1)^{k+\lambda-k'} [s'][\lambda][k'][\lambda'] W(1ks\lambda; 1k') \nonumber 
W(s'\lambda'sk'; 1\Lambda) W(k\lambda\lambda'; k'\lambda'') [k'][\lambda''] \nonumber 
\left( S^\Lambda(s,s') \cdot \left( t^k \otimes (F^\lambda(s,\mu) \otimes F^{1\lambda'}(s',\mu))^{\lambda''} \right) \right)^\Lambda. \] (3.15)

On taking the trace of the above equation, only \( \Lambda = 0 \) with \( s' = s \) contributes. Noting that \( S^0_0(s,s) = 1 \), we have
\[ \frac{d\sigma}{d\Omega} = \frac{1}{2\sqrt{3}} \sum_{s,\lambda,\lambda',k,\mu} (-1)^{\lambda}[s][\lambda][\lambda'] W(1ks\lambda; 1\lambda')(t^k \cdot (F^\lambda(s,\mu) \otimes F^{1\lambda'}(s,\mu))^k). \] (3.16)

The \( F^{\lambda}_{\nu}(s,\mu) \) are related to the complex conjugates \( F^{\lambda}_{\nu}(s,\mu)^* \) of \( F^{\lambda}_{\nu}(s,\mu) \) given by Eq.(2.54) through \( F^{\lambda}_{\nu}(s,\mu) = (-1)^{\nu} F^{\lambda}_{-\nu}(s,\mu)^* \). Noting that \( t^0_0 = 1 \) and the unpolarized differential cross section is given by
\[ \frac{d\sigma_0}{d\Omega} = \frac{1}{6} \sum_s (2s + 1) \sum_{\lambda} (-1)^{\lambda}[\lambda] \sum_{\mu} (F^\lambda(s,\mu) \cdot F^{1\lambda}(s,\mu)), \] (3.17)
we may express (3.16) in the form

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} [1 + \sum_{k=1}^{2} (t^k \cdot A^k)],
\]

(3.18)

where \(A^k\) denote the target analyzing powers which are measurable experimentally along with Eq. (3.17). Explicitly, the unpolarized differential cross section is given by

\[
\frac{d\sigma_0}{d\Omega} = \frac{8\pi^2}{3} \left[ a_0 + a_1 \cos \theta + a_2 \cos^2 \theta \right],
\]

(3.19)

where

\[
a_0 = |M_{0010}^{10}|^2 + 3\left( \frac{1}{2} |E1(0)|^2 + \frac{3}{4} |E1(1)|^2 + \frac{7}{4} |E1(2)|^2 + |M_{0111}^{01}|^2 + |E_{0121}^{01}|^2 \right),
\]

(3.20)

\[
a_1 = -3\sqrt{6} \left[ \text{Re}(E1(1)M_{0111}^{01}) + \text{Re}(E1(2)E_{0121}^{01}) \right],
\]

(3.21)

\[
a_2 = -\frac{3}{2} |E1(0)|^2 + \frac{9}{4} |E1(1)|^2 - \frac{3}{4} |E1(2)|^2,
\]

(3.22)

in terms of the partial wave multipole amplitudes. Note that Eq.(3.19) leads to the unpolarized total cross section

\[
\sigma_0 = \frac{32\pi^3}{3} \left[ |M_{0011}^{10}|^2 + |E1(0)|^2 + 3|E1(1)|^2 + 5|E1(2)|^2 + 3|M_{0111}^{01}|^2 + 3|E_{0111}^{01}|^2 \right],
\]

(3.23)

on integration with respect to \(d\Omega\). The first four terms represent the contributions from the isovector partial wave multipole amplitudes, whereas the last two terms represent the contributions from the isoscalar multipole amplitudes. Noting that Eq. (3.19) may also be written as

\[
\frac{d\sigma_0}{d\Omega} = \frac{2\pi^2}{6} [a + b \sin \theta + c \cos \theta],
\]

(3.24)
where the coefficients \( a, b, c \) are defined by Eqs. (2.80), (2.81) and (2.82) of Chapter 2. Using this form, \( \frac{d\sigma_0}{d\Omega} \) can also be determined in the same way as \( \frac{d\sigma}{d\Omega} \) given by (2.79). That is the third term can be determined by taking the difference between measurements of at two angles \( \theta(\neq \pi/2) \) and \( \pi - \theta \). It would therefore be desirable to carry out measurements at \( \theta \neq 90^\circ \) and at \( \pi - \theta \) at the lower energies, to determine \( c \). The coefficient \( b \) is readily determined by taking the difference between (2.79) and (3.24) at any angle \( \theta \neq 0 \) or \( \pi \) and for any value of \( \phi \neq \pi/4 \). Since \( b \) and \( c \) are thus known, one can determine \( a \) by measuring (2.79) or even (3.24). Thus \( a, b, c \) given by (2.80), (2.81) and (2.82) are determinable empirically without making simplifying assumptions as in [13].

### 3.1.4 Target analyzing powers

Explicitly, the target analyzing powers \( A^k_q \) are given by

\[
\frac{d\sigma_0}{d\Omega} A^2_q = 4\sqrt{2}\pi^2 \frac{3}{3}[b_0 + b_1 \cos \theta + b_2 \cos^2 \theta],
\]

(3.25)

\[
\frac{d\sigma_0}{d\Omega} A^1_1 = \pi^2 \sin \theta[c_0 - c_1 \cos \theta],
\]

(3.26)

and

\[
\frac{d\sigma_0}{d\Omega} A^2_1 = \pi^2 \sin \theta[d_0 - d_1 \cos \theta],
\]

(3.27)

while

\[
A^1_0 = A^2_\pm 2 = 0; A^1_{-2} = -A^1_1; A^2_{-1} = A^2_1.
\]

(3.28)
where

\[ b_0 = |\mathcal{M}_{00;1}^{10}|^2 + \frac{9}{4}|E1(1)|^2 + \frac{15}{4}|E1(2)|^2 - \frac{3}{2}|\mathcal{M}_{01;1}^0|^2 - \frac{3}{2}|\mathcal{E}_{01;2}^{01}|^2 \]
\[ -3\text{Re}(E1(0)E1(2)^*) - 9\text{Re}(\mathcal{M}_{01;1}^{01}\mathcal{E}_{01;2}^{01*}), \] (3.29)

\[ b_1 = 3\sqrt{\frac{3}{2}}\{\text{Re}[(E1(1) + 3E1(2))\mathcal{M}_{01;1}^{01*}] + \text{Re}[(3E1(1) + E1(2))\mathcal{E}_{01;2}^{01*}]\}, \] (3.30)

\[ b_2 = -\frac{9}{2}|E1(1)|^2 - 6|E1(2)|^2 - \frac{27}{2}\text{Re}(E1(1)E1(2)^*) + 3\text{Re}(E1(0)E1(2)^*), \] (3.31)

\[ c_0 = \sqrt{2}\{\text{Im}[(4E1(0) - 3E1(1) - 5E1(2))\mathcal{M}_{01;1}^{01*}] \]
\[ -3\text{Im}[(E1(1) + 3E1(2))\mathcal{E}_{01;2}^{01*}]\}, \] (3.32)

\[ c_1 = 2\sqrt{3}\{\text{Im}[(2E1(0)^* + E1(2)^*)E1(1)]\}, \] (3.33)

\[ d_0 = \sqrt{2}\{\text{Re}[(4E1(0) - 3E1(1) - 5E1(2))\mathcal{E}_{01;2}^{01*}] \]
\[ -\text{Re}[(3E1(1) + 9E1(2))\mathcal{M}_{01;1}^{01*}]\}, \] (3.34)

\[ d_1 = \sqrt{3}\{3|E1(1)|^2 - 4\text{Re}([-E1(0) + 3E1(1)]E1(2)^*) + 5|E1(2)|^2\}, \] (3.35)

### 3.1.5 Discussion

Clearly, \(A_1^1\) and \(A_2^1\) at \(\theta = \pi/2\) go to zero if the isoscalar amplitudes are absent. Therefore an empirical determination of \(c\) and the analyzing power \(A_0^2\) at \(\theta \neq \pi/2\) appears desirable before carrying out the more incisive analysis of the experimental data.
3.2 Photodisintegration of aligned deuterons by linearly polarized photons

3.2.1 Introduction

Polarization observables are expected to be sensitive to important dynamical details and thus allow in general much more stringent tests of theoretical models. The model independent theoretical formalism [2] for deuteron photodisintegration with linearly polarized photons, discussed in Chapter 2, revealed for the first time that the differential cross section in $d(\vec{\gamma}, n)p$ contains a term representing the interference between the $E_{1v}$ and $M_{1s}$ amplitudes, which is non zero if the three $E_{1v}$ amplitudes $E_{1v}^j$, $j = 0, 1, 2$ are unequal. In a recent publication, Blackston et al., [19] have in fact reported the first experimental observation of the splitting of the three $E_{1v}$ amplitudes.

The model independent theoretical formalism for deuteron photodisintegration with linearly polarized photons, discussed in Chapter 2, is extended to study the utility of employing aligned deuteron targets in photodisintegration, to study further the contribution of isoscalar $M_{1s}$ and $E_{2s}$ amplitudes.

3.2.2 Aligned deuterons

The deuteron is a spin-1 nucleus, which is said to be aligned, if its vector polarization is zero and tensor polarization is non-zero. Although the energy levels of spin 1 nuclei like deuteron (with non-zero electric quadrupole moment) have been studied in external electric quadrupole fields [108, 109, 110, 111], Ramachandran, Ravishankar, Sandhya and Swarnamala Sirsi [112] are probably the earliest to draw attention to the non-orientedness of the nuclear polarization in such an environment when the asymmetry parameter $\eta$ of the external electric quadrupole field with strength $A$ is non-zero.
Precise estimates of the parameters $A$ and $\eta$ when they are embedded in various compounds and at different sites are available [96, 108, 109, 110, 111]. The estimates of $t^2_q$ are also available [113] at a temperature of order mK. With the present day technology, in the case of ultra cold atoms in optical lattice, the lowest temperatures reached are of the order of nK [114] and pK [115]. It could therefore be feasible to prepare aligned deuteron targets with higher values of tensor polarization.

An external electric quadrupole field tensor $V_{\alpha\beta}$ is represented by $V_{XX}$, $V_{YY}$ and $V_{ZZ}$ in the Principal Axis Frame (PAF) [113], where $V_{\alpha\beta} \propto \delta_{\alpha\beta}$. When a spin-1 nucleus with a non-zero nuclear electric quadrupole moment $Q$ is exposed to such an external electric quadrupole field, the resulting polarization is describable in terms of the Fano statistical tensors $t^k_q$ where $t^1_q = t^2_{\pm 1} = Im(t^2_q) = 0$, which define the Principal Axes of Alignment Frame (PAAF) [104, 105, 106] which coincides with PAF in this case when no other external fields like magnetic field are present. The interaction Hamiltonian is well known and is of the form

$$H_{int} = A[(3J^2_Z - J^2) + \eta(J^2_X - J^2_Y)], \quad (3.36)$$

where $J_X$, $J_Y$ and $J_Z$ are the cartesian components of the nuclear spin $\vec{J}$ with $J^2 = \vec{J} \cdot \vec{J}$ and $A$ is proportional to the nuclear electric quadrupole moment $Q$ through

$$A = \frac{1}{4} Q V_{ZZ}, \quad \eta = \frac{V_{XX} - V_{YY}}{V_{ZZ}}; \quad |V_{ZZ}| \geq |V_{YY}| \geq |V_{XX}|. \quad (3.37)$$
The resulting eigen states \(|X>, |Y>, |Z>\), with energy eigen values \(A(1 \pm \eta)\) and 
\(-2A\) respectively, are given by

\[ |X\rangle = \frac{1}{\sqrt{2}}(|1 - 1 > -|1 1 >); \quad E_X = A(1 + \eta) \]
\[ |Y\rangle = \frac{i}{\sqrt{2}}(|1 - 1 > +|1 1 >); \quad E_Y = A(1 - \eta) \]
\[ |Z\rangle = |10 >; \quad E_Z = -2A, \] (3.38)

in terms of magnetic substates \(|1m\rangle\) defined with respect to the Z-axis of the PAAF, 
where \(t^2_{\pm 1} = 0\)

The state of the aligned deuteron is characterized by two axes \([100, 101]\) which are 
along \(n_1 = (\theta_1, \phi_1)\) and \(n_2 = (\theta_2, \phi_2)\) in PAAF. The \(\pm n_1\) and \(\pm n_2\) are determinable 
by setting \(t^2_2 = 0\), which leads to a quadratic equation having in general two solutions. 
In the particular case where \(n_1\) and \(n_2\) are collinear, the polarized spin-1 system is ori-
ented. Otherwise it is said to be non-oriented. Some typical cases [113] are illustrated 
in Table 3.1.

3.2.3 Differential cross section

Using the same notations as in Chapter 2, the differential cross section in c.m. frame 
for photodisintegration of aligned deuterons by linearly polarized photons is given by

\[ \frac{d\sigma}{d\Omega} = \frac{1}{6} \sum_{\mu} Tr[M\rho M^\dagger], \] (3.39)

where \(\rho\) is specified by Eq. (3.1) with non-zero \(t^2_q\). Using Eq. (2.56), we have

\[ M\rho M^\dagger = \frac{1}{3} \sum_{s,s',\lambda,\lambda',k} (S^\lambda(s, 1) \cdot F^\lambda(s))(S^k(1, 1) \cdot t^k)(-1)^{s'-1}[s'] \sqrt{3} \]
\[ (S^{\lambda'}(1, s') \cdot F^{\lambda'}(s')) \] (3.40)
Using the known properties of irreducible tensor operators, we have

\[ M_\rho M^\dagger = \frac{1}{\sqrt{3}} \sum_{s,s',\lambda,\lambda',k,k',\lambda'',\Lambda} (-1)^{s'-1} (-1)^{k+\lambda-k'} [s'][\lambda][k][k'][\lambda'] W(1ks\lambda; 1k') W(s'\lambda'sk'; 1\Lambda) W(k\lambda\lambda'; k'\lambda'')[\lambda''] \left( S^\Lambda(s, s') \cdot (t^k \otimes (F^\lambda(s) \otimes F^{\dagger\lambda'}(s')))^{\lambda''} \right). \]  

(3.41)

On taking the trace of the above equation, only \( \Lambda = 0 \) with \( s' = s \) contributes. Noting that \( S^0_0(s, s) = 1 \) and \( k = 2 \) for an aligned target, we have

\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{aligned}} = \frac{1}{2\sqrt{3}} \sum_{s,\lambda,\lambda'} (-1)^{\lambda}[s][\lambda'][W(12s\lambda; 1\lambda')(t^2 \cdot (F^\lambda(s) \otimes F^{\dagger\lambda'}(s')))^2], \]  

(3.42)

which may be written in the form

\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{aligned}} = \frac{d\sigma}{d\Omega}[1 + (t^2 \cdot A^2)], \]  

(3.43)

where the analyzing powers are given by

\[ A^2_q = \frac{1}{2\sqrt{3}} \sum_{s,\lambda,\lambda'} (-1)^{\lambda}(2s + 1)[s][\lambda'][W(12s\lambda; 1\lambda')(F^\lambda(s) \otimes F^{\dagger\lambda'}(s')))^2. \]  

(3.44)

### 3.2.4 Analyzing powers

Explicitly, the analyzing powers are expressible in the form,

\[ A^2_0 = \sqrt{2}\pi^2[a_0 + b_0 \sin^2 \theta + 2c_0 \sin^2 \theta \cos 2\phi + d_0 \cos \theta], \]  

(3.45)

where

\[ a_0 = \left[ \frac{4}{3} |M_1|^2 - 2|M_1|^2 - 2|E2|^2 - 3|E1(1)|^2 - 3|E1(2)|^2 \
- 18 Re[(E1(2)E1^*(1)] - 6 Re(E2^* M1^*_1) \right]. \]  

(3.46)
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\[ b_0 = 2 \left[ 3|E_{1v}(1)|^2 + 4|E_{1v}(2)|^2 + \text{Re}(2E_{1v}^*(0) + 9E_{1v}^*(1))E_{1v}(2) \right], \]  

\[ (3.47) \]

\[ c_0 = \left[ -\frac{3}{2}|E_{1v}(1)|^2 - \frac{1}{2}|E_{1v}(2)|^2 + 2\text{Re}(E_{1v}(2)E_{1v}^*(0)) \right], \]  

\[ (3.48) \]

\[ d_0 = \frac{4}{\sqrt{6}} \left[ \frac{3}{2}\text{Re}(E_{1v}(1) + 3E_{1v}(2))M_1^s + \frac{3}{2}\text{Re}(3E_{1v}(1) + E_{1v}(2))E_2^s \right], \]  

\[ (3.49) \]

and

\[ A_2^2 = 2\sqrt{3}\pi^2[a_2 + b_2 \sin^2 \theta + c_2 \sin^2 \theta e^{2i\phi} + d_2 \cos \theta], \]  

\[ (3.50) \]

\[ a_2 = \left[ \frac{2}{3}|M_{1v}|^2 - |M_1^s|^2 - |E_1^v|^2 - \frac{3}{2}|E_{1v}(1)|^2 - \frac{3}{2}|E_{1v}(2)|^2 \right. \]  

\[ \left. + 3\text{Re}(E_{1v}(2)E_{1v}^*(1)) + \text{Re}(E_2^sM_1^s) \right], \]  

\[ (3.51) \]

\[ b_2 = \left[ \frac{3}{2}|E_{1v}(1)|^2 + \frac{1}{2}|E_{1v}(2)|^2 - 2\text{Re}(E_{1v}(2)E_{1v}^*(0)) \right], \]  

\[ (3.52) \]

\[ c_2 = \left[ -|E_{1v}(2)|^2 - \text{Re}(2E_{1v}^*(0) + 3E_{1v}^*(1))E_{1v}(2) \right], \]  

\[ (3.53) \]

and

\[ d_2 = \sqrt{\frac{3}{2}}\text{Re}[(E_{1v}(2) - E_{1v}(1))(M_1^s - E_2^s)]. \]  

\[ (3.54) \]

\[ 3.2.5 \quad \text{Discussion} \]

It is important to note that the coefficients of \( \cos \theta \) in the differential cross section \( \frac{d\sigma}{d\Omega} \) as well as in the analyzing powers \( A_0^2 \) and \( A_2^2 \) (i.e., \( c, d_0 \) and \( d_2 \)) involve interference between the three isovector \( E_{1v}(\lambda) \) amplitudes and the isoscalar \( M_1^s \) and \( E_2^s \) ampli-
Target analyzing powers.......

tudes. We can express $d_0$ and $d_2$ as

\[ d_0 = \text{Re}(\alpha_0 M_1 + \beta_0 E_2), \]

(3.55) \hspace{1cm}

\[ d_2 = \text{Re}(\alpha_2 M_1 + \beta_2 E_2), \]

(3.56) \hspace{1cm}

where

\[ \alpha_0 = -4\sqrt{6}(|E_{1v}^{j=0}| - 3|E_{1v}^{j=1}| + 2|E_{1v}^{j=2}|), \]

(3.57) \hspace{1cm}

\[ \beta_0 = -4\sqrt{6}(|E_{1v}^{j=0}| + 3|E_{1v}^{j=1}| - 4|E_{1v}^{j=2}|), \]

(3.58) \hspace{1cm}

\[ \alpha_2 = -\beta_2 = \sqrt{\frac{2}{3}}(E_{1v}^{j=0} - E_{1v}^{j=2}), \]

(3.59) \hspace{1cm}

in terms of three $E_{1v}^{j}$, $j = 0, 1, 2$ amplitudes.

It is interesting to note that the above equations involve different linear combinations of the three $E_{1v}^{j}$ amplitudes and as such facilitate their study individually at lower energies of interest to astrophysics.

### 3.3 Some available quantitative estimates

The experimental finding [19] at 14 MeV and 16 MeV that all the three $E_{1v}^{j}$ amplitudes are not equal is encouraging. As discussed in Section 2.10 of Chapter 2, the approximate values of the squares of the 3 $E_{1v}^{j}$ amplitudes can be read out from Figure 7 of the first experimental observations [19] of the splittings of the $E1$ p-wave amplitudes. The linear combinations, involve their relative phase values given by

\[ \alpha_0 = -4\sqrt{6}(|E_{1v}^{j=0}|e^{i\delta_0} - 3|E_{1v}^{j=1}|e^{i\delta_1} + 2|E_{1v}^{j=2}|e^{i\delta_2}) \]

(3.60) \hspace{1cm}

\[ \beta_0 = -4\sqrt{6}(|E_{1v}^{j=0}|e^{i\delta_0} + 3|E_{1v}^{j=1}|e^{i\delta_1} - 4|E_{1v}^{j=2}|e^{i\delta_2}) \]

(3.61)
\[ \alpha_2 = -\beta_2 = \sqrt{\frac{2}{3}}(\mid E_{1_v}^{j=0} \mid e^{i\delta_0} - \mid E_{1_v}^{j=2} \mid e^{i\delta_2}) \]  

(3.62)

Since \( \mid E_{1_v}^{j=1} \mid \) appears to be leading (as seen from Table 2.2), we may write

\[ \alpha_0 = -12\sqrt{6}e^{i\delta_1}(\frac{1}{3}\mid E_{1_v}^{j=0} \mid e^{i(\delta_0 - \delta_1)} - \mid E_{1_v}^{j=1} \mid + \frac{2}{3}\mid E_{1_v}^{j=2} \mid e^{i(\delta_2 - \delta_1)}) \]  

(3.63)

\[ \beta_0 = 12\sqrt{6}e^{i\delta_1}(\frac{1}{3}\mid E_{1_v}^{j=0} \mid e^{i(\delta_0 - \delta_1)} + \mid E_{1_v}^{j=1} \mid - \frac{4}{3}\mid E_{1_v}^{j=2} \mid e^{i(\delta_2 - \delta_1)}) \]  

(3.64)

Observing that the relative phases have to lie between the extreme values \(-1\) and \(+1\), the values of \( \alpha_0, \beta_0, \alpha_2 = -\beta_2 \) are tabulated in Table 3.2 corresponding to the choices \((+1, +1), (+1, -1), (-1, +1)\) and \((-1, -1)\). We may now write \( d_0 \) and \( d_2 \) as

\[ d_0 = \mid \alpha_0 \mid \mid M_{1_s} \mid \cos(\delta_1 - \delta_M) + \mid \beta_0 \mid \mid E_{2_s} \mid \cos(\delta_1 - \delta_E) \]

\[ d_2 = \mid \alpha_2 \mid (\mid M_{1_s} \mid \cos(\delta_1 - \delta_M) - \mid E_{2_s} \mid \cos(\delta_1 - \delta_M) \]  

(3.65)

where \( \delta_M \) and \( \delta_E \) denote the phases of \( M_{1_s} \) and \( E_{2_s} \) respectively. Therefore the experimental studies on the target analyzing powers are expected to prove useful to study the role of the isoscalar amplitudes.

### 3.4 Conclusions

Using our formalism, we derived expressions for the differential cross section and target analyzing power with polarized deuterons in terms of multipole amplitudes. We noted that the unpolarized differential cross section, Eq. (3.19) itself contains a \( \cos \theta \) dependent term, whose coefficient \( c \) contains the interference of isoscalar multipole amplitudes with the dominant \( E_{1_v}^{j} \) amplitude. We have shown that, a comparison of the differential cross section with polarized photons and unpolarized differential cross section can lead to an empirical determination of all the three coefficients including \( c \).
To study further the contributions of isoscalar $M_1s$ and $E2s$ amplitudes, attention is also focussed on photodisintegration of aligned deuterons by 100% linearly polarized photons. Expressions for analyzing powers are explicitly derived.

Since the possible role of the $M_1s$ and $E2s$ amplitudes have been discussed by several authors using different theoretical formalisms over the last 50 years, we feel that it is desirable to carry out the measurements on the tensor analyzing powers $A^2_0$ and $A^2_2$ in photodisintegration of aligned deuterons in addition to the differential cross section $\frac{d\sigma}{dQ}$. Such experimental studies are expected to clarify empirically the role of the isoscalar $M_1s$ and $E2s$ amplitudes at the range of energies of interest to astrophysics.
Table 3.1: Polarization parameters [113] of aligned deuterons at temperature $T = 0.1 \text{ mK}$

<table>
<thead>
<tr>
<th>Compound</th>
<th>$A$</th>
<th>$-\eta$</th>
<th>$t_0^2$</th>
<th>$t_2^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>acetamide (ND2)</td>
<td>0.6242</td>
<td>0.4022</td>
<td>-0.4584</td>
<td>-4.6791E-02</td>
</tr>
<tr>
<td>chloroacetamide (ND2)</td>
<td>0.6260</td>
<td>0.3772</td>
<td>-0.4601</td>
<td>-4.3949E-02</td>
</tr>
<tr>
<td>formamide (ND2)</td>
<td>0.5752</td>
<td>0.3712</td>
<td>-0.4223</td>
<td>-4.1353E-02</td>
</tr>
<tr>
<td>phthalamide ($ND_2)_{II}$</td>
<td>0.6457</td>
<td>0.3722</td>
<td>-0.4749</td>
<td>-4.4035E-02</td>
</tr>
<tr>
<td>L-asparagine (ND2)</td>
<td>0.6377</td>
<td>0.3826</td>
<td>-0.4688</td>
<td>-4.4990E-02</td>
</tr>
<tr>
<td>L-asparagine hydrate (ND2)</td>
<td>0.6797</td>
<td>0.3272</td>
<td>-0.5010</td>
<td>-3.9644E-02</td>
</tr>
</tbody>
</table>
Target analyzing powers.....

Table 3.2: Values of $|\alpha_0|$, $|\beta_0|$ and $|\alpha_2|$ for the four choices

<table>
<thead>
<tr>
<th></th>
<th>Solution 1</th>
<th></th>
<th>Solution 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14 MeV</td>
<td>16 MeV</td>
<td>14 MeV</td>
</tr>
<tr>
<td>$</td>
<td>\alpha_0</td>
<td>$</td>
<td>(+1, +1)</td>
</tr>
<tr>
<td></td>
<td>(+1, −1)</td>
<td>30.593</td>
<td>31.008</td>
</tr>
<tr>
<td></td>
<td>(−1, +1)</td>
<td>12.0027</td>
<td>11.587</td>
</tr>
<tr>
<td></td>
<td>(−1, −1)</td>
<td>36.789</td>
<td>36.374</td>
</tr>
<tr>
<td>$</td>
<td>\beta_0</td>
<td>$</td>
<td>(+1, +1)</td>
</tr>
<tr>
<td></td>
<td>(+1, −1)</td>
<td>−49.183</td>
<td>−48.768</td>
</tr>
<tr>
<td></td>
<td>(−1, +1)</td>
<td>6.587</td>
<td>6.172</td>
</tr>
<tr>
<td></td>
<td>(−1, −1)</td>
<td>−42.986</td>
<td>−43.401</td>
</tr>
<tr>
<td>$</td>
<td>\alpha_2</td>
<td>$</td>
<td>(+1, +1)</td>
</tr>
<tr>
<td></td>
<td>(+1, −1)</td>
<td>0.774</td>
<td>0.740</td>
</tr>
<tr>
<td></td>
<td>(−1, +1)</td>
<td>−0.774</td>
<td>−0.740</td>
</tr>
<tr>
<td></td>
<td>(−1, −1)</td>
<td>0.258</td>
<td>0.292</td>
</tr>
</tbody>
</table>