Appendix A

Spin transition tensors

The spin transition tensor $S^\lambda_{\mu}(s_2, s_1)$ of rank $\lambda$ connecting the spin spaces of particles with arbitrary spins $s_1$ and $s_2$ is defined [92] as

$$S^\lambda_{\mu}(s_2, s_1) = (-1)^{s_1}[s_2](K^{s_2} \otimes B^{s_1})^\lambda_{\mu}, \quad (A.1)$$

where $K^{s_2}_{\mu_2} = |s_2\mu_2\rangle$ and $B^{s_1}_{-\mu_1} = (-i)^{2\mu_1}|s_1\mu_1\rangle$ are the irreducible tensors of rank $s_2$ and $s_1$ respectively and $\lambda$ takes values from $|s_2 - s_1|$ to $(s_2 + s_1)$.

When $s_1 = s_2 = s$, which refers to the same particle or nucleus, the operator $S^\lambda_{\mu}(s_2, s_1)$ reduces to the well known irreducible tensors $\tau^\lambda_{\mu}(S)$, constructed out of the spin operator $S$ i.e.,

$$S^\lambda_{\mu}(s, s) = \tau^\lambda_{\mu}(S), \quad (A.2)$$

introduced by Fano [102] but normalized here so as to be in agreement with the Madison convention [103] for spin-1 particles. Thus we have

$$\langle sm|\tau^\lambda_{\mu}(S)|sm'\rangle = [\lambda]C(s\lambda s; m'mm). \quad (A.3)$$

If the initial spin channel $s_i$ consists of two particles with spins $s'_1$ and $s'_2$ and likewise in the final spin channel $s_f$ consists of two particles with spins $s''_1$ and $s''_2$ then we have...
the following useful identity

\[ S^\lambda_{\mu}(s_f, s_i) = \sum_{\lambda' \lambda''} (S^{\lambda'}_{\mu'}(s_2', s_1') \otimes S^{\lambda''}_{\mu''}(s_2'', s_1''))_\mu \]

\[ (-1)^{s_i - s_i'} - s_i'' \begin{vmatrix} s_f & s_f' & s_f'', s_f'' \end{vmatrix} \begin{vmatrix} s_i & s_i' \end{vmatrix} \begin{vmatrix} \lambda' & \lambda'' \end{vmatrix} \]

\[ \begin{vmatrix} s_2 & s_2' \end{vmatrix} \begin{vmatrix} s_1 & s_1' \end{vmatrix} \begin{vmatrix} \lambda' & \lambda'' \end{vmatrix} \]

\[ (A.4) \]

Another useful identity involving the spin tensors is

\[ (S^\lambda(s_2', s) \otimes S^{\lambda''}(s, s_1'))_\mu = (-1)^{\lambda' + \lambda'' - \lambda} \begin{vmatrix} \lambda & \lambda'' \end{vmatrix} \begin{vmatrix} \lambda' & \lambda'' \end{vmatrix} \begin{vmatrix} \lambda' & \lambda'' \end{vmatrix} \begin{vmatrix} s_2' & s_2'' \end{vmatrix} \begin{vmatrix} s_1' & s_1'' \end{vmatrix} \]

\[ (A.5) \]