Preface

The purpose of this thesis is to present a comprehensive theoretical study of the photodisintegration of deuterons at the energies relevant to Big Bang Nucleosynthesis (BBN). Most of the earlier theoretical studies were model dependent and a few were based on effective field theory. Our approach in this thesis differs from both and is entirely model independent. It is well known that spin observables provide incisive information with regard to the role of different amplitudes when some of these contributions are small. Our approach, in this thesis, which makes use of irreducible tensor techniques is well suited to make predictions on spin observables as well as on differential cross section. In the later part of the thesis, we extend our study to Positive Operator Valued Measures (POVMs) constructed out of irreducible tensor operators, which find application in the field of quantum information processing. Major portions of the work reported in this thesis have already been published [1, 2, 3] by the author, in leading international journals like Physical Review C, Journal of Physics G: Nuclear and Particle Physics and International Journal of Modern Physics E.

Along with the Cosmic Microwave Background Radiation [4] and Hubble expansion [5], the Primordial Nucleosynthesis [6, 7, 8] provides an important test of Big Bang Cosmology, which has now entered a precision era [9]. As the Universe cools from an astounding $10^{32}$ K to approximately $10^9$ K in the first three minutes, four light nuclei namely $^2$H, $^3$He, $^4$He and $^7$Li are produced in significant amounts which depend on the primordial baryon density $\Omega_B$.

The primordial abundance of deuterium assumes high significance because it varies sharply with $\Omega_B$ through 8 orders of magnitude from $10^{-2}$ to $10^{-10}$ as compared to the variation in the case of $^3$He which is from $10^{-4}$ to $10^{-5}$, of $^7$Li which is from $10^{-9}$ to $10^{-7}$, whereas $^4$He is more or less constant. Hence primordial abundance of deuterium
has been referred to as the "Baryometer". Moreover, an accurate knowledge of the
primordial abundance of deuterium facilitates an accurate prediction of the abundances
of $^3\text{He}$ $^4\text{He}$ and $^7\text{Li}$.

Since deuterium is fragile, it is destroyed in stars even before they reach main se-
quence and as such current astronomical observations do not directly reflect primordial
deuterium abundance. It has been observed [10] that 'The ratio of the primordial abun-
dance of deuterium to that observed today could be any where between 1 and 50'.

Laboratory measurements and decisive developments in astronomical observations go
hand in hand to remove crucial ambiguities and sharpen the predictions in the astro-
red shift hydrogen clouds has been a helpful development. However, the work of
Burles et al [12] has inspired recent experimental studies [13, 14, 15, 16, 17, 18, 19]
at the Duke Free electron Laser Laboratory using 100% linearly polarized photons to
measure the beam analyzing power along with the differential cross section in photo-
disintegration of deuterons.

The photodisintegration of the deuteron, even at low energies is known to be domi-
nated by the isovector E1 transitions leading to final triplet $n - p$ states in the contin-
uum. However the inverse reaction, $n - p$ fusion has been studied right from 1930's at
thermal neutron energies and the transition is known to be dominated by an isovector
$M1$ amplitude. Considerable attention has been paid to determine theoretically the rel-
ative $M1$ and $E1$ contributions to the process at astrophysical energies. Such studies
have shown that the strength of the isovector $M1$ amplitude decreases [20, 21] by sev-
eral orders of magnitude as the energies relevant to BBN is approached. In this context,
one has to consider the contribution of the isoscalar M1 amplitude, which, though small
compared to the isovector $M1$ amplitude has been estimated to be around $401.7\text{mb}$ at
thermal neutron energies. It is interesting to note that the measured $n - p$ fusion cross
section \cite{22, 23} itself at the energies relevant to BBN is of the same order of magnitude as that of the extracted isoscalar $M1$ cross section at thermal neutron energies. Therefore one has to consider carefully, not only the relative contributions of isovector $M1$ and $E1$ contributions but also the contribution of isoscalar $M1$ amplitudes at the energies relevant to BBN. Some earlier authors \cite{24, 25} have also taken isoscalar $E2$ amplitude into consideration. We therefore include the isoscalar $E2$ amplitude also to our study.

In view of the several theoretical and ongoing experimental studies, it is felt that a detailed theoretical study of the spin structure of the amplitudes in photodisintegration of deuterons and their expansion in terms of ‘electric’ and ‘magnetic’ amplitudes is needed to analyze the measurements of spin observables, which leads to a better understanding of the problem at astrophysical energies of relevance for sharpening the predictions of BBN.

In this thesis, we express the on-energy-shell transition matrix $M$ for the reaction in a novel way in terms of spin operators $S^\lambda_\nu(s, 1)$ connecting initial and final hadron spin states and the corresponding irreducible tensor amplitudes $F^\lambda_\nu$ of rank $\lambda$ with $\nu = \lambda, \cdots, -\lambda$. The advantage of our formalism is that there is no limit to the number of partial waves that can be accommodated for the analysis, as energy increases. Irrespective of energy the number of irreducible tensor amplitudes remain the same. It is well known that, as the c.m. energy increases the number of multipole strengths contributing to the photodisintegration reaction increases. We therefore describe the photodisintegration process in terms of the irreducible tensor amplitudes $F^0_\nu(1, \mu)$, $F^1_\nu(1, \mu)$ and $F^2_\nu(1, \mu)$ for the transitions to the final triplet spin-1 states and the irreducible tensor amplitudes $F^1_\nu(0, \mu)$ for the transition to the final spin-0 state while the index $\mu$ specifies the state of photon polarization. This facilitates elegant derivation of formulae for differential cross-section, analyzing powers, neutron polarization and spin correlations in terms of
bilinears involving the irreducible tensor amplitudes.

Extending our study on irreducible tensor operators, $\tau^k_{\bar{q}}$, we construct a set of Positive Operator Valued Measures (POVMs) in a symmetric subspace of any dimension using $\tau^k_{\bar{q}}$ as detection operators. POVMs are the most general class of measurements, which find application in the field of quantum information processing. The rapidly developing quantum information theory has generated a lot of interest in the construction and experimental implementation of POVMs [26, 27, 28]. Noting the experimental significance of symmetric states, we construct POVMs in the symmetric subspace and using Neumark’s theorem, we also demonstrate the physical implementation of our POVMs.

The present thesis consists of 5 Chapters. Chapter-wise summary of the thesis is as follows.

**Chapter 1**

This Chapter presents a comprehensive background and literature survey on the problem. A discussion about the reaction kinematics is also presented. Since our formalism in the thesis is described in the c.m. frame, we explicitly derive expressions for the energies of the photon, deuteron, neutron and proton in the c.m frame. We also derive the expression for unpolarized differential cross section for photodisintegration of deuterons in the c.m. frame.
Chapter 2

In this Chapter, we outline the basic formalism developed by us to study photodisintegration of deuterons. A discussion about irreducible tensors and polarization states of the photon are presented. An elegant formula for the reaction matrix elements is first derived. Since the experimental facilities are available at the Duke Free Electron Laser Laboratory, we focus attention on $d(\gamma, n)p$ with linearly polarized photons. Explicit formulae are obtained for the irreducible tensor amplitudes for the reaction in terms of the all the relevant multipole amplitudes at energies of astrophysical interest. The expression for the differential cross section with linearly polarized photons is also obtained. Our model independent discussion shows clearly that there are three different $E1$ amplitudes leading to the final relative $p$–states between the neutron and the proton. If all of them are assumed to be equal, only one of the irreducible tensor amplitudes will survive. That is too drastic an assumption. We therefore focus attention on the problem, when all the three $E1$ amplitudes are not equal. This revealed that we can plan experiments to study the small isoscalar contributions to the reaction. As our theoretical work was going on, it so happened that experimental evidence came up from abroad that the three $E1$ amplitudes are indeed different. This encouraged us to pursue further, the problems associated with identification of small amplitudes characterizing the reaction at the energies of astrophysical interest. The full partial wave expansions are also presented to enable comparison with the experimental work.

Chapter 3

Complementary to the work on unpolarized deuteron targets presented in Chapter 2, we extend the model independent formalism to study photodisintegration of deuterons
employing polarized deuteron targets using unpolarized photons. The expression for the unpolarized differential cross section in terms of multipole amplitudes is derived. It is also noted that the unpolarized differential cross section itself contains a $\cos \theta$ dependent term, whose coefficient $c$ contains the interference of isoscalar multipole amplitudes with the dominant $E1$ amplitude. It is also shown that by comparing the unpolarized differential cross section with the differential cross section with polarized photons, one can determine all the coefficients including $c$. We then go on to show that the use of irreducible tensor amplitudes facilitates elegant derivation of formulae for the target analyzing powers also. To study further the contribution of isoscalar $M1$ and $E2$ amplitudes, attention is also focussed on aligned deuteron targets, as such spin polarized deuterons are expected to be present when they are embedded in external quadrupole fields. The formalism developed in Chapter 2 with polarized photons, is then extended to study photodisintegration of aligned deuterons by polarized photons. Expressions for analyzing powers are explicitly derived.

**Chapter 4**

In this Chapter, we extend our model independent formalism to study neutron polarization in the final state. The spin density matrix for the neutron in the final state is derived when

(i) initial photons are unpolarized

(ii) initial photons are linearly polarized.

This study has been taken up in anticipation of experimental measurements of the spin polarization of the neutron with the existing experimental set up with 100% linearly polarized photons in the Duke Free Electron Laser Laboratory. We find that the neutron spin is sensitive to the initial polarization of the photon.
We also derive the full density matrix for the n-p system in the final state as it can be used to study the nucleon-nucleon spin correlations.

**Chapter 5**

Using the irreducible tensor operators, as detection operators, we construct a set of Positive Operator Valued Measures, $E_{q}^{h}$ in a symmetric subspace of any dimension. We start with a brief discussion about symmetric states and operator representation in terms of irreducible tensors. Explicit expressions for the matrix representation of irreducible tensor operators are given. Considering the special case of spin-1 and using Neumark’s theorem, we demonstrate the physical implementation of our POVMs as the projective measurements in the 4-dimensional 2-qubit Hilbert space. These are easily identified as Hamiltonians, which can be implemented in NMR quantum computing. We calculate these explicitly in terms of the Pauli spin matrices. A discussion about the post measurement state is also presented.