ABSTRACT

The convolution or the Hadamard product of two conformal mappings is a highly studied subject. Many beautiful and interesting results are available in the literature on the convolution of conformal mappings. For example, the class of univalent convex analytic functions is closed under the operation of convolution. As harmonic mappings are natural generalization of analytic functions, it is therefore interesting to investigate those properties of convolutions of analytic functions which extend to that of harmonic mappings. The present work entitled “CONVOLUTION PROPERTIES OF SOME UNIVALENT HARMONIC FUNCTIONS” is devoted to examine the convolution properties of some univalent harmonic mappings and is divided into following six chapters.

In Chapter 1, some fundamental concepts and classes of univalent analytic functions and univalent harmonic mappings are introduced. We also constructed some particular univalent harmonic mappings by using the technique of ‘Shear Construction or Shearing’. A brief survey of the literature along with the techniques used in the present work are also presented in this chapter.

Unfortunately, all the results on convolution of conformal mappings do not carry over to convolution of univalent harmonic mappings. Although the study of convolution of harmonic mappings made some progress in the recent few years but yet very little is known in this direction. In Chapter 2, we introduced a family, \( F_a = H_a + \overline{G}_a \), of univalent harmonic right half-plane mappings given by

\[
\begin{align*}
H_a(z) + G_a(z) &= \frac{z}{(1 - z)} \\
G'_a(z) &= \frac{(a - z)}{(1 - az)}, \quad a \in (-1, 1).
\end{align*}
\]

We investigated convolution properties of mappings from the family \( \{F_a\} \) with other harmonic right half-plane mappings. Apart from obtaining some new results we also strengthened some existing results in this chapter.
As mentioned earlier, under the operation of convolution, harmonic mappings behave differently from that of conformal mappings. Unlike the case of conformal mappings the class $K_H$ of convex univalent harmonic mappings is not closed under the operation of harmonic convolution. This fact generated a lot of interest in researchers to identify those convex harmonic mappings whose convolution has some nice mapping properties. In Chapter 3, we investigated convolution properties of mappings from the families $\{F_a\}$ and $\{f_\beta\}$, where $f_\beta = h_\beta + g_\beta$ are harmonic mappings obtained by shearing of analytic strip mappings

$$h_\beta(z) + g_\beta(z) = \frac{1}{2i \sin \beta} \log \left( \frac{1 + z e^{i\beta}}{1 + z e^{-i\beta}} \right), \quad 0 < \beta < \pi$$

with dilatation $\omega(z) = e^{i\theta} z^n$ ($n = 1, 2, 3, 4, \theta \in \mathbb{R}$). We also presented the following conjecture in this chapter.

**Conjecture.** Let $f_\beta = h_\beta + g_\beta$ be as defined above. Then $F_a \ast f_\beta \in S^0_H$ and is CHD for all $n \in \mathbb{N}$ provided $a \in [(n - 2)/(n + 2), 1]$.

In Chapter 4, we settled this conjecture in the affirmative for $\beta = \pi/2$.

In Chapter 5, we identified a class of harmonic mappings whose convolution with harmonic slanted right half-plane mapping is harmonic close-to-convex. We also investigated convolutions of harmonic slanted right half-plane and harmonic square mappings. Convolution properties of a mapping in the class $K^0_H$ with some other harmonic mappings, defined by some coefficient conditions are also studied in this chapter. We proved that convolutions under such conditions belong to a well known subclasses of univalent harmonic mappings. Also, we found conditions under which harmonic convolution of two harmonic mappings from $S^0_H$ is in $C^0_H$.

In Chapter 6, we investigated linear combinations and convolution properties of mappings from the family, $S_H(\alpha)$, of locally univalent and sense-preserving harmonic mappings $f = h + g$, obtained by shearing of

$$h(z) + g(z) = \frac{z(1 - \alpha z)}{(1 - z^2)}, \quad \alpha \in [-1, 1].$$
Linear combinations of mappings $f \in S_H(\alpha)$ and mappings from the family $\{f_\beta\}$ are also studied. We also obtained a general condition on univalent harmonic mappings convex in one direction so that its harmonic convolutions with analytic functions from the class $DCP$ (Direction convexity preserving) are also convex in the same direction.