7. Unreliable Batch Arrival Retrial Queue with Positive and Negative Customers, Priority or Collisions, Delayed Repair and Orbital Search

Batch arrival retrial queue with positive and negative customers is considered. Positive customers arrive in batches according to Poisson process. If the server is idle upon the arrival of a batch, one of the customers in the batch receives service immediately and others join the orbit. If the server is busy, the arriving batch joins the orbit or collides with the customer in service resulting in all being shifted to the orbit or one of the customers in the batch interrupts the customer in service to get his own service. The arrival of negative customer brings the server down and makes the interrupted customer to leave the system. The repair of the failed server starts after a random amount of time. During the repair time and delay time, the customers are allowed to balk. After each service completion, the server searches for customers in the orbit with certain probability. Using supplementary variable technique various performance measures are derived. Stochastic decomposition property is established. Special cases are discussed and numerical results are presented.

7.1 Model Description

Single server queueing system with two types of arrivals positive and negative is considered. Positive customers arrive in batches according to Poisson process with rate $\lambda^+$. At every arrival epoch, a batch of $k$ customers arrives with probability $C_k$. The generating function of the sequence $\{C_k\}$ is $C(z)$ with first two moments $m_1$ and $m_2$. There is no waiting space in front of the server and therefore if the arriving batch of positive customers finds the server idle, then one of the customers receives the service and the others join the orbit. If the server is busy, then the arriving batch proceeds to the server with probability $\delta$ or enters the orbit with probability $\bar{\delta} (= 1 - \delta)$. In the first case, with probability $\alpha$ one of the customers interrupts the customer in
service to commence his own service and the interrupted customer along with remaining customers join the orbit. Otherwise, with probability \( \bar{\alpha} (= 1 - \alpha) \) the arriving batch collides with the customer in service resulting in all being shifted to the orbit and the server becomes idle.

Negative customers arrive independently according to Poisson process with rate \( \lambda^- \). The arrival of negative customer removes the positive customer in service from the system and causes the server breakdown. The repair of the failed server commences after a random amount of time. If the server is waiting for repair (under repair), the arriving batch enters the orbit with probability \( p(q) \) or leaves the system complementary with probability \( \overline{p}(\overline{q}) \). As soon as the service of the positive customer is completed, the server goes for search of customers in the orbit with probability \( \theta \) or remains idle with probability \( \overline{\theta} (= 1 - \theta) \). The search time is assumed to be negligible.

Distribution function, density function, Laplace Stieltjes transform and the first two moments of retrial time, service time, delay time and repair time which are generally distributed are given below.

<table>
<thead>
<tr>
<th>Time</th>
<th>Distribution function</th>
<th>Density function</th>
<th>Laplace Stieltjes transform</th>
<th>First two moments</th>
<th>Hazard rate function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retrial</td>
<td>( A(x) )</td>
<td>( a(x) )</td>
<td>( A^*(s) )</td>
<td>-</td>
<td>( \eta(x) )</td>
</tr>
<tr>
<td>Service</td>
<td>( B(x) )</td>
<td>( b(x) )</td>
<td>( B^*(s) )</td>
<td>( \mu_1, \mu_2 )</td>
<td>( \mu(x) )</td>
</tr>
<tr>
<td>Delay</td>
<td>( D(x) )</td>
<td>( d(x) )</td>
<td>( D^*(s) )</td>
<td>( \gamma_1, \gamma_2 )</td>
<td>( \gamma(x) )</td>
</tr>
<tr>
<td>Repair</td>
<td>( R(x) )</td>
<td>( r(x) )</td>
<td>( R^*(s) )</td>
<td>( \beta_1, \beta_2 )</td>
<td>( \beta(x) )</td>
</tr>
</tbody>
</table>

The stochastic behaviour of the retrial queueing system can be described by the Markov process \( \{X(t), t \geq 0\} = \{S(t), N(t), \xi(t), t \geq 0\} \) where \( S(t) \) denotes the server state 0, 1, 2 or 3 according as the server being idle, providing service, waiting for repair or under repair. \( N(t) \) corresponds to the number of customers in the orbit. If \( S(t) = 0 \), then \( \xi(t) \) represents the elapsed
retrial time. If \( S(t) = 1 \), then \( \xi(t) \) represents the elapsed service time. If \( S(t) = 2 \), then \( \xi(t) \) represents the elapsed delay time. If \( S(t) = 3 \), then \( \xi(t) \) represents the elapsed repair time.

### 7.2 Stability Condition

Let \( N(t_n^-) \) be the number of customers in the orbit just after time \( t_n \). Then the sequence of random variables \( Y_n = N(t_n^-) \) form a Markov Chain, which is the embedded Markov Chain of the system.

**Theorem 7.1**

The embedded Markov chain \( \{Y_n, n \in \mathbb{N}\} \) is ergodic if and only if

\[
[1 - B^*(\lambda^+ \delta + \lambda^-)] [\lambda^+ (\delta + m_1) + \lambda^- m_1 (p \lambda^+ \gamma_1 + q \lambda^+ \beta_1)] \\
+ m_1(1 - A^*(\lambda)) (\lambda^- + \lambda^+ \delta \overline{\alpha}) < (\lambda^+ \delta + \lambda^-) [1 - m_1(1 - A^*(\lambda^+)) \overline{\beta} B^*(\lambda^+ \delta + \lambda^-)].
\]

The theorem can be proved along similar lines as in Gomez-Corral (1999).

### 7.3 Steady State Distribution

For the process \( \{X(t), t \geq 0\} \), define the probability densities

\[
I_0(t) = P\{S(t) = 0, N(t) = n\} \\
I_n(x, t) dx = P\{S(t) = 0, N(t) = n, x \leq \xi(t) < x + dx\}, x \geq 0, n \geq 1 \\
P_n(x, t) dx = P\{S(t) = 1, N(t) = n, x \leq \xi(t) < x + dx\}, x \geq 0, n \geq 0 \\
F_{1,n}(x, t) dx = P\{S(t) = 2, N(t) = n, x \leq \xi(t) < x + dx\}, x \geq 0, n \geq 0 \\
F_{2,n}(x, t) dx = P\{S(t) = 3, N(t) = n, x \leq \xi(t) < x + dx\}, x \geq 0, n \geq 0
\]
The governing equations of the model under study are

\[ \frac{d}{dt} I_0(t) = -\lambda^+ I_0(t) + \int_0^\infty P_0(x, t) \mu(x) \, dx + \int_0^\infty F_{2,0}(x, t) \beta(x) \, dx \]  \tag{7.1} \]

\[ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) I_n(x, t) = - (\lambda^+ + \eta(x)) I_n(x, t), \quad n \geq 1 \]  \tag{7.2} \]

\[ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) P_n(x, t) = - (\lambda^+ + \lambda^- + \mu(x)) P_n(x, t) + \lambda^+ \sum_{k=1}^n C_k P_{n-k}(x, t), \quad n \geq 0 \]  \tag{7.3} \]

\[ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) F_{1,n}(x, t) = - (p \lambda^+ + \gamma(x)) F_{1,n}(x, t) + \lambda^+ p \sum_{k=1}^n C_k F_{1,n-k}(x, t), \quad n \geq 0 \]  \tag{7.4} \]

\[ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) F_{2,n}(x, t) = - (q \lambda^+ + \beta(x)) F_{2,n}(x, t) + \lambda^+ q \sum_{k=1}^n C_k F_{2,n-k}(x, t), \quad n \geq 0 \]  \tag{7.5} \]

with boundary conditions

\[ I_1(0, t) = \theta \int_0^\infty P_1(x, t) \mu(x) \, dx + \int_0^\infty F_{2,1}(x, t) \beta(x) \, dx \]  \tag{7.6} \]

\[ I_n(0, t) = \theta \int_0^\infty P_n(x, t) \mu(x) \, dx + \int_0^\infty F_{2,n}(x, t) \beta(x) \, dx + \lambda^+ \delta \sum_{k=1}^n C_k \int_0^\infty P_{n-(k+1)}(x, t) \, dx \]  \tag{7.7} \]

\[ P_0(0, t) = \lambda^+ C_1 I_0(t) + \int_0^\infty I_1(x, t) \eta(x) \, dx + \theta \int_0^\infty P_1(x, t) \mu(x) \, dx \]  \tag{7.8} \]

\[ P_n(0, t) = \lambda^+ C_{n+1} I_0(t) + \int_0^\infty I_{n+1}(x, t) \eta(x) \, dx + \theta \int_0^\infty P_{n+1}(x, t) \mu(x) \, dx \]

\[ + \lambda^+ \delta \alpha \sum_{k=1}^n C_k \int_0^\infty P_{n-k}(x, t) \, dx + \lambda^+ \sum_{k=1}^n C_k \int_0^\infty I_{n-k+1}(x, t) \, dx, \quad n \geq 1 \]  \tag{7.9} \]

\[ F_{1,n}(0, t) = \lambda^- \int_0^\infty P_n(x, t) \, dx, \quad n \geq 0 \]  \tag{7.10} \]

\[ F_{2,n}(0, t) = \int_0^\infty F_{1,n}(x, t) \gamma(x) \, dx, \quad n \geq 0 \]  \tag{7.11} \]
Taking limit as $t \to \infty$ on both sides of the equations, we get the following steady state equations.

\[ \lambda^+ I_0 = \int_0^\infty P_0(x) \mu(x) \, dx + \int_0^\infty F_{2,0}(x) \beta(x) \, dx \]  
(7.12)

\[ \frac{d}{dx} I_n(x) = - (\lambda^+ + \eta(x)) I_n(x), \quad n \geq 1 \]  
(7.13)

\[ \frac{d}{dx} P_n(x) = - (\lambda^+ + \lambda^- + \mu(x)) P_n(x) + \lambda^+ \delta \sum_{k=1}^n C_k P_{n-k}(x), \quad n \geq 0 \]  
(7.14)

\[ \frac{d}{dx} F_{1,n}(x) = - (p \lambda^+ + \gamma(x)) F_{1,n}(x) + \lambda^+ p \sum_{k=1}^n C_k F_{1,n-k}(x), \quad n \geq 0 \]  
(7.15)

\[ \frac{d}{dx} F_{2,n}(x) = - (q \lambda^+ + \beta(x)) F_{2,n}(x) + \lambda^+ q \sum_{k=1}^n C_k F_{2,n-k}(x), \quad n \geq 0 \]  
(7.16)

with boundary conditions

\[ I_1(0) = \lambda^- C_1 I_0 + \int_0^\infty I_1(x) \eta(x) \, dx \]  
(7.17)

\[ I_n(0) = \lambda^- C_{n+1} I_0 + \int_0^\infty I_n(x) \eta(x) \, dx + \lambda^+ \delta \sum_{k=1}^n C_k \int_0^\infty P_{n-k}(x) \, dx, \quad n \geq 2 \]  
(7.18)

\[ P_0(0) = \lambda^+ C_1 I_0 + \int_0^\infty I_1(x) \eta(x) \, dx + \int_0^\infty P_1(x) \mu(x) \, dx \]  
(7.19)

\[ P_n(0) = \lambda^+ C_{n+1} I_0 + \int_0^\infty I_{n+1}(x) \eta(x) \, dx + \lambda^+ \delta \alpha \sum_{k=1}^n C_k \int_0^\infty P_{n-k}(x) \, dx \]  
\[ + \lambda^+ \sum_{k=1}^n C_k \int_0^\infty I_{n-k+1}(x) \, dx + \int_0^\infty P_{n+1}(x) \mu(x) \, dx, \quad n \geq 1 \]  
(7.20)

\[ F_{1,n}(0) = \lambda^- \int_0^\infty P_n(x) \, dx, \quad n \geq 0 \]  
(7.21)

\[ F_{2,n}(0) = \int_0^\infty F_{1,n}(x) \gamma(x) \, dx, \quad n \geq 0 \]  
(7.22)
The normalising equation is

\[ I_0 + \sum_{n=1}^{\infty} \int I_n(x) \, dx + \sum_{n=0}^{\infty} \int P_n(x) \, dx + \sum_{n=0}^{\infty} \int F_{1,n}(x) \, dx + \sum_{n=0}^{\infty} \int F_{2,n}(x) \, dx = 1 \]  

(7.23)

To solve the above equations, define the probability generating functions

\[ I(x, z) = \sum_{n=1}^{\infty} I_n(x) z^n; \quad P(x, z) = \sum_{n=0}^{\infty} P_n(x) z^n; \]

\[ F_1(x, z) = \sum_{n=0}^{\infty} F_{1,n}(x) z^n \quad \text{and} \quad F_2(x, z) = \sum_{n=0}^{\infty} F_{2,n}(x) z^n \]

Multiplying equation (7.13) by \( z^n \) and summing over \( n \) we get

\[ \left( \frac{\partial}{\partial x} + \lambda^+ + \eta(x) \right) I(x, z) = 0 \]  

(7.24)

Solving the partial differential equation (7.24), we get

\[ I(x, z) = C \, e^{-\lambda^+ x} e^{-\int \eta(x) \, dx} \]

\[ = C \, e^{-\lambda^+ x} \, e^{\log (1-A(x))} \]

\[ = C \, e^{-\lambda^+ x} (1-A(x)) \]

Eliminating \( C \) by taking \( x = 0 \), we get

\[ I(x, z) = I(0, z) \, e^{-\lambda^+ x} (1-A(x)) \]  

(7.25)

Performing similar operations on equations (7.14) to (7.16) and solving the corresponding partial differential equations, we get

\[ P(x, z) = P(0, z) \, e^{-\left(\lambda^+ + \lambda^- + \lambda^+ \delta C(z)\right)x} (1-B(x)) \]  

(7.26)

\[ F_1(x, z) = F_1(0, z) \, e^{-\left(p \lambda^+ (1-C(z))\right)x} (1-D(x)) \]  

(7.27)

\[ F_2(x, z) = F_2(0, z) \, e^{-\left(q \lambda^+ (1-C(z))\right)x} (1-R(x)) \]  

(7.28)
Multiplying equations (7.17) to (7.22) by \( z^n \) and summing over \( n \), we get

\[
I(0, z) = \int_0^\infty P(x, z) \mu(x) \, dx + \int_0^\infty F_2(x, z) \beta(x) \, dx \\
+ \lambda^+ \delta \overline{\alpha} \int_0^\infty P(x, z) \, dx - \lambda^+ I_0 \tag{7.29}
\]

\[
P(0, z) = \frac{\lambda^+ C(z)}{z} I_0 + \frac{1}{z} \int_0^\infty I(x, z) \eta(x) \, dx + \frac{\lambda^+ C(z)}{z} \int_0^\infty I(x, z) \, dx \\
+ \frac{1}{z} \int_0^\infty P(x, z) \mu(x) \, dx + \lambda^+ \delta \overline{\alpha} \int_0^\infty P(x, z) \, dx \tag{7.30}
\]

\[
F_1(0, z) = \lambda^- \int_0^\infty P(x, z) \, dx \tag{7.31}
\]

\[
F_2(0, z) = \int_0^\infty F_1(x, z) \gamma(x) \, dx \tag{7.32}
\]

Using equations (7.26) and (7.28) in equation (7.29), we have

\[
I(0, z) = \int_0^\infty P(0, z) B^*(\lambda^+ (1 - \overline{\delta} C(z)) + \lambda^-) + F_2(0, z) R^* (q \lambda^+ (1 - C(z))) \\
+ \lambda^+ \delta \overline{\alpha} \int_0^\infty P(0, z) k(z) - \lambda^+ I_0 \tag{7.33}
\]

where \( K(z) = \frac{1 - B^*(\lambda^+ (1 - \overline{\delta} C(z)) + \lambda^-)}{\lambda^+ (1 - \overline{\delta} C(z)) + \lambda^-} \)

Substituting equations (7.25) and (7.26) in equation (7.30), we obtain

\[
P(0, z) = \frac{\lambda^+ C(z)}{z} I_0 + \frac{1}{z} \int_0^\infty [A^*(\lambda^+) + C(z) (1 - A^*(\lambda^+))] \\
+ \frac{1}{z} \int_0^\infty P(0, z) B^*(\lambda^+ (1 - \overline{\delta} C(z)) + \lambda^-) + \lambda^+ \delta \overline{\alpha} \int_0^\infty P(0, z) K(z) \tag{7.34}
\]

Using equation (7.26), the equation (7.31) yields

\[
F_1(0, z) = \lambda^- P(0, z) K(z) \tag{7.35}
\]
Using equations (7.27) and (7.35), $F_1(x, z)$ can be expressed in terms of $P(0, z)$ and substituting the resultant expression in equation (7.32), we get

$$F_2(0, z) = \lambda^- P(0, z) D^*(p \lambda^- - p \lambda^+ C(z)) K(z)$$  \hspace{1cm} (7.36)

Using equation (7.36) in equation (7.33) and substituting the resultant expression of $I(0, z)$ which is in terms of $P(0, z)$ in equation (7.34) and solving we obtain

$$P(0, z) = A^*(\lambda^+) (C(z) - 1) \lambda^+ I_0 / T(z)$$ \hspace{1cm} (7.37)

where

$$T(z) = z - [C(z) + A^*(\lambda^+) (1 - C(z))] \bar{\theta} B^*(\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-)$$
$$- \theta B^*(\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) - K(z) [(A^*(\lambda^+) + C(z) (1 - A^*(\lambda^+)))]$$
$$(\lambda^- D^*(p \lambda^+ (1 - C(z))) R^*(q \lambda^+(1 - C(z))) + \lambda^+ \delta \alpha z C(z))$$
$$+ \lambda^+ \delta \alpha z C(z)$$

Substituting equations (7.36) and (7.37) in equation (7.33), we have

$$I(0, z) = [\lambda^+ I_0 [C(z) B^*(\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) - z + \theta (1 - C(z))$$
$$B^*(\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) + K(z) (C(z) (\lambda^- D^*(p \lambda^+ (1 - C(z))))$$
$$R^*(q \lambda^+(1 - C(z))) + \lambda^+ \delta \alpha z C(z) + \lambda^+ \delta \alpha z))]] / T(z)$$  \hspace{1cm} (7.38)

Using equation (7.37) in equations (7.35) and (7.36), we obtain respectively

$$F_1(0, z) = \lambda^+ I_0 A^*(\lambda^+) \lambda^-(C(z) - 1) K(z) / T(z)$$  \hspace{1cm} (7.39)

and

$$F_2(0, z) = \lambda^+ I_0 A^*(\lambda^+) \lambda^-(C(z) - 1) D^*(p \lambda^+ (1 - C(z))) K(z) / T(z)$$  \hspace{1cm} (7.40)
Substituting the expressions of $I(0, z)$, $P(0, z)$, $F_1(0, z)$ and $F_2(0, z)$ in (7.25), (7.26), (7.27) and (7.28), we get

$$I(x, z) = [\lambda^+ I_0 \left[C(z) B^*(\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) - z + \theta(1 - C(z))\right]$$
$$B^*(\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) + K(z) (C(z) (\lambda^- D^*(p \lambda^+ (1 - C(z))))$$
$$R^*(q \lambda^+ (1 - C(z)) + \lambda^+ \bar{\alpha} z C(z) + \lambda^+ \delta \alpha z)]$$
$$e^{-\lambda^+ x} (1 - A(x)) / T(z)$$

(7.41)

$$P(x, z) = \lambda^+ I_0 A^+ (\lambda^+ (C(z) - 1)) e^{-(\lambda^+ + \lambda^- + \lambda^+ \bar{\delta} C(z)) x} (1 - B(x)) / T(z)$$

(7.42)

$$F_1(x, z) = \lambda^+ \lambda^- I_0 A^+ (\lambda^+ (C(z) - 1)) K(z) e^{-(p \lambda^+ (1 - C(z))) x} (1 - D(x)) / T(z)$$

(7.43)

$$F_2(x, z) = \lambda^+ \lambda^- I_0 A^+ (\lambda^+ (C(z) - 1)) D^*(p \lambda^+ (1 - C(z))) K(z) e^{-(q \lambda^+ (1 - C(z))) x} (1 - R(x)) / T(z)$$

(7.44)

The partial probability generating function of the orbit size when the server is idle is

$$I(z) = \int_0^\infty I(x, z) dx$$

$$= [I_0 (1 - A^*(\lambda^+)) [(C(z) B^*(G(z)) - z + \theta B^*(G(z)) (1 - C(z))$$
$$+ (\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) + C(z) (1 - B^*(G(z))) (\lambda^- D^*(p \lambda^+ (1 - C(z))))$$
$$R^*(q \lambda^+ (1 - C(z)) + \lambda^+ \delta \bar{\alpha} z C(z) + \lambda^+ \delta \alpha z)] / T_1(z)$$

(7.45)

where

$$G(z) = \lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-$$
$$T_1(z) = [z - (A^*(\lambda^+) + C(z) (1 - A^*(\lambda^+))) \bar{\theta} B^*(G(z)) - \theta B^*(G(z))]$$
$$= (\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) - (1 - B^*(G(z))) [(A^*(\lambda^+) + C(z) (1 - A^*(\lambda^+)))$$
$$+ (\lambda^- D^*(p \lambda^+ (1 - C(z))) R^*(q \lambda^+ (1 - C(z)) + \lambda^+ \delta \bar{\alpha} z C(z)$$
$$+ \lambda^+ \delta \alpha z C(z))]$$
The partial probability generating function of the orbit size when the server is busy is

\[ P(z) = \int_{0}^{\infty} P(x, z) dx = I_0 \lambda^+ A^*(\lambda^+) (C(z) - 1)(1 - B^* (G(z))) / T_1(z) \] (7.46)

The partial probability generating function of the orbit size when the server is waiting for repair is

\[ F_1(z) = \int_{0}^{\infty} F_1(x, z) dx = I_0 \lambda^- A^*(\lambda^+) (1 - B^* (G(z))) (D^*(p\lambda^+ (1 - C(z)) - 1) / (p T_1(z)) \] (7.47)

The partial probability generating function of the orbit size when the server is under repair is

\[ F_2(z) = \int_{0}^{\infty} F_2(x, z) dx = [I_0\lambda^+ A^*(\lambda^+) (1 - B^* (G(z))) D^*(p\lambda^+ (1 - C(z))) (R^*(q\lambda^+ (1 - C(z)) - 1))/ (q T_1(z)) \] (7.48)

Using the normalising equation (7.23) \( I_0 \) can be obtained as

\[ I_0 = \left[ (B^*(\lambda^+ \delta + \lambda^-) - 1) (\lambda^+ (\delta + m_1) + \lambda^- \lambda^+ m_1 (p \gamma_1 + q \beta_1) + m_1 (1 - A^*(\lambda^+)) \\
(\lambda^- + \lambda^+ \delta + \alpha) + (\lambda^+ \delta + \lambda^-) (1 - m_1 (1 - A^*(\lambda^+)) - \theta B^*(\lambda^+ \delta + \lambda^-)) / [A^*(\lambda^+)] \\
((\lambda^+ \delta + \lambda^-) + (B^*(\lambda^+ \delta + \lambda^-) - 1) (\lambda^+ (\delta - \lambda^- m_1 (p \gamma_1 + q \beta_1)))) \right] \] (7.49)

The probability generating function of the orbit size is

\[ P_q(z) = I_0 + I(z) + P(z) + F_1(z) + F_2(z) = I_0 A^*(\lambda^+) [\lambda^+ ((z - 1) (1 - C(z) + \delta C(z) B^* (G(z)))) + T_2(z)] / T_1(z) \] (7.50)
where

\[ T_2(z) = \lambda^{-1}[(z - B^*(G(z))) - (1 - B^*(G(z))) \]

\[ (D^*(p \lambda^+ (1 - C(z)))R^*(q \lambda^+ (1 - C(z))) + ((1 - D^*(p \lambda^+ (1 - C(z)))) / p) \]

\[ + (D^*(p \lambda^+ (1 - C(z))) (1 - R^*(q \lambda^+ (1 - C(z)))) / q))] \]

The probability generating function of the system size is

\[ P_S(z) = I_0 + I(z) + z P(z) + F_1(z) + F_2(z) \]

\[ = I_0 A^*(\lambda^+) [(\lambda^+ (z - 1)B^*(G(z)) (1 - C(z)) + \delta C(z)) + T_2(z)] / T_1(z) \]  \hspace{1cm} (7.51)

7.4 **Performance Measures**

- If the system is in steady state, then the probability that the system is empty is given by \( I_0 \)

- The probability that the server is idle during retrial time is given by

\[ I = \lim_{z \to 1} I(z) \]

\[ = I_0 (1 - A^*(\lambda^+)) [(1 - B^*(\lambda^+ \delta + \lambda^-)) (\lambda^- m_1 (1 + \lambda^+ p \gamma_1 + \lambda^+ q \beta_1) \]

\[ + \lambda^+ (m_1 + \delta + \delta m_1 \theta_1)) + (\lambda^+ \delta + \lambda^-) (\overline{\theta} m_1 B^*(\lambda^+ \delta + \lambda^-) - 1)] / T_1^\prime (1) \]

\[ T_1^\prime (1) = (B^*(\lambda^+ \delta + \lambda^-) - 1) [\lambda^+ \delta + \lambda^+ m_1 + \lambda^- \lambda^+ m_1 (p \gamma_1 + q \beta_1) \]

\[ + m_1 (1 - A^*(\lambda^+)) (\lambda^- + \lambda^+ \delta \overline{\alpha}) + (\lambda^+ \delta + \lambda^-) \]

\[ (1 - m_1 (1 - A^*(\lambda^+)) \overline{\theta} B^*(\lambda^+ \delta + \lambda^-)) \]  \hspace{1cm} (7.52)

- The probability that the server is busy is given by

\[ P = \lim_{z \to 1} P(z) \]

\[ = I_0 A^*(\lambda^+) \lambda^+ m_1 (1 - B^*(\lambda^+ \delta + \lambda^-)) / T_1^\prime (1) \]  \hspace{1cm} (7.53)
• The probability that the server is in failure mode is given by

\[
F = \lim_{z \to \infty} (F_1(z) + F_2(z)) = I_0 A^*(\lambda^+) \lambda^- \lambda^+ m_1(\gamma_1 + \beta_1) (1 - B^*(\lambda^+ \delta + \lambda^-)) / T_1'(1) \quad (7.54)
\]

• The probability of unsuccessful retrials made by the customer is given by

\[
V_R = \frac{\lambda^+ m_1 (1 + \lambda^- \gamma_1 + \lambda^- \beta_1) A^*(\lambda^+) (1 - B^*(\lambda^+ \delta + \lambda^-))}{(B^*(\lambda^+ \delta + \lambda^-) - 1)[A^*(\lambda^+) (\lambda^+ (\delta - \lambda^-) m_1 + \bar{p} \gamma_1 + \bar{q} \beta_1))]}
\]

Let \(N_r(z)\) and \(D_r(z)\) represent the numerator and denominator of \(P_q(z)\).

• The mean number of customers in the orbit \(L_q\) is given by

\[
L_q = \lim_{z \to 1} \frac{d}{dz} P_q(z) = \frac{D_r'(1) N_r^n(1) - N_r'(1) D_r^n(1)}{2D_r'(1)^2} \quad (7.56)
\]

where

\[
N_r'(1) = I_0 A^*(\lambda^+) [\lambda^+ \delta B^*(\lambda^+ \delta + \lambda^-) + \lambda^- (1 - B^*(\lambda^+ \delta + \lambda^-))
\]

\[
\lambda^+ m_1 (\bar{p} \gamma_1 + \bar{q} \beta_1))]
\]

\[
N_r^n(1) = I_0 A^*(\lambda^+) [2\lambda^+ (\delta h_3 + m_1 \delta B^*(\lambda^+ \delta + \lambda^-) - m_1) - \lambda^- (2B^*(\lambda^+ \delta + \lambda^-) \lambda^+ m_1
\]

\[
(\bar{p} \gamma_1 + \bar{q} \beta_1)) + \lambda^+ (\bar{p} h_1 + \bar{q} h_2 + 2 \bar{q} p \lambda^+ m_1^2 \gamma_1 \beta_1) (1 - B^*(\lambda^+ \delta + \lambda^-))]
\]

\[
D_r'(1) = T_1'(1)
\]
\[ \text{Dr}''(1) = (B^*(\lambda^+\delta + \lambda^-) - 1) [\lambda^+(2m_1 + m_2) + \lambda^-(p \lambda^+ h_1 + q \lambda^+ h_2)] \\
+ 2\lambda^-\lambda^+ m_1^2 \gamma_1 q \beta_1 + 2 m_1 (\lambda^-\lambda^+ m_1 (p \gamma_1 + q \beta_1)) (1 - A^*(\lambda^+)) \\
+ \lambda^+\delta \bar{\alpha} (1 + m_1)) + m_2 (\lambda^- + \lambda^+ \delta \bar{\alpha}) (1 - A^*(\lambda^+)) \\
+ 2 \lambda^- \lambda^+ \bar{\theta} m_1^2 (1-A^*(\lambda^+)) \ B^*(\lambda^+\delta + \lambda^-) - (\lambda^+\delta + \lambda^-) \ \bar{\theta} h_3 (2 m_1 (1-A^*(\lambda^+)) \\
+ m_2 \bar{\theta} (1 - A^*(\lambda^+)) B^*(\lambda^+\delta + \lambda^-)) \\
+ 2 h_3 (\lambda^-\lambda^+ m_1 (p \gamma_1 + q \beta_1)) \\
+ \lambda^+\delta (1 + m_1) + m_1 (1 - A^*(\lambda^+)) (\lambda^- + \lambda^+ \delta \bar{\alpha})] \\
\]

\[ h_1 = m_2 \gamma_1 + p \lambda^+ m_1^2 \gamma_2 \]

\[ h_2 = m_2 \beta_1 + q \lambda^+ m_1^2 \beta_2 \]

and

\[ h_3 = \lambda^+ \delta m_1 \int_0^\infty e^{-(\lambda^+ \delta + \lambda^-)x} x b_1(x) \ dx \]

- The mean number of customers in the system is given by

\[ L_S = \lim_{z \to 1} \frac{d}{dz} P_S(z) \]

\[ = L_q + P \]

(7.57)

7.5 Reliability Indices

Let \( A(t) \) be the system availability at time \( t \), that is the probability that the server is idle or working for a customer. Then under steady state condition, the availability of the server is given by

\[ A = I_0 + \lim_{z \to 1} \left[ \int_0^\infty I(x,z) \ dx + \int_0^\infty P(x,z) \ dx \right] \]

\[ = I_0 + I + P \]

\[ = \frac{(\lambda^+ \delta + \lambda^-) + \lambda^+ (\delta + \lambda^- m_1 (p \gamma_1 + q \beta_1)) (B^* (\lambda^+ \delta + \lambda^-) - 1)}{(\lambda^+ \delta + \lambda^-) + \lambda^+ (\delta - \lambda^- m_1 (p \gamma_1 + q \beta_1)) (B^* (\lambda^+ \delta + \lambda^-) - 1)} \]

(7.58)
The steady state failure frequency of the server is

\[ F = \lambda^- P \]

\[ = \frac{\lambda^- \lambda^+ m_1 (1 - B^+ (\lambda^+ \delta + \lambda^-))}{(\lambda^+ \delta + \lambda^-) + (\lambda^+ (\delta - \lambda^-) m_1 (\bar{p} \gamma_1 + \bar{q} \beta_1)) (B^+ (\lambda^+ \delta + \lambda^-) - 1)} \]

(7.59)

### 7.6 Stochastic Decomposition

#### Theorem 7.2

The expected number of customers in the system (L₅) can be expressed as the sum of two independent random variables, one of which is the expected number of customers in unreliable batch arrival classical queueing system with positive and negative customers, priority or collisions and delayed repairs (L) and the other is the expected number of customers in the orbit given that the server is idle (L₁).

#### Proof

The probability generating function \( \Phi(z) \) of the number of customers in unreliable batch arrival queue with positive and negative customers, priority or collisions and delayed repairs is given by

\[ \Phi(z) = \frac{(B^+ (\lambda^+ \delta + \lambda^-) - 1) (\lambda^+ (\delta + m_1) + \lambda^- \lambda^+ m_1 (p \gamma_1 + q \beta_1)) + (\lambda^+ \delta + \lambda^-)}{(\lambda^+ \delta + \lambda^-) + (B^+ (\lambda^+ \delta + \lambda^-) - 1) [\lambda^+ (\delta - \lambda^- m_1 (\bar{p} \gamma_1 + \bar{q} \beta_1))]} \]

\[ \times \frac{\lambda^+ (z - 1) B^*(G(z)) (1 - C(z) + \delta C(z)) + T_2(z)}{(z - B^*(G(z))) (\lambda^+ (1 - \delta C(z)) + \lambda^-) - (1 - B^*(G(z)))}
\]

\[ (\lambda^- D^*(p \lambda^+ (1 - C(z))) R^*(q \lambda^+ (1 - C(z))) + \lambda^+ \delta z C(z)) \]

(7.60)
The probability generating function \( \psi(z) \) of the number of customers in the orbit given that the server is idle is given by

\[
\psi(z) = \frac{I_0 + I(z)}{I_0 + I(1)}
\]

\[
= \frac{[(\lambda^- \lambda^+ m_1 (p \gamma_1 + q \beta_1) + \lambda^+ (m_1 + \delta)) + (\lambda^+ \delta + \lambda^-)] (B^*(\lambda^+ \delta + \lambda^-) - 1)}{[(\lambda^+ \delta + \lambda^-) + (\lambda^+ (\delta - \lambda^-) m_1 (\bar{p} \gamma_1 + \bar{q} \beta_1) (B^*(\lambda^+ \delta + \lambda^-) - 1))]} x
\]

\[
[[I_0 A^*(\lambda^+) (\lambda^+ (1 - \bar{\delta}C(z)) + \lambda^-) (z - B^*(G(z)))
- (1 - B^*(G(z))) (\bar{\lambda} D^*(p \lambda^+ (1 - C(z))) R^*(q \lambda^+ (1 - C(z)))
+ \lambda^+ \delta z C(z))] / T_1(z)]
\]

From equations (7.51), (7.60) and (7.61) we get

\[
P_S(z) = \Phi(z) \psi(z)
\]

Consequently, in terms of convolution we can state that

\[
L_S = L + L_1
\]

### 7.7 Special Cases

#### Case (i)

If \( \lambda^- = 0 \) (no negative customers), then our model reduces to \( M^X/G/1 \) retrial queue with priority or collisions and orbital search. In this case

\[
P_S(z) = I_0 A^*(\lambda^+) [\lambda^+ (z - 1) B^*(\lambda^+ (1 - \bar{\delta} C(z))) (1 - C(z) + \delta C(z))] / T_3(z)
\]

where

\[
T_3(z) = z - (A^*(\lambda^+) + C(z) (1 - A^*(\lambda^+))) \bar{\delta} B^*(\lambda^+ (1 - \bar{\delta} C(z)))
- \theta B^*(\lambda^+ (1 - \bar{\delta} C(z))) (\lambda^+ (1 - \bar{\delta} C(z))) - (1 - B^*(\lambda^+ (1 - \bar{\delta} C(z))))

[(A^*(\lambda^+) + C(z) (1 - A^*(\lambda^+))) + \lambda^+ \delta \bar{\alpha} z C(z)] + \lambda^+ \delta \alpha z C(z)
\]

\[
I_0 = [(B^*(\lambda^+ \delta) - 1) (\lambda^+ (\delta + m_1) + m_1 (1 - A^*(\lambda^+)) (\lambda^+ \delta \bar{\alpha}))
+ \lambda^+ \delta (1 - m_1 (1 - A^*(\lambda^+)) \bar{\delta} B^*(\lambda^+ \delta))] / (\lambda^+ \delta (A^*(\lambda^+ + (B^*(\lambda^+ \delta) - 1)))}
Case (ii)

\( A^*(\lambda) \rightarrow 1 \) (no retrials), in this case our model becomes \( M^x/G/1 \) queue with positive and negative customers, priority or collisions, server breakdown and delayed repairs. For this model

\[
P_s(z) = \frac{I_0[\lambda^+(z-1)B^*(G(z))(1-C(z)+\delta C(z))+T_2(z)]}{(z-B^*(G(z)))(\lambda^+(1-\delta C(z))+\lambda^-)}
- (1-B^*(G(z))\lambda^- D^*(p \lambda^+(1-C(z)))R^*(p \lambda^+(1-C(z))+\lambda^+ \delta z C(z))
\]

\[
I_0 = \frac{(B^*(\lambda^+ \delta + \lambda^-)-1)[\lambda^+ (\delta + m_1) + \lambda^- \lambda^+ m_1(p \gamma_1 + q \beta_1)] + (\lambda^+ \delta + \lambda^-)}{\lambda^+ \delta + \lambda^-} \times \frac{(B^*(\lambda^+ \delta + \lambda^-)-1)[\lambda^+ (\delta - \lambda^- m_1(p \gamma_1 + q \beta_1))] + (\lambda^+ \delta + \lambda^-)}{\lambda^+ \delta + \lambda^-}
\]

7.8 Practical Justification of the Model

A computer system (server) is meant to execute programs (customers) that are submitted to it. Prior to execution, the submitted programs remain in the computer memory (orbit).

When multiple programs are submitted, the computer executes them based on a predetermined priority. A high priority program, upon entry, interrupts the computer system and is serviced prior to the other low priority ones. The low priority programs, meanwhile, remain in the memory. When the computer system completes an execution it picks-up the next program for execution, from the orbit, if available.

A computer virus, unlike a conventional computer program, is a malicious program that can harm the system in a numerous ways and can even cause a breakdown. Computer viruses, based on their behaviour, can be either passive or active.

A passive virus, collides with the program-in-service and interrupts only the current service. As a result both the virus and the current program move to the memory. An active virus, on the other hand, causes a heavy damage by terminating all services and ultimately shuts down the computer system.
When the computer system experiences failure it has to undergo repair. The repair process begins after an initial delay. During this down time, the programs that come in for execution may be withdrawn (balking) without being executed.

7.9 Numerical Results

Performance measures are calculated numerically by assuming that the retrial time, service time, delay time and repair time follow exponential distribution with respective rates $\eta$, $\mu$, $\gamma$ and $\beta$.

For the parameters $\lambda^+ = 2$, $\lambda^- = 0.3$, $\delta = 0.6$, $\alpha = 0.7$, $p = 0.5$, $q = 0.5$, $\theta = 0.5$, $\mu = 4$, $\gamma = 2$, $\beta = 1$, $\eta = 30$, $C_1 = C_2 = 0.5$, the performance measures $I_0$ - the probability that the system is empty, $I$ - the probability that the server is idle in non-empty system, $P$ - the probability that the server is busy, $F_1$ - the probability that the server is in delay time, $F_2$ - the probability that the server is under repair, $A$ - the availability of the server, $F$ - the failure frequency of the server and $L_S$ - the mean number of customers in the system are calculated by varying the rates $\lambda^-$, $p$, $q$, $\alpha$, $\delta$, $\beta$ and $\gamma$ and presented in Tables 7.1 to 7.7 respectively.

Table 7.1 reveals that $I_0$, $P$ and $A$ monotonically decrease and $I$, $F_1$, $F_2$, $F$ and $L_S$ increase as $\lambda^-$ increases. Tables 7.2 and 7.3 indicate that increase in $p$ and $q$ decreases $I_0$ and $A$ and increases other performance measures. From Tables 7.4 and 7.5, we observe that the parameters $\alpha$ and $\delta$ do not show any effect on the performance measures other than $I_0$, $I$ and $L_S$. For increasing values of $\alpha$, $I_0$ and $L_S$ increase and $I$ decreases. As $\delta$ increases $I_0$ decreases and $I$ and $L_S$ increase. Tables 7.6 and 7.7 depict the effect of $\beta$ and $\gamma$ on the performance measures. It is noted that $I_0$, $P$, $A$ and $F$ increase and $I$ and $L_S$ decrease with increase in $\beta$ and $\gamma$. Also $F_1$ increases with $\beta$ and decreases with $\gamma$. $F_2$ increases with $\gamma$ and decreases with $\beta$. 
The effect of $I_0$, $I$ and $L_S$ against the parameters $\lambda^+$ and $\mu$ are plotted in Fig. 7.1 (a) to (d). From the figures it is observed that

- $I_0$ decreases with $\lambda^+$ and increases with $\mu$.
- $I$, $P$ and $L_S$ increase for increasing values of $\lambda^+$ and decrease for increasing $\mu$.

The variation of $I_0$, $I$ and $L_S$ with respect to the parameters $\eta$ and $\theta$ are given in Fig. 7.2 (a) to (d). Figures reveal that

- $I_0$ increases with increase in $\eta$ and $\theta$.
- $I$ and $L_S$ decrease with increase in $\eta$ and $\theta$.
- $P$ is independent of $\eta$ and $\theta$. 
Table 7.1 Performance Measures by varying $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$I_0$</th>
<th>$P$</th>
<th>$I$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$A$</th>
<th>$F$</th>
<th>$L_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1000</td>
<td>0.1589</td>
<td>0.6936</td>
<td>0.0434</td>
<td>0.0347</td>
<td>0.0694</td>
<td>0.8960</td>
<td>0.0694</td>
<td>3.0262</td>
</tr>
<tr>
<td>0.2000</td>
<td>0.1170</td>
<td>0.6452</td>
<td>0.0443</td>
<td>0.0645</td>
<td>0.1290</td>
<td>0.8065</td>
<td>0.1290</td>
<td>4.5646</td>
</tr>
<tr>
<td>0.3000</td>
<td>0.0805</td>
<td>0.6030</td>
<td>0.0451</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
<td>0.1809</td>
<td>7.3056</td>
</tr>
<tr>
<td>0.4000</td>
<td>0.0486</td>
<td>0.5660</td>
<td>0.0458</td>
<td>0.1132</td>
<td>0.2264</td>
<td>0.6604</td>
<td>0.2264</td>
<td>13.2096</td>
</tr>
<tr>
<td>0.5000</td>
<td>0.0203</td>
<td>0.5333</td>
<td>0.0464</td>
<td>0.1333</td>
<td>0.2667</td>
<td>0.6000</td>
<td>0.2667</td>
<td>34.1323</td>
</tr>
</tbody>
</table>

Table 7.2 Performance Measures by varying $p$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$I_0$</th>
<th>$P$</th>
<th>$I$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$A$</th>
<th>$F$</th>
<th>$L_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2000</td>
<td>0.1066</td>
<td>0.5871</td>
<td>0.0421</td>
<td>0.0881</td>
<td>0.1761</td>
<td>0.7358</td>
<td>0.1761</td>
<td>4.6621</td>
</tr>
<tr>
<td>0.4000</td>
<td>0.0894</td>
<td>0.5976</td>
<td>0.0441</td>
<td>0.0896</td>
<td>0.1793</td>
<td>0.7311</td>
<td>0.1793</td>
<td>6.2268</td>
</tr>
<tr>
<td>0.6000</td>
<td>0.0715</td>
<td>0.6085</td>
<td>0.0461</td>
<td>0.0913</td>
<td>0.1826</td>
<td>0.7262</td>
<td>0.1826</td>
<td>8.6935</td>
</tr>
<tr>
<td>0.8000</td>
<td>0.0530</td>
<td>0.6198</td>
<td>0.0482</td>
<td>0.0930</td>
<td>0.1860</td>
<td>0.7211</td>
<td>0.1869</td>
<td>13.1073</td>
</tr>
<tr>
<td>1.000</td>
<td>0.0338</td>
<td>0.6316</td>
<td>0.0504</td>
<td>0.0947</td>
<td>0.1895</td>
<td>0.7158</td>
<td>0.1895</td>
<td>23.0154</td>
</tr>
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</table>

Table 7.3 Performance Measures by varying $q$

<table>
<thead>
<tr>
<th>$q$</th>
<th>$I_0$</th>
<th>$P$</th>
<th>$I$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$A$</th>
<th>$F$</th>
<th>$L_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2500</td>
<td>0.1232</td>
<td>0.5769</td>
<td>0.0403</td>
<td>0.0865</td>
<td>0.1731</td>
<td>0.7404</td>
<td>0.1731</td>
<td>3.6252</td>
</tr>
<tr>
<td>0.4000</td>
<td>0.0981</td>
<td>0.5923</td>
<td>0.0431</td>
<td>0.0888</td>
<td>0.1777</td>
<td>0.7335</td>
<td>0.1777</td>
<td>5.3537</td>
</tr>
<tr>
<td>0.5500</td>
<td>0.0715</td>
<td>0.6085</td>
<td>0.0461</td>
<td>0.0913</td>
<td>0.1826</td>
<td>0.7262</td>
<td>0.1826</td>
<td>8.7271</td>
</tr>
<tr>
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<td>0.6257</td>
<td>0.0493</td>
<td>0.0938</td>
<td>0.1877</td>
<td>0.7185</td>
<td>0.1877</td>
<td>17.3063</td>
</tr>
<tr>
<td>0.8500</td>
<td>0.0139</td>
<td>0.6438</td>
<td>0.0526</td>
<td>0.0966</td>
<td>0.1931</td>
<td>0.7103</td>
<td>0.1931</td>
<td>66.4295</td>
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</table>

Table 7.4 Performance Measures by varying $\alpha$

<table>
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<tr>
<th>$\alpha$</th>
<th>$I_0$</th>
<th>$P$</th>
<th>$I$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$A$</th>
<th>$F$</th>
<th>$L_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2000</td>
<td>0.0685</td>
<td>0.6030</td>
<td>0.0572</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
<td>0.1809</td>
<td>6.6464</td>
</tr>
<tr>
<td>0.4000</td>
<td>0.0733</td>
<td>0.6030</td>
<td>0.0523</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
<td>0.1809</td>
<td>6.9361</td>
</tr>
<tr>
<td>0.6000</td>
<td>0.0781</td>
<td>0.6030</td>
<td>0.0475</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
<td>0.1809</td>
<td>7.1900</td>
</tr>
<tr>
<td>0.8000</td>
<td>0.0829</td>
<td>0.6030</td>
<td>0.0427</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
<td>0.1809</td>
<td>7.4144</td>
</tr>
<tr>
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<td>0.6030</td>
<td>0.0379</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
<td>0.1809</td>
<td>7.6141</td>
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</table>
Table 7.5 Performance Measures by varying $\delta$

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$I_0$</th>
<th>$P$</th>
<th>$I$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$A$</th>
<th>$F$</th>
<th>$L_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4000</td>
<td>0.0829</td>
<td>0.6030</td>
<td>0.0427</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
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<tr>
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<td>0.6030</td>
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<td>0.0905</td>
<td>0.1809</td>
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<td>0.1809</td>
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<tr>
<td>0.7000</td>
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<td>0.6030</td>
<td>0.0463</td>
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<td>0.1809</td>
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</tr>
<tr>
<td>0.8500</td>
<td>0.0775</td>
<td>0.6030</td>
<td>0.0481</td>
<td>0.0905</td>
<td>0.1809</td>
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</tr>
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<td>0.0499</td>
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<td>0.7286</td>
<td>0.1809</td>
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</tbody>
</table>

Table 7.6 Performance Measures by varying $\beta$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$I_0$</th>
<th>$P$</th>
<th>$I$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$A$</th>
<th>$F$</th>
<th>$L_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>0.0805</td>
<td>0.6030</td>
<td>0.0451</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
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</tr>
<tr>
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<td>0.1162</td>
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<td>0.0444</td>
<td>0.0933</td>
<td>0.1244</td>
<td>0.7824</td>
<td>0.1865</td>
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<td>0.1349</td>
<td>0.6316</td>
<td>0.0441</td>
<td>0.0947</td>
<td>0.0947</td>
<td>0.8105</td>
<td>0.1895</td>
<td>3.2996</td>
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<td>0.1464</td>
<td>0.6376</td>
<td>0.0439</td>
<td>0.0956</td>
<td>0.0765</td>
<td>0.8278</td>
<td>0.1913</td>
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<td>0.6417</td>
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<td>0.1925</td>
<td>2.6545</td>
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Table 7.7 Performance Measures by varying $\gamma$

<table>
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<tr>
<th>$\gamma$</th>
<th>$I_0$</th>
<th>$P$</th>
<th>$I$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$A$</th>
<th>$F$</th>
<th>$L_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
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<td>0.5769</td>
<td>0.0460</td>
<td>0.1731</td>
<td>0.1731</td>
<td>0.6538</td>
<td>0.1731</td>
<td>23.2899</td>
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<tr>
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<td>0.0635</td>
<td>0.5941</td>
<td>0.0454</td>
<td>0.1188</td>
<td>0.1782</td>
<td>0.7030</td>
<td>0.1782</td>
<td>9.9085</td>
</tr>
<tr>
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<td>0.0805</td>
<td>0.6030</td>
<td>0.0451</td>
<td>0.0905</td>
<td>0.1809</td>
<td>0.7286</td>
<td>0.1809</td>
<td>7.3056</td>
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<tr>
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<td>0.0449</td>
<td>0.0730</td>
<td>0.1826</td>
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<tr>
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<td>0.6122</td>
<td>0.0448</td>
<td>0.0612</td>
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<td>0.7551</td>
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<td>5.6281</td>
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</tbody>
</table>
(a) $I_0$ versus $(\mu, \lambda^*)$

(b) $I$ versus $(\mu, \lambda^*)$

(c) $P$ versus $(\mu, \lambda^*)$

(d) $L_s$ versus $(\mu, \lambda^*)$

Fig. 7.1 Performance Measures Versus $(\mu, \lambda^*)$
(a) $I_0$ versus $(\eta, \theta)$

(b) $I$ versus $(\eta, \theta)$

(c) $P$ versus $(\eta, \theta)$

(d) $L_s$ versus $(\eta, \theta)$

Fig. 7.2 Performance Measures versus $(\eta, \theta)$