3.1 Introduction

Coordination is essential to improve the performance of the Supply Chain (SC) as it involves multiple organisations and numerous activities with different policies led by different people. This creates interesting challenges for effective SC coordination. Long term effective coordination is possible by implementing proper coordination mechanisms based on the nature of the SC. Literature in the area of Supply Chain Management (SCM) shows that increasing importance is being given now to coordination in SCM to improve performance. Two popular coordination mechanisms in use are ‘price discount’ and ‘delay in payment’. The effect of these two mechanisms individually on SC performance has already been reported in literature as follows. Jaber et al (2006a) conducted a study using price discount as mechanism in a three level supply chain and Jaber et al (2006b) conducted a study using delay in payments as coordination mechanisms in a two level supply chain. In both the studies, they found out the improvement in performance due to coordination compared to ‘no coordination’. In this part of the study, we have considered a three level
supply chain with combination of these mechanisms (price discount and delay in payment) simultaneously and found further significant improvement in the performance compared to individual cases of coordination. We have further extended this study to network supply chain under different business situations and it is presented in the chapter 4.

We have also conducted a sensitivity analysis to study the effect of changes in system parameters (rate of return, order quantity in various combinations and price elasticity of demand) on the output which will be useful for the supply chain practitioners to take appropriate decisions in the concerned area.

As mentioned above, in this part of the study, the two mechanisms ‘price discount’ and ‘delay in payment’ are used simultaneously to coordinate a three level SC. ‘Price discount’ is a coordination mechanism by which the seller provides a discount to the buyer to enhance the price elastic/ price dependent demand thereby coordinating the order quantity among the SC levels. ‘Delay in payment’ is another coordination mechanism by which the seller permits a delay in payment to the buyer for which the buyer need not pay any interest to the seller on the amount to be paid. But, the buyer can avail more time than is permitted for which the buyer has to pay interest on the amount to be paid for the additional period that exceeds the permitted one. Owing to the delay in payment, the buyer’s order quantity will be enhanced because of the reduction in holding cost. So, allowing ‘delay in payment’ also coordinates the order quantity among the SC levels. The details of benefit of simultaneous implementation of ‘price discount and delay in payment’ are explained in succeeding sections.

In this study, end customer demand is assumed as price elastic. So, the end customer demand increases as price decreases and hence the phenomenon
of bullwhip effect will not occur. Further, as delay in payment is provided, the holding cost will be reduced. In mathematical modelling, annual demand rate is measured from the end side of the supply chain which is assumed to be a linear function of the discount rate given by the retailer.

In addition to the benefits of simultaneous use of ‘price discount and delay in payment’, the analysis of the effect of various system parameters on SC profit are also analysed in this part of the study which enables the SC practitioners to exercise appropriate control on the decision variables of the SC system. The SC profit (surplus) is taken as the performance measure in this study. The improvement in SC profit due to combination of ‘price discount and delay in payment’ is compared with the case of no coordination and price discount alone and the improvements reported.

### 3.2 Mathematical Modeling

The three level SC used in this study consists of one supplier, one manufacturer and one retailer as shown in Figure 1. In this case, annual customer demand is known and retailer places the order according to its Economic Order Quantity (EOQ). As the demand is assumed to be price elastic, the retailer provides an optimal discount to the customer to increase the demand. Similarly, each player provides optimal discount to their buyer in the SC to increase the demand and thereby the order quantity. At the same time, the supplier and the manufacturer allow delay in payment to their buyers due to which the holding cost of manufacturer and retailer decreases thereby increasing their order quantity. With permissible delay in payment, the retailer and the manufacturer have the opportunity to invest the unpaid balance for the period of delay in payment. The overall objective of implementation of these mechanisms (price discount and delay in payment) simultaneously is to enhance the volume of business and to improve the coordination among the SC members.
thereby improving the SC profit. Figure 3.1 shows the structure of the three-level SC and the strategic coordination mechanisms being implemented.

![Figure 3.1: Structure of the Supply Chain](image)

The work presented in this chapter deals with the modelling and analysis of a three level SC coordination with price discount along with delay in payment as mechanisms. The type of price discount used is ‘all unit price discounts’. With the all-units price discount, discount will be given to all items in a given purchase quantity irrespective of the quantity purchased. The performance is measured by taking the sum of the profits of the three players in the SC. In this case, both the ‘all unit price discount’ and ‘delay in payment’ are used to coordinate the order quantities among the SC levels.

In this chapter, the section 3.2 is for mathematical modelling of the three level SC profit function. Sub section 3.2.1 present assumptions, 3.2.2 presents notations used in this modelling and subsection 3.2.3 present analysis part of the profit functions of each player in the three-level SC. Under subsection 3.2.3, subsections 3.2.3.1 to 3.2.3.3 present individual profit functions for the retailer, manufacturer and supplier respectively. Then, sub section 3.2.3.4 present the mathematical profit model for the three level SC with coordination using price
discount along with delay in payment as coordination mechanisms. The model developed is a nonlinear mathematical model and this model approach is adopted as the operating conditions such as demand lead time etc are assumed to be static and known with certainty. Finally Section 3.3 presents numerical results; section 3.4 presents sensitivity analysis and section 3.5 presents conclusion.

3.2.1 Assumptions

This study used an SC consisting of a single supplier, single manufacturer and single retailer and considered the case of a single product. It is also assumed that no shortages are allowed at any level of the SC. Apart from this, zero lead time or instantaneous replenishment is assumed throughout the SC. Price elastic or price dependent demand is another important assumption in this study. Other relevant assumptions include the one like products have perfect quality, no rejections at any level of SC, instantaneous replenishment on placing the order, infinite planning horizon and cost parameters do not vary over time. As far as fund flow is concerned, it is also assumed that each player is financially capable of settling his/her balance with the preceding player at any point in time in a single payment. Linear storage cost per unit time is taken for the computations. In addition to this, 10% rate of return indicates recession, 15% rate of return indicates normal and 20% rate of return indicates boom situation in this study. Continuous compounding rate of return is assumed in this regard. This study also assumed that discount given by each player to its downstream player is ‘all unit price discount’ and supplier is not getting any delay in payment or price discount from its upstream player which is not considered as a part of this study.

With respect to inventory carrying cost, both holding cost and storage cost are considered in this study. Holding cost in this study is taken as cost due
to interest on working capital and storage cost means all other costs incurred for storing (other costs such as rent, product deterioration and obsolescence, insurance etc) the item. Carrying cost can be taken as the sum of storage cost and holding cost. We have made this division of Carrying cost to calculate the effect of delay in payment. Holding cost is zero for the buyer till he makes the payment to the seller. However, seller incurs an additional holding cost till he gets the payment from the buyer even though the items are reached at buyer end. More details are given in the succeeding sections.

3.2.2 Notations

\( i \) = a subscript identifying a specific player in a supply chain; \( i = s, m, r \) 

(\( s = \) supplier, \( m = \) manufacturer, \( r = \) retailer)

\( A_i \) = Order cost for player \( i \), \( h_i \) = holding cost for player \( i \), \( s_i \) = Storage cost for player \( i \)

\( c_i \) = Procurement cost for player \( i \),

\( k_i \) = Return on investment / interest to be paid for player \( i \).

Actual end customer demand \( D = D_0 + D_1 \times d_r \), Where \( D_0 = \) Initial demand, \( D_1 = \) Price elasticity of demand, \( d_r = \) Discount given by the retailer

\( p_i \) = Selling price for each player \( i \)

\( t_{ij} \) = Interest free permissible delay in payment period permitted by player \( i \) to player ‘j’ = \( t_{sm} \& t_{mr} \).
\( \tau_{ji} = \text{Maximum possible delay in payment period availed by player } \ 'j' \ \text{from player } \ 'i' = \tau_{rm} \& \tau_{ms} \), If \( \tau_{ji} > t_{ij} \), the player \( i \) \ 'charges interest on player \( j \) \ for the period of \( \tau_{ji} - t_{ij} \), Where \( i \neq j \)

\( Q_i = \text{Quantity ordered by player } i = s, m, r \)

\( T_i = \text{Inventory cycle time for player } i \), \( T_r = \frac{Q_r}{D} \), \( T_m = \frac{\lambda_m \times Q_r}{D} \), \( T_s = \frac{\lambda_m \times \lambda_s \times Q_r}{D} \)

\( \lambda_i = \text{An integer lot sizing multiplier which when multiplied with the orders received at a supply chain stage/level gives the order quantity to be placed with the immediate upstream level means an integer multiplier to set the order quantity of player } i \ \text{to that of player } j \ \text{where } i \neq j \) \ and \( \lambda_i = 1, 2, 3 \ldots \ldots \)

For example, \( Q_m = \lambda_m \times Q_r \) and \( Q_s = \lambda_s \times Q_m = \lambda_s \times \lambda_m \times Q_r \)

3.2.3 Analysis

Using the model developed and explained in the previous section, analysis has been carried out here. First, the profit functions of each of the three players are derived and this is then used to formulate the profit function for the SC. The effect of the two coordination mechanisms used has been incorporated in the profit function calculations which are given below.

3.2.3.1 Profit Function for Retailer

In this study, the retailer places his/her Economic Order Quantity (EOQ)
$Q_r$ with the manufacturer who provides an optimal discount ($d_m$) to the retailer up to a maximum of the difference between his/her purchase cost and selling price. Similarly, the retailer also provides a discount ($d_r$) to his/her customers to enhance the end customer demand as it is price elastic/price dependent. Since the end customer demand increases, the retailer’s EOQ also increases. Apart from this, the manufacturer allows the retailer a delay ($t_{mr}$) to make the payment up to a maximum of retailer’s inventory cycle time for which retailer need not pay any interest on the amount of purchase cost to be paid to the manufacturer. At the same time, the retailer can also extend this permitted period but the retailer has to pay the interest ($k_m$) for the additional period that exceeds the permitted one. Owing to this delay in payment, the retailer’s holding cost will be reduced and hence its EOQ further increases. The difference between sales revenue earned by the retailer, $Q_r p_r$ and the net cost for the retailer gives the profit (surplus) of the retailer.

Figure 3.2 illustrates the retailer’s inventory cycle. As the demand is linear throughout the cycle, the average inventory per cycle is equal to $\frac{1}{2} \times Q_r \times T_r$. The holding cost per cycle is computed by multiplying the average inventory per cycle with holding cost $h_r$. So, the holding cost is $\frac{h_r Q_r^2}{2D}$. This being the case, it is assumed that delay in payment is possible in three ways. The first one is that permitted interest free delay in payment period for the retailer by the manufacturer, $t_{mr}$ is less than or equal to retailer’s(buyer) inventory cycle time, $T_r$ and no more extension is allowed after permitted delay in payment period. In the second case, extension is allowed but interest has to be paid for the period after the
permitted period till the payment is made. This allowable extended delay in payments \( \tau_{rm} \) with interest can be less than or equal to the retailer inventory cycle time \( T_r \). In the third case also, extension is possible by paying interest to the manufacturer for the period after the permitted period till the payment is done and the allowable extended delay in payments \( \tau_{rm} \) with interest can be greater than or equal to retailer’s inventory cycle time, \( T_r \). In the second and third cases, the interest free permitted delay in payment period is less than or equal to retailer’s inventory cycle time \( T_r \) as in first case. The retailer must pay the purchase cost, \( c_rQ_r \), with the manufacturer either by the time \( t_{mr} \) or by time \( \tau_{rm} \). Thus, if the retailer avails delay in payment, retailer’s holding cost per cycle is reduced from \( \frac{h_rQ_r^2}{2D} \) to either \( \frac{h_r(Q_r - D \times t_{mr})^2}{2D} \) (case 1) or \( \frac{h_r(Q_r - D \times \tau_{rm})^2}{2D} \) (case 2) or ‘Zero’ (case 3). The holding cost is zero for the case 3 as the payment is done by the retailer to manufacturer only after the retailer’s inventory cycle time.

The retailer incurs storage cost per cycle as \( \frac{s_rQ_r^2}{2D} \).

![Figure 3.2: Illustrates the behavior of inventory in a retailer’s cycle (modified from Jaber et al, 2006)](image-url)
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The delay in payment offered by the manufacturer provides the retailer an opportunity to invest the unpaid purchase cost $c_r Q_r$, for a period $t_{mr}$ at a return rate of $k_r$. It is advantageous for the retailer to extend the delay in payment to maximum possible $\tau_{rm}$ if the return rate for the retailer exceeds that for the manufacturer ($k_r > k_m$). As mentioned earlier, continuous compounding rate of return is assumed in this study. So, the term $c_r Q_r \times \left\{ e^{(k_m \times (\tau_{rm} - t_{mr}))} - e^{(k_r \times \tau_{rm})} \right\}$ indicates the savings or additional cost for the retailer from investing an amount of $c_r Q_r$ for a period of length $\tau_{rm}$ ($\tau_{rm} > t_{mr}$).

The term $c_r Q_r \times \left\{ e^{(k_m \times (\tau_{rm} - t_{mr}))} - e^{(k_r \times \tau_{rm})} \right\}$ can be described as follows. If the retailer avails delay in payment from the manufacturer to a period $\tau_{rm}$ that exceeds the permitted interest-free period $t_{mr}$, then the retailer has to pay the interest along with the purchase cost of $c_r Q_r$ at a rate of $k_m$, for the period of $\tau_{rm} - t_{mr}$. At the same time, the retailer can invest this amount of purchase cost ($c_r Q_r$) at a rate of $k_r$ for a period $\tau_{rm}$, and can earn some additional revenue equal to $c_r Q_r \times e^{(k_r \times \tau_{rm})}$.

Profit of the retailer = Sales revenue – Net cost 3(1)

Net cost per unit cycle = $\psi_r(Q_r, d_r, t_{mr}, \tau_{rm}, d_m)$

= order cost + procurement cost + storage cost + holding cost + Interest paid to
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\[ \text{manufacturer} + \text{discount to customer} - (\text{discount from manufacturer} + \text{savings from investment}) \]

Quantity ordered by the retailer \( = Q_r \)

Discount offered by the retailer to the customer \( = d_r \)

Total Discount \( = d_r \times Q_r \)

Order cost \( = A_r \)

Procurement cost \( = c_r \times Q_r \)

Storage cost \( = \frac{s_r \times (Q_r)^2}{2D} \)

Holding cost \( = H_r (Q_r, t_{mr}, \tau_{rm}) \)

\[ = \frac{h_r (Q_r - D \times t_{mr})^2}{2D} \text{ (case1) or } \frac{h_r (Q_r - (D \times \tau_{rm}))^2}{2D} \text{ (case2) or } \] ‘Zero’ (case3)

Inventory cycle time for retailer \( T_r = \frac{Q_r}{D} \)

Discount by manufacturer \( = d_m \times Q_r \)

Interest paid to manufacturer \( = c_r \times Q_r \times e^{(k_m \times (\tau_{rm} - t_{mr}))} \)

Savings from the investments \( = c_r \times Q_r \times e^{(k_r \times \tau_{rm})} \)

Net cost of retailer per cycle \( = \psi_r(Q_r, d_r, t_{mr}, \tau_{rm}, d_m) \)
\[\psi_r(Q_r, d_r, t_{m r}, \tau_{m r}, d_m) = A_r + (c_r \times Q_r) + \left(\frac{s_r \times Q_r^2}{2D}\right) + H_r(Q_r, t_{m r}, \tau_{m r}) + (d_r \times Q_r) - (d_m \times Q_r) + c_r \times Q_r \times \left\{e^{(k_m \times (\tau_{m r} - t_{m r}) - e^{(k_r \times \tau_{m r})}}\right\} \]

Net cost of retailer per unit time = \[\frac{\psi_r(Q_r, d_r, t_{m r}, \tau_{m r}, d_m)}{T_r}\]

\[= \frac{A_r \times D}{Q_r} + (c_r \times D) + \frac{s_r \times Q_r^2}{2} + \frac{D \times H_r(Q_r, t_{m r}, \tau_{m r})}{Q_r} + d_r \times D - d_m \times D + c_r \times D \times \left\{e^{(k_m \times (\tau_{m r} - t_{m r}) - e^{(k_r \times \tau_{m r})}}\right\} \]

Sales revenue per cycle = \[Q_r \times p_r\]  

Sales revenue per unit time = \[\frac{Q_r \times p_r}{T_r} = \frac{Q_r \times p_r \times D}{Q_r} = p_r \times D\]

Net Profit for the retailer per unit time = Sales revenue per unit time – Net cost per unit time

\[= P_{ret}(Q_r, d_r, \tau_{m r}, t_{m r}, d_m) = (p_r \times D) - \left[\frac{A_r \times D}{Q_r} + (c_r \times D) + \frac{s_r \times Q_r^2}{2}\right] + \frac{D \times H_r(Q_r, t_{m r}, \tau_{m r})}{Q_r} + d_r \times D - d_m \times D + c_r \times D \times \left\{e^{(k_m \times (\tau_{m r} - t_{m r}) - e^{(k_r \times \tau_{m r})}}\right\}\]
3.2.3.2 Profit Function for Manufacturer

In this case, manufacturer places an optimal order quantity \( \lambda_m Q_r \) with the supplier and dispatches shipments of \( Q_r \) units to the retailer. It is to be noted that the replenishment is assumed as instantaneous between any two players in this study. That means the manufacturer replenishes his/her inventory instantaneously in every cycle time \( T_m = \frac{\lambda_m Q_r}{D} \). Once the shipment from the supplier reaches the manufacturer, he/she instantaneously delivers the first shipment to the retailer and thereafter his/her every inventory cycle time \( T_r = \frac{Q_r}{D} \). As the retailer gets the delay in payment and price discount from the manufacturer, the manufacturer also gets a price discount \( d_s \) as well as delay in payment \( t_{sm} \) or \( t_{ms} \) from the supplier for the purchase cost. The limit for the maximum discount is the difference between respective player’s purchase cost and selling price and for the permitted interest free delay in payment, the maximum limit is manufacturer’s (buyer’s) inventory cycle time. Owing to the permitted delay in payment provided by the manufacturer to the retailer, manufacturer will lose the opportunity to invest its profit per order over this permitted delay in payment period and this opportunity cost is equal to \( (c_r - c_m) \times \lambda_m \times Q_r \times e^{(b_m x t_{mr})} \), where it is assumed that the manufacturer pays its total purchase cost \( (c_m \times \lambda_m \times Q_r) \) at the beginning of its inventory cycle. As explained earlier, if the retailer avails the delay in payment that exceeds the permitted \( (\tau_{rm} > t_{mr}) \), then the manufacturer will
get the interest for the purchase cost amount to be obtained from the retailer at the rate of \( k_m \), for a period of \( (\tau_{rm} - t_{mr}) \).

Apart from all the above costs, unlike the retailer, the manufacturer incurs an additional holding cost. This is due to the fact that even though the shipment reaches the retailer, his/her financial burden is carried by the manufacturer till he/she gets the payment of purchase cost from the retailer by the time either by \( t_{mr} \) or \( \tau_{rm} \). This additional cost is computed by multiplying the shaded area \( (Q_r \times \tau_{rm}) \) in Figure 3.3 by the manufacturer’s holding cost. Since it occurs \( \lambda_m \) times in every manufacturer’s cycle, the total additional holding cost incurred by the manufacturer in each cycle is \( (h_m \times Q_r \times \tau_{rm} \times \lambda_m) \).

Figure 3.3 depicts the behavior of inventory level of retailer and manufacturer for a typical supply chain system for \( \lambda_m = 4 \)

![Figure 3.3: Illustrates the behavior of inventory levels for the retailer and manufacturer of a supply chain (Modified from Jaber et al, 2006 b)](image-url)
As far as delay in payments is concerned, three cases can happen between manufacturer and supplier as in the case of retailer and manufacturer (discussed in section 3.2.3.1). The manufacturer must settle his/her purchase cost, $c_m \lambda_m Q_r$, with the supplier either by the time $t_{sm}$ or by time $\tau_{ms}$, thus manufacturer’s holding cost per cycle is reduced from $\frac{h_m (\lambda_m \times Q_r)^2}{2D}$ to either $\frac{h_m (\lambda_m \times Q_r - D \times t_{sm})^2}{2D}$ (case 1) or $\frac{h_m (\lambda_m \times Q_r - D \times \tau_{ms})^2}{2D}$ (case 2) or ‘Zero’ (case 3). The holding cost is zero for the case 3 as the payment is made by the manufacturer to the supplier only after the manufacturer’s inventory cycle time.

The manufacturer incurs storage cost per cycle as $\frac{s_m \times (\lambda_m - 1) \times \lambda_m \times Q_r^2}{2D}$.

The delay in payment offered by the supplier provides an opportunity to the manufacturer to invest the unpaid balance $c_m \lambda_m Q_r$, for a period $t_{sm}$ at a return rate of $k_m$. It is advantageous for the manufacturer to extend the delay in payment to the maximum possible ($\tau_{ms}$) if the return rate for the manufacturer exceeds that for the supplier ($k_m > k_s$). As mentioned earlier, continuous compounding rate of return is assumed in this study. So, the term $c_m \lambda_m Q_r \times \left\{ e^{(k_s \times (\tau_{ms} - t_{sm}))} - e^{k_m \times \tau_{ms}} \right\}$ indicates the savings or additional cost from investing an amount of $c_m \lambda_m Q_r$ for a period of $\tau_{ms}$ ($\tau_{ms} > t_{sm}$).
The term \( c_m \times \lambda_m \times Q_r \times \{e^{(k_s \times (\tau_{ms} - t_{sm}))} - e^{k_m \times \tau_{ms}}\} \) can be described as follows. If the manufacturer avails its delay in payment from the supplier to a period \( \tau_{ms} \) that exceeds the interest-free period \( t_{sm} \), then the manufacturer has to pay the interest along with the purchase cost of \( (c_m \lambda_m Q_r) \) at a rate of \( k_s \), for the period \( \tau_{ms} - t_{sm} \). At the same time, the manufacturer can invest this amount of purchase cost at the rate of \( k_m \) for a period of \( \tau_{ms} \) and can earn some additional revenue equal to \( c_m \times \lambda_m \times Q_r \times e^{(k_s \times (\tau_{ms} - t_{sm}))} \) as in the case of retailer.

Profit of manufacturer

\[
\text{Profit of manufacturer} = \text{Sales revenue} - \text{Net cost} \quad 3(16)
\]

Net cost per unit cycle

\[
\psi_m (Q_r, d_r, d_m, d_s, \tau_{ms}, t_{sm}, \lambda_m) = \text{Order cost} + \text{Procurement cost} + \text{Storage cost} + \text{Holding cost} + \text{Interest paid to supplier} + \text{Cost due to loss of opportunity to invest the profit} + \text{Discount to retailer} - (\text{savings from investment} + \text{Interest paid by retailer} + \text{Discount by supplier}) \quad 3(17)
\]

Discount rate by manufacturer to retailer

\[
= d_m
\]

Quantity ordered by the manufacturer

\[
= Q_m = \lambda_m \times Q_r \quad 3(18)
\]

Discount by supplier

\[
= d_m \times Q_m = d_m \times \lambda_m \times Q_r \quad 3(19)
\]

Storage cost

\[
= s_m \sum_{n=1}^{\lambda} (\lambda_m - n)Q_r^2 \quad = \frac{2D}{2D}
\]
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\[
\frac{s_m \times (\lambda_m - 1) \times \lambda_m \times Q_r^2}{2D}
\]

3(20)

Holding cost = \( H_m(Q_m, t_{ms}, \tau_{ms}) \)

\[
= \frac{h_m(\lambda_m \times Q_r - D \times t_{ms})^2}{2D}
\]

(Case1),

\[
\text{Or } \frac{h_m \times ((\lambda_m \times Q_r) - (D \times \tau_{ms}))^2}{2D}
\]

(Case 2), Or ‘Zero’ (Case3) 3(21)

Additional holding cost

\[
= h_m Q_r \lambda_m \tau_{rm}
\]

3(22)

Inventory cycle time for manufacturer

\[
= T_m = \frac{\lambda_m \times Q_r}{D}
\]

3(23)

Procurement cost

\[
= c_m \times \lambda_m \times Q_r
\]

3(24)

Interest paid to supplier

\[
= c_m \times \lambda_m \times Q_r \times e^{(k_r \times (\tau_{ms} - t_{ms}))}
\]

3(25)

Savings from investment

\[
= c_m \times \lambda_m \times Q_r \times e^{(k_m \times \tau_{ms})}
\]

3(26)

Interest paid by retailer

\[
= c_r \times Q_r \times e^{(k_m \times (\tau_{rm} - t_{rm}))}
\]

3(27)

Cost due to loss of opportunity to invest the profit

\[
= (c_r - c_m) \times \lambda_m \times Q_r \times e^{(k_m \times \tau_{rm})}
\]

3(28)

Cost function per unit cycle

\[
= \psi_m (Q_r, d_r, d_s, \tau_{ms}, \tau_{rm}, t_{sm}, t_{mr}, \lambda_m)
\]

\[
= A_m + (c_m \times \lambda_m \times Q_r) + \frac{s_m \times (\lambda_m - 1) \times \lambda_m \times Q_r^2}{2D}
\]
\[ + H_m(Q_r, \lambda_m, t_{sm}, \tau_{ms}) + c_m \times \lambda_m \times Q_r \times e^{(k_x \times (\tau_{ms} - t_{sm}))} \]

\[ + c_m \times \lambda_m \times Q_r \times e^{(k_m \times \tau_{ms})} - c_r \times Q_r \times e^{(k_m \times (\tau_{mr} - t_{mr}))} \]

\[ + (c_r - c_m) \times \lambda_m \times Q_r \times e^{(k_m \times t_{mr})} + h_m Q_r \lambda_m \tau_{rm} \]

\[ - d_s \times \lambda_m \times Q_r + d_m \times \lambda_m \times Q_r \]

3(29)

Cost function per unit time

\[ \psi_m = \frac{A_m \times D}{\lambda_m \times Q_r} + c_m \times D + \frac{s_m \times (\lambda_m - 1) \times Q_r}{2} \]

\[ + \frac{D \times H_m(Q_r, \lambda_m, t_{sm}, \tau_{ms})}{\lambda_m \times Q_r} + c_m \times D \times e^{(k_x \times (\tau_{ms} - t_{sm}))} \]

\[ - c_m \times D \times e^{(k_m \times \tau_{ms})} - d_s \times D + d_m \times D + h_m \times \tau_{rm} \times D \]

\[ - \frac{c_r \times D \times e^{(k_m \times (\tau_{mr} - t_{mr}))}}{\lambda_m} + (c_r - c_m) \times D \times e^{(k_m \times t_{mr})} \]

3(30)

Sales revenue per unit cycle = \[ Q_r \times \lambda_m \times p_m \]

3(31)

Sales revenue per unit time = \[ \frac{Q_r \times \lambda_m \times p_m}{T_m} = p_m \times D \]

3(32)

Profit of manufacturer per unit time = \[ P_{mfr}(Q_r, d_m, d_s, \lambda_m, \tau_{ms}, t_{sm}) \]

\[ = \text{Sales revenue per unit time} - \text{Net cost per unit time} \]
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\[ p_m \times D - \left( \frac{A_m \times D}{\lambda_m \times Q_r} + c_m \times D + \frac{s_m \times (\lambda_m - 1) \times Q_r}{2} \right) \]

\[ + \frac{D \times H_m (Q_r, \lambda_m, t_{sm}, \tau_{ms})}{\lambda_m \times Q_r} + c_m \times D \times e^{(k_r \times (\tau_{ms} - t_{sm}))} \]

\[-c_m \times D \times e^{(k_m \times \tau_{ms})} - d_s \times D + d_m \times D + h_m \times \tau_{rm} \times D \]

\[ - \frac{c_r \times D \times e^{(k_m \times (\tau_{rm} - t_{mr})})}{\lambda_m} + (c_r - c_m) \times D \times e^{(k_m \times t_{mr})} \]

\[ 3(33) \]

3.2.3.3 Profit Function for Supplier

In this case, supplier places an optimal order quantity \((\lambda_m \lambda_s Q_r)\) on its upstream player (which is not a part of this study as such) and dispatches shipments of \(Q_m\) units to the manufacturer. As the replenishment is instantaneous in this study, the supplier replenishes his/her inventory instantaneously in every cycle time \((T_s = \frac{\lambda_m \lambda_s Q_r}{D})\). Once the shipment reaches the supplier, he/she instantaneously delivers the first shipment to the manufacturer and thereafter his/her every inventory cycle time \((T_m = \frac{Q_m}{D})\).

Owing to the permitted delay in payment \((t_{sm})\) provided by the supplier to the manufacturer, the supplier will lose the opportunity to invest his/her profit per order over this permitted delay in payment period and this opportunity cost is equal to \((c_m - c_s) \times \lambda_m \times \lambda_s \times Q_r \times e^{(k \times t_{sm})}\), where it is assumed that the supplier pays his/her total purchase cost \((c_m \times \lambda_m \times \lambda_s \times Q_r)\) at the
Chapter 3

beginning of its inventory cycle. As explained earlier, if the manufacturer avails the delay in payment that exceeds the permitted \((\tau_{ms} > t_{sm})\), then the supplier will get the interest for the purchase cost amount to be obtained from the manufacturer at the rate of \(k_s\), for a period of \((\tau_{ms} - t_{sm})\). As mentioned earlier, the supplier also provides a unit discount of \(d_s\) to the manufacturer but he/she does not get any discount from his/her upstream player in this study.

As discussed in the previous section of manufacturer’s profit function, the supplier also incurs additional holding cost in this study due to the reason that the supplier provides the delay in payment even after the items reaching the manufacturer and carries the financial burden till manufacturer makes the payment. Since this cost is incurred \(\lambda_s\) times in a supplier’s cycle, then the total additional holding cost incurred by the supplier in each cycle is \(h_s Q_r \lambda_m \lambda_s \tau_{ms}\). This holding cost is different from the normal holding cost calculated earlier.

Profit of supplier = Sales revenue – Net cost

Net cost of supplier per unit cycle = Order cost + Procurement cost + Storage cost + Holding cost + Cost due to loss of opportunity to invest the profit + Discount to manufacturer- Interest paid by manufacturer

\[\text{Cost function per unit cycle } = \psi_s (Q_r, d_r, d_m, d_s, t_{sm}, \tau_{ms}, \lambda_m, \lambda_s)\]

Discount rate by the supplier to the manufacturer = \(d_s\)

Quantity ordered by the supplier = \(\lambda_m \times \lambda_s \times Q_r\)

Discount to manufacturer = \(d_s \times Q_s = d_s \times \lambda_m \times \lambda_s \times Q_r\)
Procurement cost = $c_s \times \lambda_m \times \lambda_s \times Q_r$.  

\[ (s_s + h_s) \sum_{n=1}^{\lambda_s} (\lambda_s - n) Q_r^2 \lambda_m^2 \]

Storage cost and holding cost = \[
\frac{(s_s + h_s) \times (\lambda_m \times Q_r)^2 \times \lambda_s \times (\lambda_s - 1)}{2D}
\]

Additional holding cost = $h_s \times \lambda_m \times \lambda_s \times Q_r \times \tau_{ms}$  

Inventory time of cycle for the supplier = $T_s = \frac{\lambda_m \times \lambda_s \times Q_r}{D}$

Interest paid by manufacturer = $c_m \times \lambda_m \times Q_r \times e^{(k_s \times \tau_{sm})}$

Investment due to delay in payments = \[
(c_m - c_s) \times \lambda_m \times \lambda_s \times Q_r \times e^{(k_s \times \tau_{sm})}
\]

Net Cost per unit cycle = $\psi_s(Q_r, d_r, d_m, d_s, t_{sm}, \tau_{ms}, \lambda_m, \lambda_s)$

\[
= A_s + (c_s \times \lambda_m \times \lambda_s \times Q_r) + \frac{s_s \times (\lambda_m \times Q_r)^2 \times \lambda_s \times (\lambda_s - 1)}{2D}
\]

\[
+ \frac{h_s \times (\lambda_m \times Q_r)^2 \times \lambda_s \times (\lambda_s - 1)}{2D} + h_s \times \lambda_m \times \lambda_s \times Q_r \times \tau_{ms}
\]

\[
+ d_s \times \lambda_m \times \lambda_s \times Q_r - c_m \times \lambda_m \times Q_r \times e^{(k_s \times \tau_{sm})}
\]

\[
+(c_m - c_s) \times \lambda_m \times \lambda_s \times Q_r \times e^{(k_s \times \tau_{sm})}
\]
Net cost per unit time = \( \frac{\psi_s}{T_s} \)

\[
= \frac{A_s \times D}{\lambda_m \times Q_r \times \lambda_s} + (c_s \times D) + \frac{s_s \times Q_r \times \lambda_m \times (\lambda_s - 1)}{2} \\
+ \frac{h_s \times Q_r \times \lambda_m \times (\lambda_s - 1)}{2} + h_s \times \tau_{ms} \times D \\
- \frac{c_m \times D \times e^{(k_s \times (\tau_{ms} - t_{sm}))}}{\lambda_s} + d_s \times D \\
+(c_m - c_s) \times D \times e^{(k_s \times t_{sm})}
\]  

Sales revenue per unit cycle = \( \lambda_m \times \lambda_s \times Q_r \times p_s \)  

Sales revenue per unit time = \( \frac{\lambda_m \times \lambda_s \times Q_r \times p_s}{T_m} = p_s \times D \)  

Profit of supplier per unit time = \( P_{sup} (Q_r, d_r, d_m, d_s, \tau_{ms}, t_{sm}, \lambda_m, \lambda_s) \)

\[
= \text{Sales revenue per unit time - Net cost per unit time} \\
= p_s \times D - \left[ \frac{A_s \times D}{\lambda_m \times Q_r \times \lambda_s} + (c_s \times D) + \frac{s_s \times Q_r \times \lambda_m \times (\lambda_s - 1)}{2} \\
+ \frac{h_s \times Q_r \times \lambda_m \times (\lambda_s - 1)}{2} + h_s \times \tau_{ms} \times D \\
- \frac{c_m \times D \times e^{(k_s \times (\tau_{ms} - t_{sm}))}}{\lambda_s} + d_s \times D +(c_m - c_s) \times D \times e^{(k_s \times t_{sm})} \right]
\]
3.2.3.4 Total Supply Chain Profit Function

The total supply chain profit function \( P_{sc} \) with coordination (price discount and delay in payment simultaneously as mechanisms) can be taken as the sum of the profit functions of the retailer [equation 3(15)], manufacturer [equation 3(33)] and supplier [equation 3(47)] under the sections 3.2.3.1, 3.2.3.2 & 3.2.3.3 respectively. With coordination, the retailer, manufacturer and supplier need to agree on the following decision variables \( Q_r, \lambda_m, \lambda_s, \tau_{ms}, \tau_{rm}, t_{mr}, d_r, d_m, d_s \) that maximizes the total SC profit. The decision variables and other parameters are defined as follows:

- \( Q_r \) = Order quantity of retailer
- \( \lambda_m, \lambda_s \) = An integer lot size multiplier of manufacturer, supplier respectively
- \( \tau_{ms} \) = Delay in payment availed by manufacturer from supplier
- \( \tau_{rm} \) = Delay in payment availed by retailer from manufacturer
- \( t_{mr} \) = Interest free delay in payment permitted by manufacturer to the retailer
- \( t_{sm} \) = Interest free delay in payment permitted by supplier to the manufacturer
- \( d_r, d_m, d_s \) = Discount given by the retailer, manufacturer, supplier respectively
- \( p_r, p_m, p_s \) = Selling price of retailer, manufacturer, supplier respectively
- \( c_r, c_m, c_s \) = Procurement cost of retailer, manufacturer, supplier respectively

The mathematical model could then be written as follows:

Maximize supply chain profit \( P_{sc} = P_{ret} + P_{mfr} + P_{sup} \)

\[ \text{Subject to the constraints:} \]

\[ \lambda_m, \lambda_s \geq 1 \quad 3(48a), \quad Q_r \geq 1 \quad 3(48b), \quad \tau_{rm}, t_{mr}, \tau_{ms}, t_{sm} \geq 0 \quad 3(48c) \]

Maximum discount permitted:

\[ d_s \leq p_s - c_s, d_m \leq p_m - (p_s - d_s), d_r \leq p_r - (p_m - d_m), \quad 3(48d) \]
Maximum delay in payment:

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_r / D - t_{mr} \geq 0 ) (3(48e))</td>
<td>( Q_r / D - \tau_{rm} \geq 0 ) (3(48g))</td>
<td>( \tau_{rm} - Q_r / D \geq 0 ) (3(48i))</td>
</tr>
<tr>
<td>( \lambda_m Q_r / D - t_{sm} \geq 0 ) (3(48f))</td>
<td>( Q_r / D - t_{mr} \geq 0 ) (3(48h))</td>
<td>( t_{mr} - Q_r / D \leq 0 ) (3(48m))</td>
</tr>
<tr>
<td>( \lambda_m Q_r / D - \tau_{ms} \geq 0 ) (3(48i))</td>
<td>( \lambda_m Q_r / D - t_{sm} \geq 0 ) (3(48j))</td>
<td>( \tau_{ms} - \lambda_m Q_r / D \geq 0 ) (3(48n))</td>
</tr>
<tr>
<td>( \tau_{rm} \geq t_{mr} ), ( \tau_{ms} \geq t_{sm} ) (3(48k))</td>
<td></td>
<td>( \tau_{rm} \geq t_{mr} ), ( \tau_{ms} \geq t_{sm} ) (3(48p))</td>
</tr>
</tbody>
</table>

Equation 3(48) represents the SC profit maximization function. Constraint 3(48a) indicates that the integer lot sizing multiplier of manufacturer and supplier is greater than or equal to one and constraint 3(48b) indicates that the order quantity of retailer is greater than or equal to one. 3(48c) indicates that the delay in payment availed by the retailer from the manufacturer (\( \tau_{rm} \)), the delay in payment permitted by the manufacturer to the retailer (\( t_{mr} \)), the delay in payment availed by manufacturer from the supplier (\( \tau_{ms} \)) and the delay in payment permitted by the supplier to the manufacturer (\( t_{sm} \)) will be greater than or equal to zero. Constraint 3(48d) indicates that the discount given by each player must be less than or equal to the difference between his/her selling price and discounted unit purchase cost. Constraint 3(48e) indicates the delay in payment permitted by the manufacturer (\( t_{mr} \)) to the retailer is less than or equal to the retailer’s inventory cycle time (\( Q_r / D \)). Constraint 3(48f) indicates that the delay in payment permitted by the supplier (\( t_{sm} \)) to the manufacturer is less than or equal to the manufacturer’s inventory cycle time (\( \lambda_m Q_r / D \)). Constraint
Performance of Three Level Supply Chain While Using Price Discount and Delay in……

Constraint 3(48g) indicates that the delay in payment availed by the retailer from the manufacturer \((\tau_{rm})\) is less than the retailer’s inventory cycle time \((\frac{Q_r}{D})\).

Constraint 3(48h) indicates that the delay in payment permitted by the manufacturer to the retailer \((t_{mr})\) is less than or equal to the retailer’s inventory cycle time \((\frac{Q_r}{D})\). Constraint 3(48i) indicates that the delay in payment availed by the manufacturer from the supplier \((\tau_{ms})\) is less than or equal to the manufacturer’s inventory cycle time \((\frac{\lambda_m Q_r}{D})\). Constraint 3(48j) indicates that the delay in payment permitted by the supplier to the manufacturer \((t_{sm})\) is less than or equal to the manufacturer’s inventory cycle time \((\frac{\lambda_m Q_r}{D})\). Constraint 3(48k) indicates that the delay in payment availed by the retailer from the manufacturer \((\tau_{rm})\) is greater than or equal to delay in payment permitted by the manufacturer to the retailer \((t_{mr})\) and the delay in payment availed by the manufacturer from the supplier \((\tau_{ms})\) is greater than or equal to the delay in payment permitted by the supplier to the manufacturer \((t_{sm})\). Constraint 3(48l) indicates that the delay in payment availed by the retailer from the manufacturer \((\tau_{rm})\) is greater than or equal to retailer’s inventory cycle time \((\frac{Q_r}{D})\). Constraint 3(48m) indicates that the delay in payment permitted by the manufacturer \((t_{mr})\) to the retailer is less than or equal to the retailer’s inventory cycle time \((\frac{Q_r}{D})\). Constraint 3(48n) indicates that the delay in payment availed by the manufacturer from the supplier \((\tau_{ms})\) is greater than or
equal to manufacturer’s inventory cycle time \( \left( \frac{\lambda_m Q_r}{D} \right) \). Constraint 3(48o) indicates that the delay in payment permitted by the supplier to the manufacturer \( t_{sm} \) is less than or equal to the manufacturer’s inventory cycle time \( \left( \frac{\lambda_m Q_r}{D} \right) \). Constraint 3(48p) indicates that the delay in payment availed by the retailer from the manufacturer \( \tau_{rm} \) is greater than or equal to delay in payment permitted by the manufacturer to the retailer \( t_{mr} \) and the delay in payment availed by the manufacturer from the supplier \( \tau_{ms} \) is greater than or equal to the delay in payment permitted by the supplier to the manufacturer \( t_{sm} \).

As mentioned in the beginning, the price discounts and the delay in payments are effected in transactions between supplier & manufacturer and manufacturer & retailer. The retailer permits only price discount to the customer but no delay in payments. Under these circumstances, three cases are possible for delay in payments (case 1, 2, & 3 in the set of constraints given above) as explained below. Each case is considered separately for solving the above mathematical model and the results are compared and analyzed.

Case 1 - The seller permits interest free permissible delay in payments to the buyer up to a maximum period of buyer’s inventory cycle time and no extension of delay in payment is allowed after the permitted period.

Case 2 - The buyer can extend the delay in payment period even over the permitted interest free delay in payment period but maximum up to his/her inventory cycle time. That means, interest free delay in payment period will be less than or equal to delay in payment period with interest and both can be maximum up to buyer’s inventory cycle time.
Case 3 - Delay in payments period can be extended even over the buyer’s inventory cycle time. But, the maximum interest free delay in payment period is up to the buyer’s inventory cycle time only as in above two cases.

3.3 Numerical Results

The nonlinear mathematical model developed for the three level supply chain coordination with case 1, case 2 and case 3 are solved using ‘Excel solver’. This study shows that the profit is more for the case 3 and the same is used for the comparison with results obtained when using price discounts only as mechanism used by Jaber et al. (2006).

The table 3.1 consists of data adopted from Munson and Rosenblatt (2001) is used for analysis. It is assumed that a fixed annual demand of 150000 units and it increases with increase in discount offered by the retailer and the elasticity of demand. This means that the chain is driven by the retailer’s annual sales volume. Thus, the actual demand \( D \) is equal to \( D_0 + D_1 \times d_r \) where \( D_1 = 1000 \ldots 5000 \ldots 10000 \ldots 15000 \), etc. The return on investment (ROI) is taken as 15% per annum (normal state of the business) in all cases except in the sensitivity analysis of this SC system under different market conditions/ROI of each player. Unit time in this study is taken as a year.

### Table 3.1: Munson and Rosenblatt’s (2001) data set used for this study

<table>
<thead>
<tr>
<th>Player</th>
<th>Set up cost (Rs)</th>
<th>Purchase cost (Rs)</th>
<th>Holding cost (Rs)</th>
<th>Storage Cost* (Rs)</th>
<th>Profit Margins (%)</th>
<th>Selling Price (Rs)</th>
<th>Return on Investment* (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier</td>
<td>400</td>
<td>200</td>
<td>10</td>
<td>2</td>
<td>25</td>
<td>250.00</td>
<td>15</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>200</td>
<td>250</td>
<td>12</td>
<td>3</td>
<td>25</td>
<td>312.50</td>
<td>15</td>
</tr>
<tr>
<td>Retailer</td>
<td>30</td>
<td>312.50</td>
<td>16</td>
<td>4</td>
<td>25</td>
<td>390.63</td>
<td>15</td>
</tr>
</tbody>
</table>

*added in this study
Table 3.2 shows the supply chain performance for the three cases of delay in payment along with price discount. This analysis shows that case 2 provides slightly more profit than case 1 and case 3 provides significantly higher profit than case 1 & 2. The reasons for this phenomenon are as follows. When the delay in payment is implemented along with price discount, the net inventory carrying cost decreases due to both the reduction in holding cost and the increase in end customer demand as the demand is price elastic. It further reduces in the third case in which the buyer can avail delay in payment more than its inventory cycle time (interest to be paid by the buyer to the seller for exceeding period than the permitted interest free period). Apart from this when the buyer gets more time than its inventory cycle time for the investment, the amount that is available for investment will be relatively more and the net savings for the third case will be also more than the first two cases. The optimal value of interest free delay in payment permitted by the supplier to the manufacturer is obtained as zero as supplier does not get any delay in payment from its upstream player in this study. In the first case, the delay in payment is not permitted more than buyer’s inventory cycle time. So, the net savings from its investment is very less compared to other two cases. In the second and third case, it is obtained that the optimal values of the (0.009) delay in payment permitted (without interest) is equal to the delay in payment (0.009) availed.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda_m$</th>
<th>$\lambda_s$</th>
<th>$t_{mr}$</th>
<th>$t_{rm}$</th>
<th>$t_{sm}$</th>
<th>$t_{ms}$</th>
<th>$d_r$</th>
<th>$d_m$</th>
<th>$d_s$</th>
<th>Supply chain profit (Rs.)</th>
<th>Increase in profit (%) in case 2 &amp; 3 (with base as case 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.009</td>
<td>-</td>
<td>0.028</td>
<td>-</td>
<td>21.28</td>
<td>10.0</td>
<td>3.69</td>
<td>29107526</td>
<td>0.009</td>
<td>0</td>
<td>0.009</td>
</tr>
<tr>
<td>2</td>
<td>0.009</td>
<td>0</td>
<td>0</td>
<td>0.028</td>
<td>21.28</td>
<td>10.0</td>
<td>3.69</td>
<td>29143672</td>
<td>0.009</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>0.009</td>
<td>0</td>
<td>0</td>
<td>0.050</td>
<td>20.61</td>
<td>10.0</td>
<td>0</td>
<td>29250439</td>
<td>0.009</td>
<td>0.51</td>
<td>0.51</td>
</tr>
</tbody>
</table>
(with interest) between retailer and manufacturer as both the parties have equal opportunities for investment with same rate of return. The optimal value of price discount is obtained as minimum (zero) for supplier and maximum (20.61) for retailer in the third case (maximum SC profit case). The reasons for these observations are supplier is not getting any discount from its upstream player and the discount given by the retailer decides the end customer demand respectively. So, the third case of delay in payment can be implemented to have maximum improvement in SC profit among these three cases considering all practical aspects.

**Table 3.3:** Profit comparison when using price discounts alone and along with ‘delay in payments’ Vs ‘no coordination’ (\(D_1 =1000, \lambda_m = 3, \lambda_s = 1, \ k_r = 15\%\),\( k_m = 15\%, \ k_s = 15\%\))

<table>
<thead>
<tr>
<th>SC Profit under no coordination (a)</th>
<th>SC profit under discount alone (b)</th>
<th>Increase in profit (%) under discount alone compared to no coordination ((b-a)/a)x100</th>
<th>Profit under combination of mechanisms (Price discounts with delay in payments) (Case 3) (c)</th>
<th>Increase in profit (%) under combination of mechanisms (case3) compared to ‘no coordination’ ((c-a)/a)x100</th>
</tr>
</thead>
<tbody>
<tr>
<td>28538426</td>
<td>28950375</td>
<td>1.44</td>
<td>29250439</td>
<td>2.49</td>
</tr>
</tbody>
</table>

Table 3.3 shows the profit comparison while using ‘price discount alone and along with delay in payment’ versus ‘no coordination’. The analysis shows the use of price discount along with delay in payment (case3) enhances the SC profit significantly and this hike in profit from price discount alone is approximately equal to the hike in profit between no coordination and price discount alone. So, this study suggests incorporating price discount along with delay in payment as strategic coordination mechanisms improve the SC performance than the case of price discount alone. So, this analysis also
relevant as it is useful to take appropriate decision on the implementation of coordination mechanism(s) considering the effort (cost aspects) required for the same. The increase in profit in the case of delay in payment is due to the decrease in inventory carrying cost and net saving from the investment due to the delay in payment. The increase in profit in the case of price discount is due to the increase in end customer demand as it is price elastic. The combined effect is reflected in the case of combination of both price discount and delay in payment.

3.4 Sensitivity Analysis

The effect of change in various parameters on SC performance is also analyzed to understand the sensitivity of this supply chain system and it will enable the decision maker to take proper decisions accordingly. The parameters considered for the sensitivity analysis are elasticity of demand, return on investment and order cost/set up cost. In any business, boom and recession are a common phenomenon depending on various external and internal factors. That means, the players (supplier, manufacturer or retailer) can be either in recession, normal or boom state of market. So, the return on investment can vary in the case of any of these players in an SC system based on the existing market conditions. The following table shows such an analysis how difference in return on investment or the market conditions affects the profit of individual players and overall SC profit.
Table 3.4: Analysis of supply chain profit (case 3 - maximum profit case) for the different market conditions/values of Return on investment (ROI) of various players

\[ D_1 = 1000, \lambda_m = 3, \lambda_s = 1, A_r = 30, A_m = 200, A_r = 400 \]

<table>
<thead>
<tr>
<th>Case</th>
<th>Return on investment</th>
<th>(\lambda_m)</th>
<th>(\lambda_s)</th>
<th>Retailer profit (Rs)</th>
<th>Manufacturer profit (Rs)</th>
<th>Supplier profit (Rs)</th>
<th>Supply chain profit (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(k_r = 10) (k_m = 15) (k_s = 20)</td>
<td>3</td>
<td>1</td>
<td>11562235</td>
<td>8811524</td>
<td>8858144</td>
<td>29231904</td>
</tr>
<tr>
<td>II</td>
<td>(k_r = 10) (k_m = 20) (k_s = 15)</td>
<td>3</td>
<td>1</td>
<td>111562235</td>
<td>9043522</td>
<td>8798461</td>
<td>29040219</td>
</tr>
<tr>
<td>III</td>
<td>(k_r = 20) (k_m = 15) (k_s = 10)</td>
<td>3</td>
<td>1</td>
<td>11632264</td>
<td>9020682</td>
<td>8643984</td>
<td>29296930</td>
</tr>
<tr>
<td>IV</td>
<td>(k_r = 20) (k_m = 10) (k_s = 15)</td>
<td>3</td>
<td>1</td>
<td>111708837</td>
<td>8820650</td>
<td>8704013</td>
<td>29235000</td>
</tr>
<tr>
<td>V</td>
<td>(k_r = 15) (k_m = 10) (k_s = 20)</td>
<td>3</td>
<td>1</td>
<td>11601901</td>
<td>8745856</td>
<td>8789760</td>
<td>29137517</td>
</tr>
<tr>
<td>VI</td>
<td>(k_r = 15) (k_m = 20) (k_s = 10)</td>
<td>3</td>
<td>1</td>
<td>11586459</td>
<td>9172128</td>
<td>8669856</td>
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</tr>
<tr>
<td>VII</td>
<td>(k_r = 10) (k_m = 10) (k_s = 10)</td>
<td>3</td>
<td>1</td>
<td>11562236</td>
<td>8922942</td>
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<td>29103613</td>
</tr>
<tr>
<td>VII</td>
<td>(k_r = 15) (k_m = 15) (k_s = 15)</td>
<td>3</td>
<td>1</td>
<td>11587056</td>
<td>8912452</td>
<td>8750931</td>
<td>29250439</td>
</tr>
<tr>
<td>IX</td>
<td>(k_r = 20) (k_m = 20) (k_s = 20)</td>
<td>3</td>
<td>1</td>
<td>11610695</td>
<td>8914532</td>
<td>8927451</td>
<td>29452678</td>
</tr>
</tbody>
</table>

10% ROI – Recession state, 15% ROI – Normal state, 20% ROI – Boom state

Table 3.4 shows the performance of players under different market conditions/values of return on investment. The above analysis reveals that the state of the manufacturing sector (cases II & VI – boom and cases IV & V – recession) affects more on the total SC profit. This is because the manufacturer is getting the chance to invest the amount to be paid to the supplier and at the same time to receive the interest from the retailer for delaying the payment. But, the retailer has no opportunity to get the interest due to delay in payment from its downstream player even though he has the opportunity to invest the amount to be paid to the manufacturer. Similarly, supplier has the opportunity to receive the interest from the manufacturer but no opportunity to delay the payment to its upstream player and to make the investment. The rate of increase in the SC profit as the whole market condition improves (cases VII, VIII & IX) is almost constant.
Chapter 3

The order cost/set up cost is normally highest for manufacturer or supplier and lowest for other players in an SC system. The Table 3.5 shows the analysis of how the variation in order cost/set up cost of various players affects the SC profit. The other parameters such as Elasticity of demand, ROI, Lot size multiplier remains same. It shows that as retailer’s order cost increases, the overall SC profit also increases. This study also reveals that mutual change in the order cost between supplier and manufacturer does not affect the overall SC performance provided the retailer’s order cost is the same. This phenomenon can be seen in the cases of I &II, III & IV and V & VI. The order/set up costs given for players in each case of VII, VIII & IX are equal but are in an increasing order from VII to IX case. The SC profit is also found to be increasing in the same order compared to the case 1. All these findings show that the order cost of the retailer plays a major role in the performance of the SC. This is due to the fact that when the retailer order cost is high, the retailer’s order quantity increases and as a result the retailer’s cycle time increases. Consequently, the optimal delay in payment of all players changes in such a way that the net savings from investment increases in the case of retailer and supplier. When the order cost of manufacturer or supplier increases (Case V & IX), it affects the SC slightly adverse. The reason is that the order quantity of manufacturer or supplier does not affect the order quantity and their per unit order cost increases. Similar to the above results, the optimal values of delay in payment and price discount changes only when retailer order cost changes. The effect is that the optimal values of delay in payment increases as the order cost of retailer increases and the optimal value of discount given by the retailer increases as the order cost of the retailer increases. But it does not affect the discount given by the manufacturer and the optimal value of discount given by the supplier remains same as zero. The maximum increase in profit with respect to the base case is 2.52% when the retailer order cost is maximum (case VI) and the minimum increase in profit with respect to the base case when the retailer order cost is 0.06% (case VII). The above results and these kind of related insights are also useful for taking appropriate decisions to suit the situations.
Table 3.5: Performance of the players and supply chain profit for different order/set up cost

<table>
<thead>
<tr>
<th>Case</th>
<th>order/set up cost</th>
<th>Delay in payment</th>
<th>Discount given by each player</th>
<th>Retailer profit (Rs)</th>
<th>Manufacturer profit (Rs)</th>
<th>Supplier profit (Rs)</th>
<th>Supply chain profit (Rs.)</th>
<th>Increase in SC profit (With base as case I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>30 200 400</td>
<td>0.000 0.009 0.050 0.009</td>
<td>20.61 10.00 0.00</td>
<td>11587056</td>
<td>8912452</td>
<td>8750931</td>
<td>29250439</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>30 400 200</td>
<td>0.000 0.009 0.050 0.009</td>
<td>20.61 10.00 0.00</td>
<td>11587056</td>
<td>8905342</td>
<td>8758041</td>
<td>29250439</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>200 400 30</td>
<td>0.000 0.024 0.129 0.024</td>
<td>21.11 10.00 0.00</td>
<td>11639946</td>
<td>8802569</td>
<td>9171962</td>
<td>29694477</td>
<td>1.52</td>
</tr>
<tr>
<td>IV</td>
<td>200 30 400</td>
<td>0.000 0.024 0.129 0.024</td>
<td>21.11 10.00 0.00</td>
<td>11639946</td>
<td>8887671</td>
<td>9166860</td>
<td>29694477</td>
<td>1.52</td>
</tr>
<tr>
<td>V</td>
<td>400 200 30</td>
<td>0.000 0.034 0.184 0.034</td>
<td>21.45 10.00 0.00</td>
<td>11676371</td>
<td>8864368</td>
<td>9454440</td>
<td>29995179</td>
<td>2.54</td>
</tr>
<tr>
<td>VI</td>
<td>400 30 200</td>
<td>0.000 0.034 0.184 0.034</td>
<td>21.45 10.00 0.00</td>
<td>11676371</td>
<td>8866027</td>
<td>9452781</td>
<td>29995179</td>
<td>2.54</td>
</tr>
<tr>
<td>VII</td>
<td>30 30 30</td>
<td>0.000 0.009 0.050 0.009</td>
<td>20.61 10.00 0.00</td>
<td>11499830</td>
<td>8962616</td>
<td>8806642</td>
<td>29269088</td>
<td>0.06</td>
</tr>
<tr>
<td>VIII</td>
<td>200 200 200</td>
<td>0.000 0.024 0.129 0.024</td>
<td>21.11 10.00 0.00</td>
<td>11665083</td>
<td>8902844</td>
<td>9184979</td>
<td>29694906</td>
<td>1.52</td>
</tr>
<tr>
<td>IX</td>
<td>400 400 400</td>
<td>0.000 0.034 0.184 0.034</td>
<td>21.45 10.00 0.00</td>
<td>11677210</td>
<td>8861996</td>
<td>9450402</td>
<td>29989608</td>
<td>2.52</td>
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</tbody>
</table>
As part of the sensitivity analysis, the SC profit while using price discounts along with delay in payments for different price elasticity of demand is also found out and compared with the SC profit under price discounts alone.

**Table 3.6: Analysis of supply chain profit under no coordination with price discounts alone and price discounts in conjunction with delay in payments for various cases of elasticity**

<table>
<thead>
<tr>
<th>SI No</th>
<th>Elasticity of demand</th>
<th>Supply chain profit under price discounts alone as mechanism Jaber.M.Y. et.al (2006) (a)</th>
<th>Supply chain profit under Price discounts along with delay in payments as mechanisms (b)</th>
<th>Percentage of Increase in profit (%) ((b-a)/a)x100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>28950375</td>
<td>29250439</td>
<td>1.036</td>
</tr>
<tr>
<td>2</td>
<td>5000</td>
<td>60743508</td>
<td>61293115</td>
<td>0.905</td>
</tr>
<tr>
<td>3</td>
<td>10000</td>
<td>105566572</td>
<td>106317166</td>
<td>0.711</td>
</tr>
<tr>
<td>4</td>
<td>15000</td>
<td>150698412</td>
<td>151659658</td>
<td>0.638</td>
</tr>
<tr>
<td>5</td>
<td>20000</td>
<td>195992817</td>
<td>197120813</td>
<td>0.576</td>
</tr>
<tr>
<td>6</td>
<td>25000</td>
<td>241328917</td>
<td>242570598</td>
<td>0.515</td>
</tr>
<tr>
<td>7</td>
<td>30000</td>
<td>286686783</td>
<td>288012060</td>
<td>0.462</td>
</tr>
<tr>
<td>8</td>
<td>35000</td>
<td>332057668</td>
<td>333536940</td>
<td>0.445</td>
</tr>
<tr>
<td>9</td>
<td>40000</td>
<td>377437082</td>
<td>379013058</td>
<td>0.418</td>
</tr>
<tr>
<td>10</td>
<td>45000</td>
<td>422822466</td>
<td>424484781</td>
<td>0.393</td>
</tr>
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<td>11</td>
<td>50000</td>
<td>468212240</td>
<td>469953006</td>
<td>0.372</td>
</tr>
</tbody>
</table>

The Table 3.6 shows that as elasticity increases, the profit increases but the rate profit increase, decreases. This indicates that if the price elasticity of demand is very high, there will not be much benefit by implementing these coordination mechanisms simultaneously to enhance the profit.

**3.5 Conclusion**

The performance of a three – level supply chain with price discount and delay in payment jointly using mathematical modelling is analysed in this study. From this, it can be concluded that the implementation of the two coordination mechanisms: ‘price discount’ along with ‘delay in payment’
Improves SC profit significantly compared to price discount alone (about 1.5% in the case studied). It is also found that the SC profit reaches the maximum in case 3 where the delay in payment period taken from the seller exceeds the buyer's inventory cycle time. So, the 'delay in payment' can be provided more than the buyer’s inventory cycle time considering the financial status and past performance of the buyer. Sensitivity analysis reveals the effect of variation of order cost of different players on SC performance provided other’s order cost remains same. The overall supply chain profit is seen to increase with the increase in the order cost of the retailer. This indicates that the retailer has a major role in this supply chain system. Sensitivity analysis of return on investment (ROI) shows that the SC profit is most sensitive to the manufacturer’s ROI. Rate of increase in profit in the case of a combination of coordination mechanisms studied compared to the case of ‘price discount’ alone was found to be decreasing with increase in price elasticity of demand. SC managers must understand that using both price discounts and delay in payment produces nearly the same increase in SC profit, over use of only price discount, as that between no coordination and use of price discounts. The SC profit is more sensitive to retailer order cost and manufacturer ROI. The overall analysis indicated that combination of mechanism can be implemented to enhance the profit further after conducting a detailed analysis and considering the related practical aspects.