Chapter 8

On Special $g_{s\Lambda}$-functions

Introduction

In chapter 8, special functions such as perfectly $g_{s\Lambda}$-continuous function and strongly $g_{s\Lambda}$-continuous function are identified and few of their properties are discussed. It is been proved that perfectly $g_{s\Lambda}$-continuous function is a weaker form of continuous function but stronger form of $g_{s\Lambda}$-continuous function and strongly $g_{s\Lambda}$-continuous function is stronger form of continuous function and $g_{s\Lambda}$-continuous function. Also it is proved that strongly $g_{s\Lambda}$-continuous function is stronger form of perfectly $g_{s\Lambda}$-continuous function.

8.1 Perfectly $g_{s\Lambda}$-continuous function

Definition 8.1.1 A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is called perfectly $g_{s\Lambda}$-continuous function if the inverse image of each open set in $Y$ is $g_{s\Lambda}$-clopen in $X$.

Theorem 8.1.2 Every continuous function is perfectly $g_{s\Lambda}$-continuous.

Proof: Let a function $f:(X,\tau)\rightarrow(Y,\sigma)$ be continuous and $U$ be a open set in $(Y,\sigma)$. Then $f^{-1}(U)$ is open in $(X,\tau)$. Since $f^{-1}(U)$ is both $g_{s\Lambda}$-open and $g_{s\Lambda}$-closed [by Theorems 2.3.4 and 2.3.5] in $(X,\tau)$, $f$ is perfectly $g_{s\Lambda}$-continuous.
Remark 52 Converse of the above Theorem 8.1.2 need not be true as seen from the following example.

Example 60 Let \( X = Y = \{a, b, c, d, e\} \) and \( \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, e\}, \{a, b, d, e\}, \{a, b, c, e\}\} \) and \( \sigma = \{\emptyset, Y, \{a\}, \{a, b\}, \{c, d, e\}, \{a, c, d, e\}\} \). The identity function \( f: (X, \tau) \longrightarrow (Y, \sigma) \) is perfectly \( gs\Lambda \)-continuous, but not continuous. Since \( A = \{c, d, e\} \) is open in \( (Y, \sigma) \) but \( f^{-1}(A) = \{c, d, e\} \) is not open in \( (X, \tau) \).

Theorem 8.1.3 Every contra continuous function is perfectly \( gs\Lambda \)-continuous.

Proof: Let a function \( f: (X, \tau) \longrightarrow (Y, \sigma) \) be contra continuous and \( U \) be an open set in \( (Y, \sigma) \). Then \( f^{-1}(U) \) is closed in \( (X, \tau) \). Since \( f^{-1}(U) \) is both \( gs\Lambda \)-open and \( gs\Lambda \)-closed by Theorems 2.1.5 and 2.1.7 \( (X, \tau) \), \( f \) is perfectly \( gs\Lambda \)-continuous.

Remark 53 Converse of the above Theorem 8.1.3 need not be true as seen from the following example.

Example 61 Let \( X = Y = \{a, b, c, d, e\} \), \( \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, e\}, \{a, b, d, e\}, \{a, b, c, e\}\} \) and \( \sigma = \{\emptyset, Y, \{a\}, \{a, b\}, \{c, d, e\}, \{a, c, d, e\}\} \). The identity function \( f: (X, \tau) \longrightarrow (Y, \sigma) \) is perfectly \( gs\Lambda \)-continuous, but not contra continuous. Since \( A = \{a\} \) is open in \( (Y, \sigma) \) but \( f^{-1}(A) = \{a\} \) is not closed in \( (X, \tau) \).

Theorem 8.1.4 Every perfectly \( gs\Lambda \)-continuous function is \( gs\Lambda \)-continuous function.

Proof: Let a function \( f: (X, \tau) \longrightarrow (Y, \sigma) \) be perfectly \( gs\Lambda \)-continuous and \( U \) be an open set in \( (Y, \sigma) \). Then \( f^{-1}(U) \) is \( gs\Lambda \)-clopen in \( (X, \tau) \) as \( f \) is perfectly \( gs\Lambda \)-continuous. That is \( f^{-1}(U) \) is both \( gs\Lambda \)-open and \( gs\Lambda \)-closed in \( (X, \tau) \). Thus \( f \) is \( gs\Lambda \)-continuous.
**Remark 54** Converse of the above Theorem 8.1.4 need not be true as seen from the following example.

**Example 62** Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, e\}, \{a, b, d, e\}, \{a, b, c, d\}, \{a, b, c, e\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}, \{a, b, c, d\}\}$. The identity function $f:(X, \tau) \rightarrow (Y, \sigma)$ is gs$\Lambda$-continuous, but not perfectly gs$\Lambda$-continuous function. Since $A = \{b, c, d\}$ is open in $(Y, \sigma)$, but $f^{-1}(A) = \{b, c, d\}$ is not gs$\Lambda$-clopen in $(X, \tau)$.

**Theorem 8.1.5** Every perfectly gs$\Lambda$-continuous function is contra gs$\Lambda$-continuous function.

**Proof:** Proof follows directly from Definition.

**Remark 55** Converse of the above Theorem 8.1.5 need not be true as seen from the following example.

**Example 63** Let $X = Y = \{a, b, c, d, e\}$, 
$\tau = \{\emptyset, X, \{a\}, \{b\}, \{c, d\}, \{d, e\}, \{a, d\}, \{c, d, e\}, \{a, c, d\}, \{a, d, e\}, \{a, c, d, e\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. The identity function $f:(X, \tau) \rightarrow (Y, \sigma)$ is contra gs$\Lambda$-continuous, but not perfectly gs$\Lambda$-continuous function.

**Theorem 8.1.6** A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is perfectly gs$\Lambda$-continuous if and only if $f$ is both gs$\Lambda$-continuous and contra gs$\Lambda$-continuous.

**Proof:** **Necessary:** Let $U$ be an open set in $(Y, \sigma)$ and $f$ be both gs$\Lambda$-continuous and contra gs$\Lambda$-continuous. Since $f$ is both gs$\Lambda$-continuous and contra gs$\Lambda$-continuous $f^{-1}(U)$ is both gs$\Lambda$-open and gs$\Lambda$-closed in $(X, \tau)$. Thus $f$ is perfectly gs$\Lambda$-continuous.

**Sufficient:** Let $U$ be an open set in $(Y, \sigma)$ and $f$ be perfectly gs$\Lambda$-continuous. Then $f^{-1}(U)$ is gs$\Lambda$-clopen. Hence $f$ is both gs$\Lambda$-continuous and contra gs$\Lambda$-continuous.
Theorem 8.1.7 A function $f:(X,\tau)\to (Y,\sigma)$ is perfectly gs\(\Lambda\)-continuous if $f$ is both continuous and contra continuous.

The proof follows directly as every open set is $gs\Lambda$-open and every closed set is $gs\Lambda$-closed.

Theorem 8.1.8 A function $f:(X,\tau)\to (Y,\sigma)$ is perfectly $gs\Lambda$-continuous if $f$ is both $\lambda$-continuous and contra $\lambda$-continuous.

**Proof:** The proof follows directly as every $\lambda$-open set is $gs\Lambda$-open and every $\lambda$-closed set is $gs\Lambda$-closed.

Theorem 8.1.9 If a function $f:(X,\tau)\to (Y,\sigma)$ is a g continuous($\hat{g}$ continuous and gs continuous) and $(X,\tau)$ is a $T_{1/2}$ space (resp.$T\hat{g}$ space,$T_b$ space) then $f:(X,\tau)\to (Y,\sigma)$ is perfectly $gs\Lambda$-closed function.

**Proof:** Let $F$ is a closed set in $(Y,\sigma)$. Since $f$ is g continuous($\hat{g}$ continuous and gs continuous),$f^{-1}(F)$is g closed ($\hat{g}$ closed, gs closed) in $(X,\tau)$. As $(X,\tau)$ is a $T_{1/2}$ space (resp.$T\hat{g}$ space,$T_b$ space), we have $f^{-1}(F)$ is closed in $(X,\tau)$, Hence $f^{-1}(F)$ is a $gs\Lambda$- open and $gs\Lambda$-closed in $Y$ as every closed set is $gs\Lambda$- open and $gs\Lambda$-closed. Thus $f$ is a perfectly $gs\Lambda$-continuous function.

Remark 56 Composition of perfectly $gs\Lambda$-continuous functions is not perfectly $gs\Lambda$-continuous.

Example 64 Let $X = Y ={a,b,c,d,e}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{e\}, \{a,e\}, \{b,c\}, \{b,d\}, \{b,d,e\}, \{a,b,c,d\}\}$. $(Z, \eta) = \{\emptyset,Z,\{a\},\{b\},\{c\}\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$. $f:X\to Y$ and $g:Y\to Z$ are perfectly $gs\Lambda$-continuous functions but $gof:X\to Z$ is not perfectly $gs\Lambda$-continuous function as $A = \{c\}$ is open in $Z$, but $(gof)^{-1}(\{c\})= \{c\}$ is not $gs\Lambda$ clopen in $X$.

Theorem 8.1.10 A map $f:(X,\tau)\to (Y,\sigma)$ is called perfectly $gs\Lambda$-continuous function if the inverse image of each closed set in $Y$ is $gs\Lambda$- clopen in $X$. 

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Proof: Proof follows from the definition 8.1.1 and the concept of complementary sets.

8.2 Strongly gsΛ-continuous function

Definition 8.2.1 A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called strongly gsΛ-continuous function if the inverse image of each gsΛ-closed set in Y is closed in X.

Theorem 8.2.2 A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called strongly gsΛ-continuous function if the inverse image of each gsΛ-open set in Y is open in X.

Proof: Proof follows from the definition 8.2.1 and the concept of complementary sets.

Theorem 8.2.3 Every strongly gsΛ-continuous function is continuous function.

Proof: Let a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is strongly gsΛ-continuous function and $U$ be a open set in $(Y,\sigma)$, then $U$ is gsΛ-open in $(Y,\sigma)$. Since $f$ is strongly gsΛ-continuous function, $f^{-1}(U)$ is open in $(X,\tau)$. Hence $f$ is continuous.

Remark 57 Converse of the above Theorem 8.2.3 need not be true as seen from the following example.

Example 65 Let $X = Y = \{a,b,c,d,e\}$, $\tau = \{\emptyset, X, \{a\}, \{b,c\}, \{d,e\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d,e\}\}$. The identity function $f:(X,\tau) \rightarrow (Y,\sigma)$ is continuous function, but not strongly gsΛ-continuous function. Since $A=\{c\}$ is gsΛ-closed in $(Y, \sigma)$, but $f^{-1}(A)=\{c\}$ is not closed in $(X,\tau)$.

Theorem 8.2.4 Every strongly gsΛ-continuous function is contra continuous function.

Proof: Let a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is strongly gsΛ-continuous and $U$ be
a closed set in $(Y,\sigma)$, then $U$ is $gs\Lambda$-open in $(Y,\sigma)$ by theorem 2.3.5. Since $f$ is strongly $gs\Lambda$-continuous function, $f^{-1}(U)$ is open in $(X,\tau)$. Hence $f$ is contra continuous.

**Remark 58** Converse of the above Theorem 8.2.4 need not be true as seen from the following example.

**Example 66** Let $X = Y = \{a,b,c,d,e\}$, $\tau = \{\emptyset, X, \{a\}, \{b,c\}, \{d,e\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d,e\}\}$. The identity function $f:(X,\tau) \to (Y,\sigma)$ is contra continuous function, but not strongly $gs\Lambda$-continuous function. Since $A = \{c\}$ is $gs\Lambda$-closed in $(Y,\sigma)$, but $f^{-1}(A) = \{c\}$ is not closed in $(X,\tau)$.

**Theorem 8.2.5** Every strongly $gs\Lambda$-continuous function is $gs\Lambda$-continuous function.

**Proof:** Let a function $f:(X,\tau) \to (Y,\sigma)$ be strongly $gs\Lambda$-continuous and $U$ be a open set in $(Y,\sigma)$, then $U$ is $gs\Lambda$-open in $(Y,\sigma)$. Since $f$ is strongly $gs\Lambda$-continuous function, $f^{-1}(U)$ is open in $(X,\tau)$. Since every open set is $gs\Lambda$-open, $f^{-1}(U)$ is $gs\Lambda$-open in $(X,\tau)$. Hence $f$ is $gs\Lambda$-continuous.

**Remark 59** Converse of the above Theorem 8.2.5 need not be true as seen from the following example.

**Example 67** Let $X = Y = \{a,b,c,d,e\}$, $\tau = \{\emptyset, X, \{a\}, \{b,c\}, \{d,e\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d,e\}\}$. The identity function $f:(X,\tau) \to (Y,\sigma)$ is $gs\Lambda$-continuous function, but not strongly $gs\Lambda$-continuous function. Since $A = \{c\}$ is $gs\Lambda$-closed in $(Y,\sigma)$, but $f^{-1}(A) = \{c\}$ is not closed in $(X,\tau)$.

**Theorem 8.2.6** Every strongly $gs\Lambda$-continuous function is contra $gs\Lambda$-continuous function.
**Proof:** Let a function \( f:(X,\tau) \rightarrow (Y,\sigma) \) be strongly gs\( \Lambda \)-continuous and \( U \) be an open set in \( (Y,\sigma) \), then \( U \) is gs\( \Lambda \)-open in \( (Y,\sigma) \). Since \( f \) is strongly gs\( \Lambda \)-continuous function, \( f^{-1}(U) \) is open in \( (X,\tau) \). Since every open set is gs\( \Lambda \)-closed, \( f^{-1}(U) \) is gs\( \Lambda \)-closed in \( (X,\tau) \). Hence \( f \) is contra gs\( \Lambda \)-continuous.

**Remark 60** Converse of the above Theorem 8.2.6 need not be true as seen from the following example.

**Example 68** Let \( X = Y = \{a,b,c,d,e\} \) and \( \tau = \{\emptyset, X, \{a\}, \{b,c\}, \{d,e\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}\} \), and \( \sigma = \{\emptyset, Y, \{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d,e\}\} \). The identity function \( f:(X,\tau) \rightarrow (Y,\sigma) \) is contra gs\( \Lambda \)-continuous function, but not strongly gs\( \Lambda \)-continuous function. Since \( A=\{c\} \) is gs\( \Lambda \)-closed in \( (Y,\sigma) \), but \( f^{-1}(A) = \{c\} \) is not closed in \( (X,\tau) \).

**Theorem 8.2.7** Every strongly gs\( \Lambda \)-continuous function is gs\( \Lambda \)-irresolute function.

**Proof:** Let a function \( f:(X,\tau) \rightarrow (Y,\sigma) \) be strongly gs\( \Lambda \)-continuous and \( U \) be a gs\( \Lambda \)-open set in \( (Y,\sigma) \). Since \( f \) is strongly gs\( \Lambda \)-continuous function, \( f^{-1}(U) \) is open in \( (X,\tau) \). Since every open set is gs\( \Lambda \)-open, \( f^{-1}(U) \) is gs\( \Lambda \)-open in \( (X,\tau) \). Hence \( f \) is gs\( \Lambda \)-irresolute.

**Remark 61** Converse of the above Theorem 8.2.7 need not be true as seen from the following example.

**Example 69** Let \( X = Y = \{a,b,c,d,e\} \), \( \tau = \{\emptyset, X, \{a\}, \{b,c\}, \{d,e\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}\} \), and \( \sigma = \{\emptyset, Y, \{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d,e\}\} \). The identity function \( f:(X,\tau) \rightarrow (Y,\sigma) \) is gs\( \Lambda \)-irresolute function, but not strongly gs\( \Lambda \)-continuous function. Since \( A=\{c\} \) is gs\( \Lambda \)-closed in \( (X,\tau) \), but \( f^{-1}(A) = \{c\} \) is not closed in \( (X,\tau) \).
Theorem 8.2.8  Every strongly gsΛ-continuous function is λ-continuous function.

Proof: Let a function $f:(X,\tau)\rightarrow (Y,\sigma)$ is a strongly gsΛ-continuous function and $U$ be a open set in $(Y,\sigma)$, which is by definition gsΛ-open set in $(Y,\sigma)$. Since $f$ is strongly gsΛ-continuous function, $f^{-1}(U)$ is open in $(X,\tau)$. Since every open set is λ-open, $f^{-1}(U)$ is λ-open in $(X,\tau)$. Hence $f$ is λ-continuous function.

Remark 62 Converse of the above Theorem 8.2.8 need not be true as seen from the following example.

Example 70 Let $X = Y = \{a,b,c,d,e\}$, $\tau = \{\emptyset, X, \{a\}, \{b,c\}, \{d,e\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}\}$, and $\sigma= \{\emptyset,Y,\{a\},\{b,c\},\{a,b,c\},\{b,c,d,e\}\}$. The identity function $f:(X,\tau)\rightarrow (Y,\sigma)$ is λ-continuous function, but not strongly gsΛ-continuous function. Since $A=\{c\}$ is gsΛ-closed in $(Y,\sigma)$, but $f^{-1}(A) = \{c\}$ is not closed in $(X,\tau)$.

Theorem 8.2.9 Every strongly gsΛ-continuous function is λ- irresolute function.

Proof: Let a function $f:(X,\tau)\rightarrow (Y,\sigma)$ is a strongly gsΛ-continuous function and $U$ be a λ-open set in $(Y,\sigma)$, which is by definition gsΛ-open set in $(Y,\sigma)$. Since $f$ is strongly gsΛ-continuous function, $f^{-1}(U)$ is open in $(X,\tau)$. Since every open set is λ-open, $f^{-1}(U)$ is λ-open in $(X,\tau)$. Hence $f$ is λ- irresolute function.

Remark 63 Converse of the above Theorem need 8.2.9 not be true as seen from the following example.

Example 71 Let $X = Y = \{a,b,c,d,e\}$, $\tau = \{\emptyset, X, \{a\}, \{b,c\}, \{d,e\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}\}$ and $\sigma= \{\emptyset,Y,\{a\},\{b,c\},\{a,b,c\},\{b,c,d,e\}\}$. The identity function $f:(X,\tau)\rightarrow (Y,\sigma)$
is \( \lambda \)- irresolute function, but not strongly \( g_{s\Lambda} \)-continuous function. Since \( A = \{ c \} \) is \( g_{s\Lambda} \)-closed in \((Y, \sigma)\), but \( f^{-1}(A) = \{ c \} \) is not closed in \((X, \tau)\).

**Theorem 8.2.10** Every strongly \( g_{s\Lambda} \)-continuous function is perfectly \( g_{s\Lambda} \)-continuous function.

**Proof:** Let a function \( f:(X, \tau) \longrightarrow (Y, \sigma) \) be strongly \( g_{s\Lambda} \)-continuous and \( U \) be a open set in \((Y, \sigma)\). By Theorem 2.3.4 \( U \) is \( g_{s\Lambda} \)-open in \((Y, \sigma)\). Since \( f \) is strongly \( g_{s\Lambda} \)-continuous function, \( f^{-1}(U) \) is open in \((X, \tau)\). Since every open set is \( g_{s\Lambda} \)-closed and \( g_{s\Lambda} \)-closed, \( f^{-1}(U) \) is \( g_{s\Lambda} \)-clopen in \((X, \tau)\). Hence \( f \) is perfectly \( g_{s\Lambda} \)-continuous.

**Remark 64** Converse of the above Theorem need 8.2.10 not be true as seen from the following example.

**Example 72** Let \( X = Y = Z = \{ a, b, c, d, e \} \), \( \tau = \{ \emptyset, X, \{ a \}, \{ b \}, \{ a, b \}, \{ b, c \}, \{ a, b, c \}, \{ b, c, d \}, \{ b, c, d, e \} \} \), and \( \sigma = \{ \emptyset, Y, \{ a \}, \{ e \}, \{ a, e \}, \{ b, c, d \}, \{ b, c, d, e \}, \{ a, b, c, d \} \} \). Here the identity function \( f:(X, \tau) \longrightarrow (Y, \sigma) \) is perfectly \( g_{s\Lambda} \)-continuous function but not strongly \( g_{s\Lambda} \)-continuous, since \( A = \{ c \} \) is \( g_{s\Lambda} \)-closed in \((Y, \sigma)\), but \( f^{-1}(A) = \{ c \} \) is not closed in \((X, \tau)\).

**Theorem 8.2.11** Composition of strongly \( g_{s\Lambda} \)-continuous functions is strongly \( g_{s\Lambda} \)-continuous.

**Proof:** Let a function \( f:(X, \tau) \longrightarrow (Y, \sigma) \) be strongly \( g_{s\Lambda} \)-continuous and a function \( g:(Y, \sigma) \longrightarrow (Z, \eta) \) be strongly \( g_{s\Lambda} \)-continuous and \( U \) be a open set in \((Z, \eta)\). By Theorem 2.3.4 \( U \) is \( g_{s\Lambda} \)-open in \((Z, \eta)\). Since \( g \) is strongly \( g_{s\Lambda} \)-continuous function, \( g^{-1}(U) \) is open in \((Y, \sigma)\). Now by Theorem 2.3.4 \( g^{-1}(U) \) is \( g_{s\Lambda} \)-open in \((Y, \sigma)\). Since \( f \) is strongly \( g_{s\Lambda} \)-continuous function, we get \( f^{-1}(g^{-1}(U)) = (gof)^{-1}(U) \) is open in \((X, \tau)\). Thus the theorem.

Hence we can conclude that strongly \( g_{s\Lambda} \)-continuous function is the strong form.