Chapter 5

Contra $gs\Lambda$-continuous function

Introduction

The notion of contra continuous[15] functions was introduced and investigated by Dontchev. Another class of function called contra $gs\Lambda$-continuous function is introduced in this chapter. This new class contains the class of contra continuous and contra $\lambda$-continuous functions. In this chapter some of the separation axioms like $gs\Lambda$-$T_0$ and $gs\Lambda$-$T_1$ are introduced. In addition some special spaces like $gs\Lambda$-Urysohn space and $gs\Lambda$-Hausdorff space are also introduced and investigated their properties. Strongly $gs\Lambda$ closed graph is also introduced.

5.1 Properties of contra $gs\Lambda$-continuous function

Definition 5.1.1 A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is called contra $gs\Lambda$-continuous map if $f^{-1}(V)$ is $gs\Lambda$-closed in $(X,\tau)$ for each open set in $V$ of $(Y,\sigma)$.

Theorem 5.1.2 A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is contra $gs\Lambda$-continuous map if and only if the inverse image of each closed set in $Y$ is $gs\Lambda$- open in $X$.

Proof: Let $U$ be a closed set in $Y$. Then $(X\setminus U)$ is open in $Y$. By definition $f^{-1}(X\setminus U)=Y\setminus f^{-1}(U)$ is $gs\Lambda$-closed in $X$. Thus $f^{-1}(U)$ is $gs\Lambda$- open in $X$. 

Converse part follows from Definition 5.1.1.

**Definition 5.1.3** A topological space \((X, \tau)\) is said to be a \(gs\Lambda\)-space if the union (intersection) of \(gs\Lambda\)-closed \((gs\Lambda\)-open\) sets is \(gs\Lambda\)-closed \((gs\Lambda\)-open\) and the intersection (union) of \(gs\Lambda\)-closed \((gs\Lambda\)-open\) sets is \(gs\Lambda\)-closed \((gs\Lambda\)-open\).

**Theorem 5.1.4** For a bijective function \(f:(X, \tau) \rightarrow (Y, \sigma)\), the following are equivalent. Assume that \((X, \tau)\) is a \(gs\Lambda\)-space.

1. \(f\) is contra \(gs\Lambda\) continuous.

2. For every closed subset \(F\) of \(Y\), \(f^{-1}(F)\) is \(gs\Lambda\)-open in \(X\).

3. For each \(x \in X\) and each closed subset \(F\) of \(Y\) with \(f(x) \in F\), there exist a \(gs\Lambda\)-open set \(U\) of \(X\) with \(x \in U\), \(f(U) \subseteq F\).

4. \(f(gs\Lambda \text{Cl}(A)) \subseteq \ker(f(A))\) for every subset \(A\) of \(X\).

5. \(gs\Lambda \text{Cl}(f^{-1}(B)) \subseteq f^{-1}(\ker(B))\) for every subset \(B\) of \(Y\).

**Proof:** (1) \(\implies\) (2)

It follows from Theorem 5.1.2.

(2) \(\implies\) (3)

Let \(F\) be any closed subset of \(Y\) and let \(f(x) \in F\) where \(x \in X\). Then by (2) \(f^{-1}(F)\) is \(gs\Lambda\)-open in \(X\). Also \(x \in f^{-1}(F)\). Let \(U = f^{-1}(F)\). Then \(U\) is \(gs\Lambda\)-open set containing \(x\) and \(f(U) \subseteq F\).

(3) \(\implies\) (2)

Let \(F\) be any closed subset of \(Y\). If \(x \in f^{-1}(F)\), then \(f(x) \in F\). Hence by (3), there exist a \(gs\Lambda\)-open set \(U_x\) of \(X\) with \(x \in U_x\) such that \(f(U_x) \subseteq F\). Then \(f^{-1}(F) = \bigcup \{U_x : x \in f^{-1}(F)\}\) and hence by assumption \(f^{-1}(F)\) is \(gs\Lambda\)-open in \(X\).
Let $A$ be any subset of $X$. Let $y \in f(gs \Lambda \text{Cl}(A))$ and suppose that $y \notin \text{ker}(f(A))$. Then by Lemma [1.1.3] there exist $V \in C(Y,y)$ such that $f(A) \cap V = \emptyset$. Thus we have $A \cap f^{-1}(V) = \emptyset$. Since $f^{-1}(V)$ is $gs \Lambda$- open by (2), we have $gs \Lambda \text{cl}(A) \cap f^{-1}(V) = \emptyset$. Hence we get $f(gs \Lambda \text{Cl}(A)) \cap V = \emptyset$. So $y \notin f(gs \Lambda \text{Cl}(A))$ is a contradiction. Thus $y \in \text{ker}(f(A))$, which implies that $f(gs \Lambda \text{cl}(A)) \subset \text{ker}(f(A))$.

(4) $\implies$ (5)

Let $F$ be any subset of $Y$. By (4), we have $f(gs \Lambda Cl(f^{-1}(F))) \subset \text{ker}(f(F))$ and $gs \Lambda cl(f^{-1}(F)) \subset f^{-1}(\text{ker}(F))$.

(5) $\implies$ (1)

Let $U$ be any open set of $Y$. Then $gs \Lambda Cl(f^{-1}(U)) \subset f^{-1}(\text{ker}(U)) = f^{-1}(U)$ and $gs \Lambda \text{Cl}(f^{-1}(U)) = f^{-1}(U)$. By assumption $f^{-1}(U)$ is $gs \Lambda$-closed in $X$. This shows that $f$ is contra $gs \Lambda$-continuous.

**Theorem 5.1.5** Every continuous function is contra $gs \Lambda$-continuous.

**Proof:** Let $F$ be a closed set in $(Y,\sigma)$ and a function $f:(X,\tau) \rightarrow (Y,\sigma)$ be a continuous function. Hence $f^{-1}(F)$ is closed in $(X,\tau)$. As every closed set is $gs \Lambda$- open set by Theorem 2.3.5, we have $f^{-1}(F)$ is $gs \Lambda$- open in $X$. Thus $f$ is contra $gs \Lambda$-continuous.

**Remark 31** Converse of the above Theorem 5.1.5 need not be true as seen from the following example.

**Example 37** Let $X = Y = \{a,b,c,d,e\}$, $\tau = \{\emptyset,X,\{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}, \{c,d,e\}, \{a,b,c,d\}, \{a,c,d,e\}, \{b,c,d,e\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a,b\}, \{a,b,e\}, \{a,b,d,e\}, \{a,b,c,e\}\}$. The identity function $f:(X,\tau) \rightarrow (Y,\sigma)$ is a contra $gs \Lambda$-continuous function but not continuous functions as $A = \{c\}$ is closed in $(Y,\sigma)$ but $f^{-1}(A) = \{c\}$ is not closed in $(X,\tau)$. 

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Remark 32 **Theorem 5.1.6** Every contra continuous function is contra gs$\Lambda$-continuous.

**Proof:** Proof follows as every open set is gs$\Lambda$-open set by Theorem 2.3.4.

Remark 33 Converse of the above Theorem 5.1.6 need not be true as seen from the following example.

**Example 38** Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, \{a, b, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. The identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra gs$\Lambda$-continuous but not contra continuous as $A = \{c\}$ is closed in $(Y, \sigma)$ but $f^{-1}(A) = \{c\}$ is not open in $(X, \tau)$.

**Theorem 5.1.7** If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra gs$\Lambda$-continuous and $(X, \tau)$ is $T_1$ and $\lambda$-space then $f$ is contra continuous.

**Proof:** Let $F$ be a closed set in $(Y, \sigma)$ and a function $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra gs$\Lambda$-continuous, where $(X, \tau)$ is $T_1$ and $\lambda$-space. Hence $f^{-1}(F)$ is gs$\Lambda$-open in $(X, \tau)$. Since $(X, \tau)$ is $T_1$ space $f^{-1}(F)$ is $\lambda$-open in $X$. Also since $(X, \tau)$ is a $\lambda$-space $f^{-1}(F)$ is open in $X$ by Lemma 1.1.9. Thus $f$ is contra continuous.

**Theorem 5.1.8** If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra gs$\Lambda$-continuous and $(X, \tau)$ is $T_1$ and $\lambda$-$S$-space then $f$ is contra semi continuous.

**Proof:** Proof follows as in $\lambda$-$S$-space every $\lambda$-open set is semi open [by Lemma 1.1.9].

**Theorem 5.1.9** Every contra $\lambda$-continuous function is contra gs$\Lambda$-continuous.

**Proof:** It follows by Definition of contra $\lambda$-continuous function and the fact that every $\lambda$-closed set is gs$\Lambda$ closed.

**Remark 34** Converse of the above Theorem 5.1.9 need not be true as seen from the following example.
**Example 39** Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, b, c, d\}, \{a, c, d, e\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{a, b, e\}, \{a, b, c, d\}\}$. The identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is a contra gs$\Lambda$-continuous function but not contra $\lambda$-continuous functions as $A = \{c\}$ is closed in $(Y, \sigma)$ but $f^{-1}(A) = \{c\}$ is not $\lambda$-open in $(X, \tau)$.

**Theorem 5.1.10** If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra gs$\Lambda$-continuous and $(X, \tau)$ is $T_1$ then $f$ is is contra $\lambda$-continuous.

**Proof:** Let $F$ be a open set in $(Y, \sigma)$ and a function $f: (X, \tau) \rightarrow (Y, \sigma)$ be a contra gs$\Lambda$ continuous function where $(X, \tau)$ is $T_1$. As $f$ is a contra gs$\Lambda$ continuous function $f^{-1}(F)$ is gs$\Lambda$-closed in $(X, \tau)$ which is $\lambda$-closed as $(X, \tau)$ is $T_1$ [by Theorem 2.2.4]. Thus $f$ is a contra $\lambda$-continuous function.

**Remark 35** The following Examples shows that $\lambda$-continuous and contra gs$\Lambda$-continuous functions are in general independent.

**Example 40** Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, b, c, d\}, \{a, c, d, e\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{a, b, e\}, \{a, b, c, d\}\}$. The identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is a contra gs$\Lambda$-continuous function but not $\lambda$-continuous functions as $A = \{c\}$ is closed in $(Y, \sigma)$ but $f^{-1}(A) = \{c\}$ is not $\lambda$-closed in $(X, \tau)$.

**Example 41** Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, e\}, \{a, b, d, e\}, \{a, b, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, b, c, d\}, \{a, c, d, e\}, \{b, c, d, e\}\}$. The identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\lambda$-continuous functions but not contra gs$\Lambda$-continuous function as $A = \{b, e\}$ is closed in $(Y, \sigma)$ but $f^{-1}(A) = \{b, e\}$ is not gs$\Lambda$- open in $(X, \tau)$. 
**Theorem 5.1.11** If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\lambda$ continuous and $(X, \tau)$ is $\lambda$-space then $f$ is contra $gs\Lambda$ continuous.

**Proof:** Let $F$ be a open set in $(Y, \sigma)$ and a function $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\lambda$ continuous function where $(X, \tau)$ is a $\lambda$- space. As $f$ is $\lambda$ continuous, $f^{-1}(F)$ is $\lambda$-open in $(X, \tau)$ which is open as $(X, \tau)$ is a $\lambda$- space [Lemma 1.1.9]. Now by Theorem 2.1.5 $f^{-1}(F)$ is $gs\Lambda$-closed in $(X, \tau)$. Thus $f$ is a contra $gs\Lambda$ continuous function.

**Remark 36** $gs\Lambda$-continuous functions and contra $gs\Lambda$-continuous functions are in general independent as it can be seen from the following examples.

**Example 42** Let $X = Y = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, e\}, \{a, d, e\}, \{b, c, e\}, \{b, c, d\}, \{c, d\}\}$, $(Y, \sigma)=\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\}\}$. The identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is $gs\Lambda$-continuous functions but not contra $gs\Lambda$-continuous function if $A=\{a, c, d\}$ is open in $(Y, \sigma)$ but $f^{-1}(A) = \{a, c, d\}$ is not $gs\Lambda$-closed in $(X, \tau)$.

**Example 43** Let $X = Y = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, d, e\}\}$, $(Y, \sigma)=\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\}\}$. The identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $gs\Lambda$-continuous function but not $gs\Lambda$-continuous function as $A=\{b, e\}$ is closed in $(Y, \sigma)$ but $f^{-1}(A) = \{b, e\}$ is not $gs\Lambda$-closed in $(X, \tau)$.

**Theorem 5.1.12** If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\lambda$ continuous and $(Y, \sigma)$ is locally indiscrete, then $f$ is contra $gs\Lambda$ continuous.

**Proof:** Let $F$ be a open set in $(Y, \sigma)$. Since $(Y, \sigma)$ is locally indiscrete, [by Lemma 1.1.12] $F$ is closed in $Y$. As $f$ is $\lambda$ continuous $f^{-1}(F)$ is $\lambda$-closed in
As every \(\lambda\)-closed set is \(gs\Lambda\)-closed set by Theorem 2.1.2, we have \(f^{-1}(F)\) is \(gs\Lambda\)-closed in \((X,\tau)\). Thus \(f\) is contra \(gs\Lambda\)-continuous.

**Theorem 5.1.13** If a function \(f:(X,\tau)\rightarrow(Y,\sigma)\) is \(gs\Lambda\)-continuous and \((Y,\sigma)\) is locally indiscrete, then \(f\) is contra \(gs\Lambda\) continuous.

**Proof:** Let \(F\) be a open set in \((Y,\sigma)\). Since \((Y,\sigma)\) is a locally indiscrete space, [by Lemma 1.1.12] \(F\) is closed in \(Y\). As \(f\) is a \(gs\Lambda\) continuous function \(f^{-1}(F)\) is \(gs\Lambda\)-closed in \((X,\tau)\). Thus \(f\) is contra \(gs\Lambda\)-continuous.

**Theorem 5.1.14** If a function \(f:(X,\tau)\rightarrow(Y,\sigma)\) is contra \(gs\Lambda\)-continuous and \((X,\tau)\) is \(T_1\), then \(f\) is contra \(\lambda\)-continuous.

**Proof:** The proof is obvious [by Theorem 2.2.4] as in \(T_1\) space every \(gs\Lambda\)-closed set is \(\lambda\)-closed.

**Theorem 5.1.15** Any function \(f:(X,\tau)\rightarrow(Y,\sigma)\) is contra \(gs\Lambda\)-continuous function, if \((X,\tau)\) is a \(T_{1/2}\) space.

**Proof:** The Proof is obvious [by Theorem 2.2.5] as in \(T_{1/2}\) space every subset is \(gs\Lambda\)-closed (\(gs\Lambda\)-open).

**Theorem 5.1.16** If a function \(f:(X,\tau)\rightarrow(Y,\sigma)\) is a \(g\) continuous(\(\hat{g}\) continuous and \(gs\) continuous) and \((X,\tau)\) is a \(T_{1/2}\) space( resp. \(T\hat{g}\)space, \(T_b\) space) then \(f\) is contra \(gs\Lambda\)-closed.

**Proof:** Let \(F\) is a closed set in \((Y,\sigma)\). Since \(f\) is \(g\) continuous (\(\hat{g}\) continuous and \(gs\) continuous), \(f^{-1}(F)\)is \(g\) closed (\(\hat{g}\) closed, \(gs\) closed) in \((X,\tau)\). As \((X,\tau)\) is a \(T_{1/2}\) space (resp. \(T\hat{g}\)space, \(T_b\) space), [by Lemma 1.1.11] we have \(f^{-1}(F)\) is closed in \((X,\tau)\). Hence \(f^{-1}(F)\) is a \(gs\Lambda\)-open in \(Y\) as every closed set is \(gs\Lambda\)-open. Thus \(f\) is a contra \(gs\Lambda\)-continuous map.
Theorem 5.1.17 If a function \( f:(X,\tau) \rightarrow (Y,\sigma) \) is a contra gs\( \Lambda \)-continuous function and irresolute, then \( f \) is contra \( \lambda \)-continuous.

Proof: Let \( V \) be a open set of \( Y \). As every open set is semi open, \( V \) is a semi open set in \( Y \). Since \( f \) is a contra gs\( \Lambda \)-continuous function and irresolute function, \( f^{-1}(V) \) is gs\( \Lambda \)-closed and semi open in \( (X,\tau) \). Now by Theorem 2.1.3 \( f^{-1}(V) \) is \( \lambda \)-closed. Thus \( f \) is contra \( \lambda \)-continuous function.

Theorem 5.1.18 If a function \( f:(X,\tau) \rightarrow (Y,\sigma) \) is a contra gs\( \Lambda \)-continuous function and semi continuous function then \( f \) is contra \( \lambda \)-continuous function.

Proof: It is similar to the above proof and follows by Lemma 2.1.3.

Theorem 5.1.19 If a function \( f:(X,\tau) \rightarrow (Y,\sigma) \) is a contra semi continuous function and \( (X,\tau) \) is globally disconnected then \( f \) is contra gs\( \Lambda \)-continuous function.

Proof: Let \( V \) be a closed set of \( Y \). Since \( f \) is a contra semi continuous function, \( f^{-1}(V) \) is semi open in \( (X,\tau) \). Now since \( (X,\tau) \) is globally disconnected by Lemma 2.1.12 \( f^{-1}(V) \) is open in \( (X,\tau) \), which is gs\( \Lambda \)-open[by Theorem 2.3.4]. Thus \( f \) is a contra gs\( \Lambda \)-continuous function.

Theorem 5.1.20 If a function \( f:(X,\tau) \rightarrow (Y,\sigma) \) is a semi continuous function and \( (X,\tau) \) is globally disconnected then \( f \) is contra gs\( \Lambda \)-continuous.

Proof: Let \( V \) be a open set of \( Y \). Since \( f \) is a semi continuous function, \( f^{-1}(V) \) is semi open in \( (X,\tau) \). Now since \( (X,\tau) \) is globally disconnected \( f^{-1}(V) \) is open in \( (X,\tau) \), which is gs\( \Lambda \)-closed [by Theorem 2.1.5]. Thus \( f \) is contra gs\( \Lambda \)-continuous.

Theorem 5.1.21 If a function \( f:(X,\tau) \rightarrow (Y,\sigma) \) is irresolute and \( (X,\tau) \) is globally disconnected then \( f \) is contra gs\( \Lambda \)-continuous.

Proof: It follows from the definitions.
Theorem 5.1.22 If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is contra $gs\Lambda$-continuous and $Y$ is regular then $f$ is $gs\Lambda$-continuous.

\textbf{Proof:} Let $x$ be an arbitrary point of $X$ and $N$ an open set of $Y$ containing $f(x)$. Since $Y$ is regular, there exist an open set $U$ in $Y$ containing $f(x)$ such that $\text{cl}(U) \subseteq N$. Since $f$ is contra $gs\Lambda$-continuous function by Theorem [5.1.4] there exist a $gs\Lambda$- open set $W$ of $X$ with $x \in W$ such that $f(W) \subseteq \text{Cl}(U)$. Then $f(W) \subseteq N$. Hence by Theorem 4.1.16, $f$ is $gs\Lambda$-continuous.

Remark 37 Recall that in a submaximal space [53] every preopen set is open. As every open set is $gs\Lambda$- open and $gs\Lambda$-closed, it is clear to observe that in a submaximal space every preopen set is $gs\Lambda$- open and $gs\Lambda$-closed

Theorem 5.1.23 If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is contra pre continuous and $X$ is submaximal then $f$ is both $gs\Lambda$-continuous and contra $gs\Lambda$-continuous.

\textbf{Proof:} It follows from remark 37.

Similarly we can prove the following Theorem

Theorem 5.1.24 If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is pre continuous and $X$ is submaximal then $f$ is both $gs\Lambda$-continuous and contra $gs\Lambda$-continuous.

Theorem 5.1.25 If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is a contra $\lambda$-continuous function and $A$ is a open subset of $X$, then the restriction $f_A:A \rightarrow Y$ is also contra $gs\Lambda$-continuous.

\textbf{Proof:} Let $V$ be a open set of $Y$ and $A$ be a open subset of $X$. As every open set is $\lambda$-closed, $A$ is $\lambda$-closed in $X$ and since $f$ is contra $\lambda$-continuous $f^{-1}(V)$ is $\lambda$-closed in $X$. Hence we have $f^{-1}(V) \cap A$ is $\lambda$-closed in $X$, which is also $gs\Lambda$-closed in $(X,\tau)$. Since $f^{-1}(V) \cap A \subseteq A \subseteq X$ where $A$ is a open subset of $X$, by Theorem 2.2.23 $f^{-1}(V) \cap A = (f_A^{-1}(V))$ is $gs\Lambda$-closed in $A$. Thus the restriction $f_A:A \rightarrow Y$ is also contra $gs\Lambda$-continuous.
**Definition 5.1.26** A topological space $X$ is said to be

1. $gs\Lambda\text{-}T_0$ if for each $x, y \in X$ such that $x \neq y$ there exist a $gs\Lambda$-open set $U$ of $X$ containing $x$ but not $y$ or a $gs\Lambda$-open set $V$ of $X$ containing $y$ but not $x$.

2. $gs\Lambda\text{-}T_1$ if for each $x, y \in X$ such that $x \neq y$ there exist a $gs\Lambda$-open set $U$ of $X$ containing $x$ but not $y$ and a $gs\Lambda$-open set $V$ of $X$ containing $y$ but not $x$.

3. $gs\Lambda\text{-}T_2$ if for each $x, y \in X$ such that $x \neq y$ there exist a $gs\Lambda$-open set $U$ of $X$ containing $x$ and a $gs\Lambda$-open set $V$ of $X$ containing $y$ such that $U \cap V = \emptyset$.

**Definition 5.1.27** A topological space $X$ is said to be

1. $gs\Lambda\text{-}Urysohn$ if for each $x, y \in X$ such that $x \neq y$ there exist a $gs\Lambda$-open set $U$ of $X$ containing $x$ and a $gs\Lambda$-open set $V$ of $X$ containing $y$ such that $gs\Lambda Cl(U) \cap gs\Lambda Cl(V) = \emptyset$.

2. $gs\Lambda\text{-}normal$ if for each pair of non empty disjoint closed sets $A$ and $B$ of $X$ there exist disjoint $gs\Lambda$-open sets $U$ and $V$ such that $A \subseteq U$ and $B \subseteq V$.

**Theorem 5.1.28** If $X$ is a topological space and for each pair of distinct points $x_1$ and $x_2 \in X$, there exist a map $f$ of $X$ into a Urysohn topological space $Y$ such that $f(x_1) \neq f(x_2)$ and $f$ is contra $gs\Lambda$-continuous at $x_1$ and $x_2$, then $X$ is $gs\Lambda\text{-}T_2$.

**Proof:** Let $x_1$ and $x_2 \in X$ such that $x_1 \neq x_2$. Then by hypothesis there is a Urysohn space $Y$ and a function $f: X \rightarrow Y$, such that $f(x_1) \neq f(x_2)$ and $f$ is contra $gs\Lambda$-continuous at $x_1$ and $x_2$. Let $y_i = f(x_i), i=1,2$. Then $y_1 \neq y_2$. Since $Y$ is Urysohn, there exist open neighbourhoods $U_{y_1}$ and $U_{y_2}$ of $y_1$ and $y_2$ respectively, in $Y$ such that $Cl(U_{y_1}) \cap Cl(U_{y_2}) = \emptyset$. Since $f$ is contra $gs\Lambda$-continuous at $x_i$, there exist a $gs\Lambda$-open neighbourhood $W_{x_i}$ of $x_i$ in $X$ such that $f(W_{x_i}) \subseteq Cl(U_{y_i})$ for $i=1,2$. Hence we get $W_{x_1} \cap W_{x_2} = \emptyset$ since $Cl(U_{y_1}) \cap Cl(U_{y_2}) = \emptyset$. Hence $X$ is $gs\Lambda Cl\text{-}T_2$. 

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Corollary 5.1.29 If $f$ is a contra $gs\Lambda$-continuous injection of a topological space $X$ into a Urysohn space $Y$, then $X$ is $gs\Lambda$-$T_2$.

Theorem 5.1.30 If $f$ is a contra $gs\Lambda$-continuous injection of a topological space $X$ into an Ultra Hausdroff space $Y$, then $X$ is $gs\Lambda$-$T_2$.

**Proof:** Let $x_1$ and $x_2 \in X$ be a pair of distinct points. Since $f$ is injective and $Y$ is Ultra Hausdroff, we get $f(x_1) \neq f(x_2)$ and there exist clopen sets $V_1$, $V_2$ in $Y$, such that $f(x_1) \in V_1, f(x_2) \in V_2$ and $V_1 \cap V_2 = \emptyset$. Then $x_i \in f^{-1}(V_i)$ for $i=1,2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$ where $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $gs\Lambda$-open in $X$. Hence by the above Theorem $X$ is $gs\Lambda$-$T_2$.

Definition 5.1.31 A topological space $X$ is said to be

1. $gs\Lambda$-Hausdroff if for each pair of distinct points $x$ and $y$ in $X$ there exist disjoint $gs\Lambda$-open subsets $U$ and $V$ of $X$ containing $x$ and $y$ respectively, such that $U \cap V = \emptyset$.

2. $gs\Lambda$-ultra Hausdroff if for each pair of distinct points $x$ and $y$ in $X$ there exist disjoint $gs\Lambda$-clopen subsets $U$ and $V$ of $X$ containing $x$ and $y$ respectively, such that $U \cap V = \emptyset$.

Theorem 5.1.32 If $f$ is a contra $gs\Lambda$-continuous injection of a topological space $X$ into an Urysohn space $Y$, then $X$ is $gs\Lambda$-Hausdroff.

**Proof:** Let $x_1$ and $x_2 \in X$ be a pair of distinct points. Suppose $y_1 = f(x_1)$ and $y_2 = f( x_2)$. Since $f$ is injective and $Y$ is Urysohn, we get $f(x_1) \neq f(x_2)$ and there exist open sets $V_1$ and $V_2$ in $Y$ containing $f(x_1)$ and $f(x_2)$ respectively, such that $Cl(V_1) \cap Cl(V_2) = \emptyset$. Since $f$ is contra $gs\Lambda$-continuous there exist $gs\Lambda$-open sets $U_1$ and $U_2$ in $X$ containing $x_1$ and $x_2$ respectively, such that $f(U_1) \subset Cl(V_1)$ and $f(U_2) \subset Cl(V_2)$. Therefore we have $U_1 \cap U_2 = \emptyset$. Hence $X$ is $gs\Lambda$-Hausdroff.
Theorem 5.1.33 If $f$ is a contra $g_{s\Lambda}$-continuous injection of a topological space $X$ into an Ultra Hausdroff space $Y$, then $X$ is $g_{s\Lambda}$-Hausdroff.

Proof: Let $x_1$ and $x_2 \in X$ be a pair of distinct points. Suppose $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since $f$ is injective and $Y$ is Ultra Hausdroff, we get $f(x_1) \neq f(x_2)$ and there exist clopen sets $V_1$ and $V_2$ in $Y$ containing $f(x_1)$ and $f(x_2)$ respectively, such that $V_1 \cap V_2 = \emptyset$. Since $f$ is contra $g_{s\Lambda}$-continuous $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $g_{s\Lambda}$-open sets containing $x_1$ and $x_2$ respectively, such that $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. Hence $X$ is $g_{s\Lambda}$-Hausdroff.

Let us define that the product space $X = X_1 \times X_2 \times X_3 \times \ldots \times X_n$ has the property $P_L$ if $U_i$ is a $g_{s\Lambda}$-open set in a topological set $X_i$ for $i=1,2,\ldots,n$, then $U_1 \times U_2 \times U_3 \times \ldots \times U_n$ is also $g_{s\Lambda}$-open set in the product space $X = X_1 \times X_2 \times X_3 \times \ldots \times X_n$.

Theorem 5.1.34 Let $f_1 : X_1 \rightarrow Y$ and $f_2 : X_2 \rightarrow Y$ be two functions, where

1. $X=X_1 \times X_2$ have the property $P_L$

2. $Y$ is a Urysohn space

3. $f_1$ and $f_2$ are contra $g_{s\Lambda}$-continuous.

Then $\{(x_1,x_2) : f_1(x_1) = f_2(x_2)\}$ is $g_{s\Lambda}$-closed in the product space $X = X_1 \times X_2$.

Proof: Let $A$ denote the set $\{(x_1,x_2) : f_1(x_1) = f_2(x_2)\}$. Let us first show that $(X_1 \times X_2) \setminus A$ is $g_{s\Lambda}$-open. Let $(x_1,x_2) \notin A$. Then $f_1(x_1) \neq f_2(x_2)$. Since $Y$ is Urysohn space there exists open sets $V_1$ and $V_2$ of $f_1(x_1)$ and $f_2(x_2)$ respectively such that $\text{Cl}(V_1) \cap \text{Cl}(V_2) = \emptyset$. Since $f_i, i=1,2$ is contra $g_{s\Lambda}$-continuous, $f_i^{-1}(\text{Cl}(V_i)), i=1,2$ is a $g_{s\Lambda}$-open set containing $x_i$ in $X_i, i=1,2$. Hence by (1) $f_1^{-1}(\text{Cl}(V_1)) \times f_2^{-1}(\text{Cl}(V_2))$ is $g_{s\Lambda}$-open. Further more $(x_1,x_2) \notin f_1^{-1}(\text{Cl}(V_1)) \times f_2^{-1}(\text{Cl}(V_2)) \subset (X_1 \times X_2) \setminus A$. It follows that $(X_1 \times X_2) \setminus A$ is $g_{s\Lambda}$-open. Thus $A$ is $g_{s\Lambda}$-closed in the product space $X_1 \times X_2$. 

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Theorem 5.1.35 Assume that the product space $X \times X$ has the property $P_L$. If $f:X \to Y$ is a contra $gs\Lambda$-continuous and $Y$ is a Urysohn space, then \{(x_1, x_2): f(x_1) = f(x_2)\} is $gs\Lambda$-closed in the product space $X \times X$.

**Proof:** It follows from Theorem 5.1.34.

Theorem 5.1.36 If $(X, \tau) \to (Y, \sigma)$ is a closed contra $gs\Lambda$-continuous injection and $Y$ is ultra normal, then $X$ is $gs\Lambda$- normal.

**Proof:** Let $V_1$ and $V_2$ be non empty disjoint closed subsets of $X$. Since $f$ is closed and injective, $f(V_1)$ and $f(V_2)$ are non empty disjoint closed subsets of $Y$. Since $Y$ is Ultra normal, $f(V_1)$ and $f(V_2)$ can be separated by disjoint clopen sets $W_1$ and $W_2$ respectively. Hence $V_i \subseteq f^{-1}(W_i), i=1,2$ and since $f$ is contra $gs\Lambda$-continuous $f^{-1}(W_i), i=1,2$ are $gs\Lambda$- open sets of $X$ and $f^{-1}(W_1) \cap f^{-1}(W_2) = \emptyset$. Thus $X$ is $gs\Lambda$- normal.

5.2 On composition of contra $gs\Lambda$ continuous functions

Theorem 5.2.1 The composition of contra continuous functions is contra $gs\Lambda$-continuous.

**Proof:** Let $f:(X, \tau) \to (Y, \sigma)$ and $g:(Y, \sigma) \to (Z, \xi)$ are contra continuous functions. Let $F$ be a open set of $(Z, \xi)$. Then $g^{-1}(F)$ is a closed set in $(Y, \sigma)$ as $g:(Y, \sigma) \to (Z, \xi)$ is a contra continuous function and $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is a open set in $(X, \tau)$ as $f:(X, \tau) \to (Y, \sigma)$ is a contra continuous function. Since every open set is $gs\Lambda$-closed, $(gof)^{-1}(F)$ is a $gs\Lambda$-closed set in $(X, \tau)$. Thus $gof:(X, \tau) \to (Z, \xi)$ is a contra $gs\Lambda$-continuous function.

Theorem 5.2.2 The composition of continuous functions is contra $gs\Lambda$-continuous.

**Proof:** It follows from definitions.
Remark 38 But composition of contra $gs\Lambda$-continuous functions need not be contra $gs\Lambda$-continuous. This is verified in the following example.

Example 44 Let $X = Y = Z = \{a,b,c,d,e\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$ and $\xi = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$. The identity function $f:(X,\tau)\rightarrow(Y,\sigma)$ and the identity function $g:(Y,\sigma)\rightarrow(Z,\xi)$ are contra $gs\Lambda$-continuous functions but $gof:(X,\tau)\rightarrow(Z,\xi)$ is not contra $gs\Lambda$-continuous function for $A=\{b,e\}$ is open in $(Z,\xi)$ but $f^{-1}(A)=\{b,e\}$. $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is a closed set in $(X,\tau)$ as $f:(X,\tau)\rightarrow(Y,\sigma)$ is a contra continuous function. Since every closed set is $gs\Lambda$-closed, $(gof)^{-1}(F)$ is a $gs\Lambda$-closed set in $(X,\tau)$. Thus $gof:(X,\tau)\rightarrow(Z,\xi)$ is a contra $gs\Lambda$-continuous function.

Theorem 5.2.3 If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a continuous function and $g:(Y,\sigma)\rightarrow(Z,\xi)$ is a contra continuous function, then $gof:(X,\tau)\rightarrow(Z,\xi)$ is a contra $gs\Lambda$-continuous function.

Proof: Let $F$ be a open set of $(Z,\xi)$. Then $g^{-1}(F)$ is a closed set in $(Y,\sigma)$ as $g:(Y,\sigma)\rightarrow(Z,\xi)$ is a contra continuous function and $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is a closed set in $(X,\tau)$ as $f:(X,\tau)\rightarrow(Y,\sigma)$ is a continuous function. Since every closed set is $gs\Lambda$-closed, $(gof)^{-1}(F)$ is a $gs\Lambda$-closed set in $(X,\tau)$. Thus $gof:(X,\tau)\rightarrow(Z,\xi)$ is a contra $gs\Lambda$-continuous function.

Theorem 5.2.4 If $f:(X,\tau)\rightarrow(Y,\sigma)$is contra continuous and $g:(Y,\sigma)\rightarrow(Z,\xi)$ is continuous, then $gof:(X,\tau)\rightarrow(Z,\xi)$is contra $gs\Lambda$-continuous.

Proof: Let $F$ be a open set of $(Z,\xi)$. Then $g^{-1}(F)$ is a open set in $(Y,\sigma)$ as $g:(Y,\sigma)\rightarrow(Z,\xi)$ is a continuous function and $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is a closed set in $(X,\tau)$ as $f:(X,\tau)\rightarrow(Y,\sigma)$ is a contra continuous function. Since every closed set is $gs\Lambda$-closed, $(gof)^{-1}(F)$ is a $gs\Lambda$-closed set in $(X,\tau)$. Thus $gof:(X,\tau)\rightarrow(Z,\xi)$ is a contra $gs\Lambda$-continuous function.
Theorem 5.2.5 If \( f: (X, \tau) \to (Y, \sigma) \) is gs\( \Lambda \)-continuous and \( g: (Y, \sigma) \to (Z, \xi) \) is contra continuous, then \( gof: (X, \tau) \to (Z, \xi) \) is contra gs\( \Lambda \)-continuous.

**Proof:** Let \( F \) be a open set of \((Z, \xi)\). Then \( g^{-1}(F) \) is a closed set in \((Y, \sigma)\) and \( f^{-1}(g^{-1}(F)) = (gof)^{-1}(F) \) is a gs\( \Lambda \)-closed set in \((X, \tau)\) as \( f: (X, \tau) \to (Y, \sigma) \) is a gs\( \Lambda \)-continuous function. Thus \( gof: (X, \tau) \to (Z, \xi) \) is a contra gs\( \Lambda \)-continuous function.

Theorem 5.2.6 If \( f: (X, \tau) \to (Y, \sigma) \) is contra gs\( \Lambda \)-continuous and \( g: (Y, \sigma) \to (Z, \xi) \) is continuous, then \( gof: (X, \tau) \to (Z, \xi) \) is contra gs\( \Lambda \)-continuous.

**Proof:** Let \( F \) be a open set of \((Z, \xi)\). Then \( g^{-1}(F) \) is a open set in \((Y, \sigma)\) as \( g: (Y, \sigma) \to (Z, \xi) \) is a continuous function and \( f^{-1}(g^{-1}(F)) = (gof)^{-1}(F) \) is a gs\( \Lambda \)-closed set in \((X, \tau)\) as \( f: (X, \tau) \to (Y, \sigma) \) is a contra gs\( \Lambda \)-continuous function. Thus \( gof: (X, \tau) \to (Z, \xi) \) is a contra gs\( \Lambda \)-continuous function.

Theorem 5.2.7 If \( f: (X, \tau) \to (Y, \sigma) \) is contra \( \lambda \)-continuous and \( g: (Y, \sigma) \to (Z, \xi) \) is continuous, then \( gof: (X, \tau) \to (Z, \xi) \) is contra gs\( \Lambda \)-continuous.

**Proof:** Let \( F \) be a open set of \((Z, \xi)\). Then \( g^{-1}(F) \) is a open set in \((Y, \sigma)\) as \( g: (Y, \sigma) \to (Z, \xi) \) is a continuous function and \( f^{-1}(g^{-1}(F)) = (gof)^{-1}(F) \) is a \( \lambda \)-closed set in \((X, \tau)\) as \( f: (X, \tau) \to (Y, \sigma) \) is a contra \( \lambda \)-continuous function. Since every \( \lambda \)-closed set is gs\( \Lambda \)-closed by preposition [2.5], \( (gof)^{-1}(F) \) is a gs\( \Lambda \)-closed set in \((X, \tau)\). Thus \( gof: (X, \tau) \to (Z, \xi) \) is a contra gs\( \Lambda \)-continuous function.

Theorem 5.2.8 If \( f: (X, \tau) \to (Y, \sigma) \) is \( \lambda \)- irresolute and \( g: (Y, \sigma) \to (Z, \xi) \) is contra \( \lambda \)-continuous, then \( gof: (X, \tau) \to (Z, \xi) \) is contra gs\( \Lambda \)-continuous.

**Proof:** Let \( F \) be a open set of \((Z, \xi)\), \( f: (X, \tau) \to (Y, \sigma) \) is a \( \lambda \)- irresolute function and \( g: (Y, \sigma) \to (Z, \xi) \) is a Contra \( \lambda \)-continuous function. Then \( g^{-1}(F) \) is a \( \lambda \)-closed set in \((Y, \sigma)\) as \( g: (Y, \sigma) \to (Z, \xi) \) is a Contra \( \lambda \)-continuous function and \( f^{-1}(g^{-1}(F)) = (gof)^{-1}(F) \) is a \( \lambda \)-closed set in \((X, \tau)\) as \( f: (X, \tau) \to (Y, \sigma) \) is a
\(\lambda\)- irresolute function. Since every \(\lambda\)-closed set is \(gs\Lambda\)-closed by, \((gof)^{-1}(F)\) is a \(gs\Lambda\)-closed set in \((X,\tau)\). Thus \(gof:(X,\tau)\rightarrow (Z,\xi)\) is a contra \(gs\Lambda\)-continuous function.

**Theorem 5.2.9** The composition of \(\lambda\)- irresolute functions is contra \(gs\Lambda\)-continuous.

**Proof:** As every \(\lambda\)-closed set is \(gs\Lambda\)-closed by the proof is clear.

**Theorem 5.2.10**

1. Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be a \(gs\Lambda\)-continuous function and \(g:(Y,\sigma)\rightarrow (Z,\xi)\) is a continuous function, then \(gof:(X,\tau)\rightarrow (Z,\xi)\) is a \(\lambda\)-continuous function if \((X,\tau)\) is a \(T_1\) space.

2. Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be a precontinuous function and \(g:(Y,\sigma)\rightarrow (Z,\xi)\) is a continuous function, then \(gof:(X,\tau)\rightarrow (Z,\xi)\) is a contra \(gs\Lambda\)-continuous function if \((X,\tau)\) is a submaximal space.

3. Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be a contra pre continuous function and \(g:(Y,\sigma)\rightarrow (Z,\xi)\) is a continuous function, then \(gof:(X,\tau)\rightarrow (Z,\xi)\) is a contra \(gs\Lambda\)-continuous function if \((X,\tau)\) is a submaximal space.

4. Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be a contra pre continuous function and \(g:(Y,\sigma)\rightarrow (Z,\xi)\) is a contra continuous function, then \(gof:(X,\tau)\rightarrow (Z,\xi)\) is a contra \(gs\Lambda\)-continuous function if \((X,\tau)\) is a globally disconnected space.

5. Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be a semi continuous function and \(g:(Y,\sigma)\rightarrow (Z,\xi)\) is a continuous function, then \(gof:(X,\tau)\rightarrow (Z,\xi)\) is a contra \(gs\Lambda\)-continuous function if \((X,\tau)\) is a globally disconnected space.

6. Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be a contra semi continuous function and \(g:(Y,\sigma)\rightarrow (Z,\xi)\) is a continuous function, then \(gof:(X,\tau)\rightarrow (Z,\xi)\) is a contra \(gs\Lambda\)-continuous function if \((X,\tau)\) is a globally disconnected space.

7. Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be a contra semi continuous function and \(g:(Y,\sigma)\rightarrow (Z,\xi)\) is a contra continuous function, then \(gof:(X,\tau)\rightarrow (Z,\xi)\) is a contra \(gs\Lambda\)-continuous function if \((X,\tau)\) is a globally disconnected space.
**Proof:**

1. The proof is clear as in a $T_1$ space every $gs\Lambda$-closed set is $\lambda$-closed.

2. Let $F$ be open in $(Z, \xi)$. Then $g^{-1}(F)$ is open in $(Y, \sigma)$ as $g$ is a continuous function and $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is a pre open set in $(X, \tau)$ as $f$ is a pre continuous function. $(gof)^{-1}(F)$ is a closed set in $(X, \tau)$ as $(X, \tau)$ is a submaximal space. Since every closed set is $gs\Lambda$-open, $(gof)^{-1}(F)$ is a $gs\Lambda$-open set in $(X, \tau)$. Thus $gof:(X, \tau) \rightarrow (Z, \xi)$ is contra $gs\Lambda$-continuous.

3. Let $F$ be closed in $(Z, \xi)$. Then $g^{-1}(F)$ is closed in $(Y, \sigma)$ and $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is preopen in $(X, \tau)$ as $f$ is contra precontinuous. $(gof)^{-1}(F)$ is a open set in $(X, \tau)$ as $(X, \tau)$ is a submaximal space. Since every open set is $gs\Lambda$-closed, $(gof)^{-1}(F)$ is a $gs\Lambda$-closed set in $(X, \tau)$. Thus $gof:(X, \tau) \rightarrow (Z, \xi)$ is contra $gs\Lambda$-continuous.

4. Let $F$ be open in $(Z, \xi)$. Then $g^{-1}(F)$ is closed in $(Y, \sigma)$ as $g$ is contra continuous and $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is preopen in $(X, \tau)$ as $f$ is contra precontinuous. $(gof)^{-1}(F)$ is a open set in $(X, \tau)$ as $(X, \tau)$ is a submaximal space. Since every open set is $gs\Lambda$-closed, $(gof)^{-1}(F)$ is $gs\Lambda$-closed in $(X, \tau)$. Thus $gof:(X, \tau) \rightarrow (Z, \xi)$ is contra $gs\Lambda$-continuous.

(5),(6),(7) can be proved in similar lines as in a globally disconnected space every semi open set is open.

**Theorem 5.2.11** Let $f:(X, \tau) \rightarrow (Y, \sigma)$ and $g:(Y, \sigma) \rightarrow (Z, \xi)$ be bijective. Then the following are true:

1. If $gof:(X, \tau) \rightarrow (Z, \xi)$ is contra continuous and $f:(X, \tau) \rightarrow (Y, \sigma)$ is $gs\Lambda$-closed then $g:(Y, \sigma) \rightarrow (Z, \xi)$ is contra $gs\Lambda$-continuous.
2. If $g: (X, \tau) \rightarrow (Z, \xi)$ is contra $\lambda$-continuous and $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\lambda$-closed then $g: (Y, \sigma) \rightarrow (Z, \xi)$ is contra $g s \Lambda$-continuous.

3. If $g: (X, \tau) \rightarrow (Z, \xi)$ is $\lambda$-irresolute and $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\lambda$-closed then $g: (Y, \sigma) \rightarrow (Z, \xi)$ is contra $g s \Lambda$-continuous.

4. If $g: (X, \tau) \rightarrow (Z, \xi)$ is contra continuous function and $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\lambda$-closed then $g: (Y, \sigma) \rightarrow (Z, \xi)$ is contra $g s \Lambda$-continuous.

5. If $g: (X, \tau) \rightarrow (Z, \xi)$ is contra $g s \Lambda$-continuous and $f: (X, \tau) \rightarrow (Y, \sigma)$ is irresolute and $\lambda$-closed then $g: (Y, \sigma) \rightarrow (Z, \xi)$ is contra $g s \Lambda$-continuous.

6. If $g: (X, \tau) \rightarrow (Z, \xi)$ is contra $g s \Lambda$-continuous and $f: (X, \tau) \rightarrow (Y, \sigma)$ is $M. g s \Lambda$-closed then $g: (Y, \sigma) \rightarrow (Z, \xi)$ is contra $g s \Lambda$-continuous.

**Proof:**

1. Let $F$ be a open set in $(Z, \xi)$. Since $g o f$ is a contra continuous function $(g o f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is closed in $(X, \tau)$. Now since $f$ is a $g s \Lambda$-closed map $f(f^{-1}(g^{-1}(F))) = g^{-1}(F)$ is $g s \Lambda$-closed set in $(Y, \sigma)$. Thus $g: (Y, \sigma) \rightarrow (Z, \xi)$ is a contra $g s \Lambda$-continuous function.

2. Let $F$ be a open set in $(Z, \xi)$. Since $g o f$ is a contra $\lambda$-continuous function $(g o f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\lambda$-closed in $(X, \tau)$. Now since $f$ is a $\lambda$-closed map $f(f^{-1}(g^{-1}(F))) = g^{-1}(F)$ is $\lambda$-closed which is also $g s \Lambda$-closed set in $(Y, \sigma)$. Thus $g: (Y, \sigma) \rightarrow (Z, \xi)$ is a contra $g s \Lambda$-continuous function.

3. Let $F$ be a open set in $(Z, \xi)$ which is also $\lambda$-closed in $Z$. Since $g o f$ is a $\lambda$-irresolute function $(g o f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\lambda$-closed in $(X, \tau)$. Now since $f$ is a $\lambda$-closed map $f(f^{-1}(g^{-1}(F))) = g^{-1}(F)$ is $\lambda$-closed which is also $g s \Lambda$-closed set in $(Y, \sigma)$. Thus $g: (Y, \sigma) \rightarrow (Z, \xi)$ is a contra $g s \Lambda$-continuous function.
4. Let $F$ be an open set in $(Z, \xi)$. Since $g$ is a contra continuous function $(gof)^{-1}(F) = f^{-1}(g^{-1}(F))$ is closed in $(X, \tau)$, which is also $\lambda$-closed in $X$. Now since $f$ is a $\lambda$-closed map $ff^{-1}(g^{-1}(F)) = g^{-1}(F)$ is $\lambda$-closed which is also $\text{gs}\Lambda$-closed set in $(Y, \sigma)$. Thus $g: (Y, \sigma) \rightarrow (Z, \xi)$ is a $\text{gs}\Lambda$-continuous function.

5. Let $F$ be an open set in $(Z, \xi)$. Since $g$ is a contra $\text{gs}\Lambda$-continuous function $(gof)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\text{gs}\Lambda$ closed in $(X, \tau)$. Now we have $ff^{-1}(g^{-1}(F)) = g^{-1}(F)$ is $\text{gs}\Lambda$-closed set in $(Y, \sigma)$. Thus $g: (Y, \sigma) \rightarrow (Z, \xi)$ is a contra $\text{gs}\Lambda$-continuous function.

6. Let $F$ be an open set in $(Z, \xi)$. Since $g$ is a $\text{gs}\Lambda$-continuous function $(gof)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\text{gs}\Lambda$-closed in $(X, \tau)$. Now since $f$ is a $\text{M. gs}\Lambda$-closed map $ff^{-1}(g^{-1}(F)) = g^{-1}(F)$ is $\text{gs}\Lambda$-closed. Thus $g: (Y, \sigma) \rightarrow (Z, \xi)$ is a contra $\text{gs}\Lambda$-continuous function.

**Theorem 5.2.12** Let $\{X_i : i \in \Delta\}$ be any family of topological spaces. If $f: X \rightarrow \prod X_i$ is a contra $\text{gs}\Lambda$-continuous function, then $Pr_i of: X \rightarrow \prod X_i$ is a contra $\text{gs}\Lambda$-continuous function for each $i \in \Delta$ where $Pr_i$ is the projection of $\prod X_i$ onto $X_i$.

**Proof:** We shall consider a fixed $i \in \Delta$. Suppose $U_i$ is an arbitrary open set in $X_i$. Since $Pr_i$ is continuous function, $Pr_i^{-1}(U_i)$ is open in $\prod X_i$. Since $f$ is contra $\text{gs}\Lambda$-continuous function, we have $f^{-1}(Pr_i^{-1}U_i) = (Pr_i o f)_i^{-1}$ is $\text{gs}\Lambda$-closed in $X$. Therefore $Pr_i of$ is contra $\text{gs}\Lambda$-continuous function.

**Theorem 5.2.13** Let $\{X_i : i \in \Delta\}$ be any family of topological spaces. If $f: X \rightarrow \prod X_i$ is a contra continuous(continuous) function, then $Pr_i of: X \rightarrow \prod X_i$ is a contra $\text{gs}\Lambda$-continuous function for each $i \in \Delta$ where $Pr_i$ is the projection of $\prod X_i$ onto $X_i$.

**Proof:** We shall consider a fixed $i \in \Delta$. Suppose $U_i$ is an arbitrary open set
in $X_i$. Then $Pr_i^{-1}(U_i)$ is open in $\prod X_i$. Since $f$ is contra continuous (continuous) function, we have $f^{-1}(Pr_i^{-1}(U_i))= (Pr_i of)^{-1}(U_i)$ is closed (open) in $X$, which is $gs\Lambda$-closed in $X$. Therefore $Pr_i of$ is contra $gs\Lambda$-continuous function.

### 5.3 Contra $gs\Lambda$-closed graph

**Definition 5.3.1** If $f:(X,\tau) \longrightarrow (Y,\sigma)$ is any function, then the subset $G(f)= \{(x,f(x)) \mid x \in X\}$ of the product space $(X \times Y, \tau \times \sigma)$ is called the graph of $f$.

**Definition 5.3.2** A function $f:(X,\tau) \longrightarrow (Y,\sigma)$ is said to have a strongly $gs\Lambda$-closed graph if for each $(x,y) \in (X \times Y) \setminus G(f)$, there exist $U \in gs\Lambda O(X,x)$ and $V \in gs\Lambda O(Y,y)$ such that $f(U) \cap gs\Lambda Cl(V) = \emptyset$.

**Definition 5.3.3** A graph $G(f)$ of a function $f:X \longrightarrow Y$ is said to be contra $gs\Lambda$-closed graph if for each $(x,y) \in (X \times Y) \setminus G(f)$, there exist $gs\Lambda$-open set $U$ in $X$ containing $x$ and a closed set $V$ in $Y$ containing $y$ such that $f(U) \cap V = \emptyset$.

**Theorem 5.3.4** If $f:X \longrightarrow Y$ is contra $gs\Lambda$-continuous injective and $Y$ is Urysohn, then $G(f)$ is contra $gs\Lambda$-closed in $X \times Y$.

**Proof:** Let $(x,y) \in (X \times Y) \setminus G(f)$, then $f(x) \neq y$ as $Y$ is Urysohn and there exist open sets $V, W$ such that $f(x) \in V, y \in W$ and $Cl(V) \cap Cl(W) = \emptyset$. Since $f$ is contra $gs\Lambda$-continuous there exist $gs\Lambda$-open set $U$ in $X$, such that $f(U) \subseteq Cl(V)$. Therefore, we obtain $f(U) \cap Cl(W) = \emptyset$. This shows that $G(f)$ is contra $gs\Lambda$-closed in $X \times Y$. 

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**Theorem 5.3.5** Let \( f: X \rightarrow Y \) have a contra gs\(\Lambda\)-graph. If \( f \) is injective, then \( X \) is gs\(\Lambda\)-T\(_1\).

**Proof:** Let \( x \) and \( y \) be any two distinct points in \( X \). Then we have \((x, f(y)) \in (X \times Y) \setminus G(f)\). Then there exist a gs\(\Lambda\)-open set \( U \) in \( X \) containing \( x \) and a closed set \( F \) in \( Y \) containing \( f(y) \) such that \( f(U) \cap F = \emptyset \). Hence \( U \cap f^{-1}(F) = \emptyset \). Therefore, we have \( y \notin U \). This implies that \( X \) is gs\(\Lambda\)-T\(_1\).

**Theorem 5.3.6** Let \( f: X \rightarrow Y \) be a function and \( g: X \rightarrow X \times Y \) the graph function with \( g(x) = (x, f(x)) \) for every \( x \in X \). Then \( f \) is contra gs\(\Lambda\)-continuous if and only if \( g \) is contra gs\(\Lambda\)-continuous function. Assume that \( X \) is a gs\(\Lambda\)-space.

**Proof:** Let us assume that \( f \) is contra gs\(\Lambda\)-continuous function. Let \( x \in X \) and let \( W \) be a closed subset of \( X \times Y \) containing \( g(x) \). Then \( W \cap (\{ x \} \times Y) \) is closed in \( \{ x \} \times Y \) containing \( g(x) \). Also \( \{ x \} \times Y \) is homogeneous to \( Y \). Hence \( \{ y \in Y : (x, y) \in W \} \) is a closed subset of \( Y \). Since \( f \) is contra gs\(\Lambda\)-continuous \( f^{-1}(y) \) where \((x, y) \in W \) is gs\(\Lambda\)-open subset of \( X \). Further we have \( x \in \cup \{ f^{-1}(y) \text{ where } (x, y) \in W \} \subseteq g^{-1}(W) \). Hence \( g^{-1}(W) \) is gs\(\Lambda\)-open. Then \( g \) is contra gs\(\Lambda\)-continuous.

Conversely, let us assume that \( f \) is contra gs\(\Lambda\)-continuous function and let \( F \) be a closed subset of \( Y \). Then \( X \times F \) is a closed subset of \( X \times Y \). Since \( g \) is contra gs\(\Lambda\)-continuous, \( g^{-1}(X \times F) \) is a gs\(\Lambda\)-open subset of \( X \). Also \( g^{-1}(X \times F) = f^{-1}(F) \). Hence \( f \) is contra gs\(\Lambda\)-continuous.