CHAPTER-VI

TO DISCUSS THE EFFECT OF MEMBRANE FLUIDITY UPON THE MECHANICAL PROPERTIES OF BIOLOGICAL CELL SUSPENSIONS
Introduction:

The biological cell membrane is fluid under physiologically active conditions. The deformability of biological cell should be intimately associated with fluidity of cell membrane. Recently, it was experimentally reported that the cell membrane of sea urchin eggs was fluid [1]. We have been studying the viscoelasticity of dispersed system of spherical cell structures [2-9]. In the present work we calculate the rigidity of the same system taking account of interfacial tension at both the interfaces of the shell by an approach similar to the previous one [5]. Using this result, we discuss the viscoelasticity of cell suspensions regarding the cell membrane as fluid substance.

In order to consider the effect of membrane fluidity upon the mechanical property of biological cell suspensions, we have calculated the complex intrinsic viscosity $[n^*] = [n'] - i [n'']$ of spherical cell structures with material incompressibility in suspension as the function of the dimensionless frequency $\chi = \frac{\omega \eta' a}{\gamma'}$ together with the parameters of $hm = \frac{\eta m}{\eta'}$, $g = \frac{\gamma}{\gamma'}$, $\sigma = \frac{d}{a}$ where $a$ is the radius of the cell, $d$ is the width of the membrane, $\eta, \eta m$ and $\eta'$ are viscosities of the medium of the membrane and
of the internal region of the cell, γ and γ' the surface tensions at the outer and inner side of the membrane respectively and ω the angular frequency. The result is simply represented by two dispersions as follows: \[
\frac{[\eta^*]}{[\eta]} = \frac{A_1 + B_1}{1 + i\omega \tau_1} + \frac{B_2}{1 + i\omega \tau_2}.
\]
Here \(i\) is imaginary unit, \(A_1 = \frac{2(1-h)}{(2+3h)+o(\delta)}\), \(B_1 = \frac{3h}{(5+5h)+o(\delta)}\), \(B_2 = \frac{h(19+16h)}{(2+3h)+o(\delta)}\), \(\tau_1 = \left\{\left(\frac{5}{24}\right)(1+h)\left(1 + \frac{1}{g}\right)\delta^{-2} + o(\delta^{-1})\right\}^{\frac{\eta'}{\gamma'}}\), \(\tau_2 = \left\{\frac{(2+3h)(19+16h)}{(40(1+h)(1+g))}\right\}^{\frac{\eta'}{\gamma'}} + o(\delta')\frac{\eta'}{\gamma'}\) and \([\eta] = \left\{\frac{96hmg+32g(5+5h-12hm)}{2(96hmg+32g(5+2h-12hm))}\right\}^{\frac{\eta'}{\gamma'}}\).

**Theory:**

A spherical shell structure is chosen as a model of biological cell. We consider a spherical free volume of radius b around the spherical cell structure and homogeneous continuum with the rigidity \(\mu^*\) in the outermost region which is equivalent to the whole dispersed system. As shown in fig. 1, suppose a dispersed system of spherical shell structures consisting of an inside region \((r < a')\) with the rigidity \(\mu'\), a shell region \((a' < r < a)\) with the rigidity \(\mu m'\), a medium \((a < r < b)\) with the rigidity \(\mu\) and a continuum \((b < r)\) equivalent to the microscopic dispersed system with the rigidity \(\mu^*\) to be calculated, in which \(r\)
is the radial distance from the origin at the centre of the cell structure, a' and a are the inner and outer radii of the cell and b is determined from the volume concentration $c = \left( \frac{a}{b} \right)^3$ of shell structure s. The interfacial tensions at the interfaces $r = a$ and $r = a'$ are $\gamma$ and $\gamma'$ respectively. We denote the inside region by $S'$, the shell region by $s_m'$, the medium by s and the continuum by $S^*$. The continuum $s^*$ is regarded as the total composite system of $S'$, $s_m$, $s$ & $S^*$.

![Figure 6.1: Model of dispersed system of spherical shell structure](image)

In the medium S, the following equations are derived from the equation of motion, Hook’s is law and strain displacement relation neglecting the inertial term in the limit of incompressibility –
\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u_r}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta \left( \frac{\partial u_r}{\partial \theta} - 2u_\theta \right) \right] - \frac{2u_r}{r^2} = \frac{1}{\mu} \frac{\partial P}{\partial r} \quad (6.1)
\]

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial u_\theta}{\partial \theta} \right) + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2 \sin^2 \theta} = \frac{1}{\mu} \frac{\partial p}{\partial \theta} \quad (6.2)
\]

Where \( u_r \) and \( u_\theta \) are the components of the displacement, \( P \) is the pressure. In the limit of incompressibility the dilation is equal to zero. This gives –

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u_r}{\partial r} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( u_\theta \sin \theta \right) = 0 \quad (6.3)
\]

An external shear stress in cartesian co-ordinates –

\[
P = \begin{bmatrix}
2T & 0 & 0 \\
0 & -T & 0 \\
0 & 0 & -T
\end{bmatrix} \quad (6.4)
\]

is applied to the system. The homogeneous and the composite microscopic systems are now considered to be subjected to a state of stress (4).

For the homogenous system the component of stress in spherical polar co-ordinates are -

\[
\sigma_{rr} = T(3 \cos^2 \theta - 1), \sigma_{\theta\theta} = T(2 - 3 \cos^2 \theta),
\]

\[
\sigma_{\phi\phi} = -T, \quad \sigma_{\theta\phi} = \sigma_{\phi\theta} = 0,
\]
\[ \sigma_{\theta r} = -3T \cos \theta \sin \theta \] \hspace{1cm} (6.5)

On \( r = R \), for sufficiently large \( R \). The components of displacement defined by –

\[ u_r = \frac{1}{2\mu^*} T_r (3 \cos^2 \theta - 1), \quad u_\theta = -\frac{3}{2\mu^*} T_r \sin \theta \cos \theta \] \hspace{1cm} (6.6)

Correspond to the stress (5). For the composite system for example, in \( S \), the components of displacement and stress are given in the limit of incompressibility by –

\[ u_r = \left( \frac{1}{7} A r^3 + \frac{1}{2} B r^{-2} + 2C - 3D r^{-4} \right) \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \] \hspace{1cm} (6.7)

\[ u_\theta = -\left( \frac{5}{14} A r^3 + 3C + 3D r^{-4} \right) \sin \theta \cos \theta \] \hspace{1cm} (6.8)

\[ \sigma_{rr} = \mu \left( -\frac{1}{7} A r^2 - 3B r^{-3} + 4C + 24D r^{-5} \right) \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \] \hspace{1cm} (6.9)

\[ \sigma_{r\theta} = -\mu \left( \frac{8}{7} A r^2 + \frac{3}{2} B r^{-3} + 6C - 24D r^{-5} \right) \sin \theta \cos \theta \] \hspace{1cm} (6.10)

Exactly similar expressions for the displacements and stresses, in which the symbols \( A, B, C, D \) and \( \mu \) are distinguished by primes, subscripts \( m \) and asterisks are valid in \( S', S_m \) and \( S^* \).

The displacement at the origin is finite only when –

\( B' = 0, \quad D' = 0 \) \hspace{1cm} (6.11)
The components of stress and displacement at \( r = R \) can be made to agree with eqns. (5) and (6) with proportional error of order of the \( \frac{D^*}{C^* R^5} \) which can be made as small as we please by taking sufficiently large \( R \) provided that -

\[
A^* = 0, \quad B^* = 0, \quad C^* = \frac{T}{2\mu^*} \quad (6.12)
\]

We can write the continuity conditions of \( u_r, u_\theta, \sigma_{rr} \) and \( \sigma_{r\theta} \) at \( r = a' \) as

\[
\frac{1}{7} a'^2 A_m + \frac{1}{2} a'^{-3} B_m + 2C_m - 3a'^{-5} D_m = \frac{1}{7} a'^2 A' + 2C' \quad (6.13)
\]

\[
\frac{5}{14} a'^2 A_m + 3 C_m + 3a'^{-5} D_m = \frac{5}{14} a'^2 A' + 3C' \quad (6.14)
\]

\[
\mu_m \left( \frac{1}{7} a'^2 A_m - 3a'^{-3} B_m + 4C_m + 24a'^{-5} D_m \right) = \frac{1}{7} \left( 4a'\gamma' - a'^2 \mu' \right) A' + 4 \left( 2a'\gamma'^{-1} + \mu' \right) C' \quad (6.15)
\]

\[
\mu_m \left( \frac{8}{7} a'^2 A_m + \frac{3}{2} a'^{-3} B_m + 6C_m - 24a'^{-5} D_m \right) = \mu' \left( \frac{8}{7} a'^2 A' + 6C' \right) \quad (6.16)
\]

At \( r = a \) as
\[
\frac{1}{7}a^2 A + \frac{1}{2}a^{-3}B + 2C - 3a^{-5}D \\
= \frac{1}{7}a^2 A_m + \frac{1}{2}a^{-3}B_m + 2C_m - 3a^{-5}D_m \quad (6.17)
\]

\[
\frac{5}{14}a^2 A + 3C + 3a^{-5}D = \frac{5}{14}a^2 A_m + 3C_m + 3a^{-5}D_m \quad (6.18)
\]

\[
\mu \left( -\frac{1}{7}a^2 A - 3a^{-3}B + 4C + 24a^{-5}D \right) \\
= \frac{1}{7}a^2 (4a^{-1}\gamma - \mu_m)A_m + a^{-3}(2a^{-1}\gamma - 3\mu_m)B_m \\
+ 4(2a^{-1}\gamma + \mu_m)C_m \\
- 12a^{-5}(a^{-1}\gamma - 2\mu_m)D_m \quad (6.19)
\]

\[
\mu \left( \frac{8}{7}a^2 A + \frac{3}{2}a^{-3}B + 6C - 24a^{-5}D \right) \\
= \mu_m \left( \frac{8}{7}a A_m + \frac{3}{2}a^{-3}B_m + 6C_m - 24a^{-5}D_m \right) \quad (6.20)
\]

at \( r = b \) as

\[
\frac{1}{7}b^2 A + \frac{1}{2}b^{-3}B + 2C - 3b^{-5}D = 2C^* - 3b^{-5}D^* \quad (21)
\]

\[
\frac{5}{14}b^2 A + 3C + 3b^{-5}D = 3C^* + 3b^{-5}D^* \quad (22)
\]

\[
\mu \left( -\frac{1}{7}b^2 A - 3b^{-3}B + 4C + 24b^{-5}D \right) \\
= \mu^*(4C^* + 24b^{-5}D^*) \quad (23)
\]
\[
\mu \left( \frac{8}{7} b^2 A + \frac{3}{2} b^{-3} B + 6C - 24b^{-5} D \right) \\
= \mu^* (6C^* - 24b^{-5} D^*) \quad (24)
\]

where \( A', C', A_m, B_m, C_m, A, B, C, D, C^* \) and \( D^* \) are functions of the time only.

**Result:**

An expression for \( \mu^* \) can be obtained by using the fact that \( C \) is small and solving eqns. (13) to (24). Rejecting an alternative solution in which \( \frac{\mu^*}{\mu} \) does not tend to one as \( C \) tends to 0, we find that

\[
\frac{\mu^*}{\mu} = \left\{ \frac{\Gamma - \left( \frac{3}{2} \right) C \Gamma'}{\Gamma + C \Gamma'} \right\} 
\]

(6.25)

\[
\Gamma = \Gamma_0 + \Gamma_3 A^3 + \Gamma_5 A^5 + \Gamma_7 A^7 + \Gamma_{10} A^{10} \quad (6.26)
\]

\[
\Gamma' = \Gamma'_0 + \Gamma'_3 A^3 + \Gamma'_5 A^5 + \Gamma'_7 A^7 + \Gamma'_{10} A^{10} \quad (6.27)
\]

\[
\Gamma_0 = -\left\{ 40(h_m + 1) + A(3h_m + 2)(16h_m + 19)x \right\} \\
\times \left\{ 20(h_m + h)g \\
+ \left( h_m + \frac{3}{2} h \right)(19h_m + 16h)x \right\} 
\]

(6.28)
\[ \Gamma_3 = 25 \{-4(2h_m + 5) + A(h_m - 1)(16h_m + 19)x\} \]
\[ \times \{-2(2h_m + 5h)g \]
\[ + 3(h_m - h)(3h_m + 4h)x\} \]  
(6.29)

\[ \Gamma_5 = -84 \{-20 + A(h_m - 1)(16h_m + 19)x\} \]
\[ \times \{-5hg + 2(h_m - h)(2h_m + 3h)x\} \]  
(6.30)

\[ \Gamma_7 = 50 \{4(2h_m - 5) + A(h_m - 1)(18h_m + 19)x\} \]
\[ \times \{(2h_m - 5h)g \]
\[ + 2(h_m - h)(2h_m + 3h)x\} \]  
(6.31)

\[ \Gamma_{10} = 8 \{20 - 19A(h_m - 1)x(h_m - 1)\} \]
\[ \times \{-5g + 6(h_m - h)x(h_m - h) \} \]  
(6.32)

\[ \Gamma'_0 = \{40(h_m + 1) + A(3h_m + 2)(16h_m + 19)x\} \]
\[ \times \{4(5h_m + 2h)g \]
\[ + (h_m - h)(19h_m + 16h)x\} \]  
(6.33)

\[ \Gamma'_3 = -25 \{-4h_m + A(h_m - 1)(16h_m + 19)x\} \]
\[ \times \{-4(h_m + h)g \]
\[ + (4h_m^2 - 10h_mh + 8h^2)x\} \]  
(6.34)

\[ T'_5 = 168 \{-20 + A(h_m - 1)(16h_m + 19)x\} \]
\[ \times \{-hg + 2(h_m - h)^2x\} \]  
(6.35)

\[ \Gamma'_7 = -100 \{4(2h_m - 5) + A(h_m - 1)(18h_m + 19)x\} \]
\[ \times \{g + 2(h_m - h)x\} (h_m - h) \]  
(6.36)
\[ \Gamma'_{10} = -8\{20 - 19A(h_m - 1) \times\}(h_m - 1) \]
\[ \times \{-5(h_m - 2h)g \]
\[ + 2(h_m - h)(3h_m + 2h)x\} \]  
(6.37)

Where \( h = \frac{\mu}{\mu'} \), \( h_m = \frac{\mu_m}{\mu'} \), \( g = \frac{\gamma}{\gamma'} \), \( A = \frac{a'}{a} \), \( x = \frac{a_m'}{\gamma'} \)  
(6.38)

**Discussion**:

We compare the above result with that for spheres in the special case i.e. a dilute emulsion of liquid droplets, where

\[ \mu_m = \mu', \text{and} \quad a = a' \]

\[ \frac{\mu*}{\mu} = \frac{40(\mu + \mu')(\gamma + \gamma') + a(3\mu + 2\mu')(16\mu + 19\mu') + 3c(4(2\mu + 5\mu')(\gamma + \gamma') - a(\mu - \mu')(16\mu + 19\mu'))}{40(\mu + \mu')(\gamma + \gamma') + a(3\mu + 2\mu')(16\mu + 19\mu') - 2c(4(2\mu + 5\mu')(\gamma + \gamma') - a(\mu - \mu')(16\mu + 19\mu'))} \]  
(6.39)

The equation (6.39) completely agrees with the result already obtained by Oldroyed. [10]

The present result should be used as a model a suspension of biological cell with fluid membrane. We exert an external stress of eqn. [6.4] varying sinusoidally with time that is proportional to \( \exp(i \omega t) \), where \( i \) is the imaginary unit \( \sqrt{-1} \), \( \omega \) the angular frequency and \( t \) the time. We can calculate the complex viscosity of this system using our result by putting.

\[ \mu^* = i\omega \eta^*, \mu' = i\omega \eta', \mu_m = i\omega \eta_m, \mu = i\omega \eta \]  
(6.40)
Where \( \eta^*, \eta', \eta_m \) are the viscosities of the suspension, the protoplasm, the membrane and the medium respectively. The complex intrinsic viscosity as defined by –

\[
[\eta^*] = \lim_{c \to 0} \frac{\eta^* - \eta}{\eta c}
\]  \hspace{1cm} (6.41)

Instead of \( [\eta^*] \) itself, we define the dimensionless function –

\[
\frac{[\eta^*]}{[\eta]} = -\frac{\Gamma'}{\Gamma}
\]  \hspace{1cm} (6.42)

Where \( \Gamma \) and \( \Gamma' \) are given by eqns. (6.26) and (6.27) with replacing \( \mu \) and \( \mu' \) by \( i\omega \eta \) and \( i\omega \eta' \) and \( [\eta] \) is the steady intrinsic viscosity which is equal to the value of \( [\eta^*] \) for \( \omega \to 0 \).

The result of the numerical calculation of eqn. (6.41) are shown in fig. (2) to fig. (5) for various values of parameters \( \delta, h, h_m \) and \( g \), where \( \delta = \frac{(a-a')}{a}, \ h = \frac{\eta}{\eta'}, \ h_m = \frac{\eta_m}{\eta'} \) and \( g = \frac{\gamma'}{\gamma} \).

The real part \( \frac{[\eta']}{\eta} \) & the imaginary parts \( \frac{[\eta'']}{\eta} \) of the real parts \( \frac{[\eta'']}{[\eta]} \) of eqn. (6.4) are plotted against \( \log \frac{\omega \eta' a}{\gamma'} \). For \( \delta \ll 1 \), \( \frac{[\eta^*]}{[\eta]} \) is written as–

\[
\frac{[\eta^*]}{[\eta]} = \frac{\Gamma A \Gamma'}{\Gamma A' \Gamma} = A_1 + \frac{B_1}{1 + i\omega \tau_1} + \frac{B_2}{1 + i\omega \tau_2}
\]  \hspace{1cm} (6.43)
\[ [\eta^*] = -\frac{5}{2} \Gamma', [\eta] = -\frac{5}{2} \Gamma_A' \]

(6.44)

Fig. 6.2: Dependence of the real and the imaginary part of the intrinsic viscosity on frequency for \( h = h_m = g = 1 \).

Fig. 6.3: Dependence of the real and the imaginary part of the intrinsic viscosity on frequency for \( h_m = g = 1 \) and \( \delta = 0.01 \).

Fig. 6.4: Dependence of the real and the imaginary part of the intrinsic viscosity on frequency for \( h = g = 1 \) and \( \delta = 0.01 \).
\[ \Gamma = \Gamma_A + \Gamma_B x + \Gamma_c x^2, \Gamma' = \Gamma_A' + \Gamma_B' x + \Gamma_c' x^2 \]  \quad (6.45)

\[ \Gamma_A = \{-96h_m g - 32g(5 + 5h - 12h_m)\delta + 0(\delta^2)\}\delta^2 \]  \quad (6.46)

Fig. 6.5 : Dependence of the real and the imaginary part of the intrinsic viscosity on frequency for \( h = h_m = g = 1 \) and \( \delta = 0.01 \).

Fig. 6.6 : Dependence of \([\eta]\) on \( h \) for \( \delta = 0.01 \) and \( g = 1 \).
\( \Gamma_{A'} = \{96h_m g + 32g(5 + 2h - 12h_m) \delta + 0(\delta^2)\} \delta^2 \quad (6.47) \)

\( \Gamma_B = -20(1 + h)h_m(1 + g) + 4 \{h_m(25 + 27g) - (25h + 22h_m^2)(1 + g) + hh_m(22 + 30g)\} \delta + 0(\delta^2) \quad (6.48) \)

\( \Gamma_{B'} = 4(5 + 2h)h_m(1 + g) - 4 \{h_m(25 + 27g) - (10h + 22h_m^2)(1 + g) + hh_m(28 + 12g)\} \delta + 0(\delta^2) \quad (6.49) \)

\( \Gamma_c = -h_m \left(19 + \frac{89}{2} h + 24h^2\right) + \left\{(114 + \frac{519}{2} h + 144h^2)h_m - (95 + 120h)(h + h_m^2)\right\} \delta + 0(\delta^2) \quad (6.50) \)

\( \Gamma_c' = h_m(19 - 3h - 16h^2) - \{3(38 + h - 32h^2)h_m - (38 - 80h)h - (95 - 32h)h_m^2\} \delta + 0(\delta^2) \quad (6.51) \)

\[ x = \frac{i\omega \eta'a}{\gamma'} \quad (6.52) \]

and \( A_1 = \frac{2(1-h)}{2+3h} + 0(\delta) \)

\( B_1 = \frac{3h}{5(1+h)} + 0(\delta), \quad B_2 = \frac{h(19 + 16h)}{5(1+h)(2 + 3h)} + 0(\delta) \quad (6.53) \)
\[ \tau_1 = \left\{ \frac{5}{24} (1+h) \left( 1 + \frac{1}{g} \right) \delta^{-2} + 0(\delta - 1) \right\} \frac{\eta'}{\gamma'} \]

\[ \tau_2 = \left\{ \frac{(2+3h)(19+16h)}{40(1+h)(1+g)} + 0(\delta) \right\} \frac{\eta'}{\gamma'} \] (6.54)

The equations (6.40 to (6.51) are different from the results of Oosterbroek et al. [11]. For \( \delta = 0 \), i.e., for the liquid sphere

\[ [\eta^*] = \frac{5}{2} \frac{4(5+2h)(1+g) + (1-h)(19+16h)}{20(1+h)(1+g) + (2+3h)(19+16h)/z_h} \] (6.55)

\[ [\eta] = \frac{5}{2} \frac{(5+2h)}{5(1+h)} \] (6.56)

\[ \frac{[\eta^*]}{[\eta]} = A_0 + \frac{B_0}{1 + i\omega \tau} \] (6.57)

\[ A_0 = \frac{5(1+h)(1-h)}{(5+2h)(1+3/2h)} \]

\[ B_0 = \frac{h(19+16h)}{(5+2h)(2+3h)} \]

\[ \tau = \frac{(19+16h)(2+3h)}{40(1+h)(1+g)} \frac{\eta'}{\gamma'} \] (6.58)

The eqns. (53), (54) and (58) show that in the limit of \( \delta = 0 \), the first relaxation moves to the infinitely low frequency region as seem in fig. (2), \( \tau \) in eqn. (58) agrees with \( \tau_2 \) is eqn. (54) and

\[ A_0 = \frac{A_1}{(A_1+B_2)}, B_0 = \frac{B_2}{(A_1+B_2)} \] in which \( A_1 + B_2 = \frac{(5+2h)}{(5+5h)} \)
We compare the present result with that for suspensions of spherical cells with elastic membrane. For the case of elastic membrane there exist four relaxations with increasing h, a pair of relaxation amplitudes increase on the low frequency side; and the other pair of amplitudes decrease on the high frequency side. For the case of viscous membrane the relaxation amplitudes increase with increasing h. For both the cases all the relaxations shift to the lower frequency as h increases. Moreover, if we could change the radius of cells all the relaxation times would be much influenced by the radius in the viscous membrane on the other hand the relaxation times would not explicitly depend on it in elastic membrane.

Complex viscosity of bovine red blood cells in suspensions is measured in [12]. In the last chapter the results are compared with the theory for elastic shells [5], where the membrane of blood cells is regarded as an elastic substance.

References:


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